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A Study on Motion of a Free Falling Body in Kinematic Equation

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Abstract: The motion of a free falling object in kinematic equation is studied using its characteristics and strategy. The paper is based on the definition of Newton's law of motion. By appropriate examples, it is shown how to solve a free falling object with the given parameters in kinematic equation. Some sufficient conditions to determine the motion of a free falling object, based on the four kinematic equations are established. Some examples are illustrated to solve the problems.
Keywords- Displacement, Velocity, Acceleration, Free Falling Body, Kinematic Equation.

I. INTRODUCTION

Kinematic equations are among the linchpins of modern mathematics, which along with physics are essential for analyzing and solving complex problems in engineering and natural sciences. The ultimate purpose of using kinematic equation is to represent the motion of objects. These types of equations are called as Kinematic equations. There are many quantities such as displacement (distance), velocity(speed), acceleration and time that are associated with the motion of objects. These quantities provide enough information about the motion of an object. Only a few parameters regarding the motion of an object are known, while the rest of the parameters remain unknown. These unknown parameters can be found using the kinematic equations. A kinematic equation includes four variables. If any of the three values are known, the fourth value can be calculated. Thus a kinematic equation provides a useful means of calculating the motion of an object, if any of the variables is unknown. The basic concepts of gravity and free fall are described in [1] by Kavanagh C & Sneider C, The Italien Astronomer Giovanni Battista Riccioli [2] was probably the first who measured with precision the falling speed of objects differing in mass, size and material. the paper [3] published by Champagne A.B, Klopfer L.E & Andersson J.H shows that heavier objects fall faster than lighter objects. The fact that the speed of an object descending along an incline increases with its mass is explained in [4] by Halloun I.A & Hestenes D and in [5] by Karpp E.R & Anderson N.H. the relation between mass & speed of a falling object has been investigated in [6], [7] by Sequeira M & Leite L and Shanon B. The various methods used for construction of analytical solutions- the traditional approach is explained in [8], [9] by Benacka J, Vial A. In this paper, the motion of a free falling object is studied. We define the characteristics and problem solving strategy of free fall objects. Some examples illustrating the concepts are also given

II. MATHEMATICAL ANALYSIS

An object is said to be at free fall if it is falling under the influence of gravity. Such objects are said to be free falling objects.=These objects will have a downward acceleration of -9.8 m/s^2. These are four kinematic equations that describe the motion of an object.

$$d = v_i t + ½\, at^2$$

$$v_f{}^2 = v_i{}^2 + 2ad$$

$$v_f = v_i + at$$

$$d = \frac{v_{i+v_f}}{2} * t$$

Where

d = displacement of the object

t = time taken

a = acceleration

v_i = initial velocity

v_f = final velocity.

A. Derivation Of The Kinematic Equation- Constant Acceleration

Here we consider the case of motion that exhibits constant acceleration. The acceleration does not change for constant acceleration and it is the same as average acceleration. Here we consider x as the position of an object and Δx as displacement of the object, v as the velocity and Δv as the change in velocity (final velocity v_f -initial velocity v_i) and acceleration as a.By the definition of acceleration, we've

$$\text{Acceleration} = \frac{change\ in\ velocity}{change\ in\ time} = \frac{\Delta v}{\Delta t}$$

$$a = \frac{v_{f-}v_i}{t_{f-}t_i}$$

$$a\Delta t = v_{f-}v_i$$

$$\boldsymbol{v_f = v_i + a\Delta t} \quad \ldots\ldots\ldots\ldots..(1)$$

By the definition of average velocity that studies the initial and final position of an object(displacement).

$$v_{avg} = \frac{\Delta x}{\Delta t} \quad \ldots\ldots\ldots\ldots.(2)$$

The average velocity can also be found by adding the initial and final velocities.

$$v_{avg} = \frac{v_{i+}v_f}{2} \quad \ldots\ldots\ldots\ldots.(3)$$

By (2) and (3),

$$\frac{\Delta x}{\Delta t} = \frac{v_{i+}v_f}{2}$$

Hence the displacement is

$$\boldsymbol{\Delta x = ½ (v_i + v_f)\, \Delta t} \quad \ldots\ldots..(4)$$

Here we describe the motion of an object by combining equations (1) and (4)

$$\Delta x = ½ (v_i + (v_i + a\Delta t))\, \Delta t$$

$$= ½ (2\, v_i + a\Delta t)\, \Delta t$$

$$\boldsymbol{\Delta x = v_i \Delta t + ½\, a\, (\Delta t)^2} \quad \ldots\ldots.(5)$$

On solving (1) for Δt, we get

$$v_f = v_i + a\Delta t$$

$$\Delta t = \frac{v_{f-}v_i}{a}$$

On substituting Δt in (1), we obtain

$$\Delta x = ½ (v_i + v_f)\, \Delta t$$

$$\Delta x = ½ (v_i + v_f) \left(\frac{v_{f-}v_i}{a}\right)$$

$$2a\Delta x = vf^2 - vi^2$$

$$\boldsymbol{v_f^2 = v_i^2 + 2a\Delta x} \quad \ldots\ldots\ldots..(6)$$

Equations (1), (4), (5) and (6) are known as the kinematic equations with constant acceleration.

B. Problem Solving Strategy

The strategy for solving a problem involves the following steps

1) To form a diagram of the situation given in the problem.
2) Listing the known variables.
3) Listing the unknown variables.
4) To identify the correct equation using the known and unknown variables.
5) By substituting the known variables in the equation and to find the unknown variable.

C. Characteristics Of Free Fall Motion

There are some characteristics that define the motion of a free fall body. These characteristics are explained as follows

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1) A free fall body has an acceleration of -9.8 m/s^2(Negative sign describes downward acceleration). However, the acceleration of a free falling body in kinematic equation is -9.8 m/s^2.
2) The initial velocity is 0 m/s when an object is dropped from a particular height.
3) An object will slowly rise upwards if an object is projected upward in vertical direction. The velocity is 0 m/s when it reaches the peak of its trajectory.

III. EXAMPLE

A. A terrorist drops a bomb from the top of a mountain located 90 metres above the ground. The bomb explodes when it touches the ground. Determine the time taken by the bomb to explode when it touches the ground. Also draw the path of the bomb at different heights.

1) *Solution:* The first step of the problem is forming a diagram of the situation. In the second step, we identify the known variables. The known variables from the statement is displacement. The displacement of the bomb is -90 metres (The negative sign is used because the displacement is downwards). The initial velocity is 0 m/s since the bomb is dropped from a particular height and not thrown. The acceleration of the bomb is -9.8 m/s^2 since the value of acceleration for any kinematic equation is -9.8 m/s^2.
2) *Diagram*

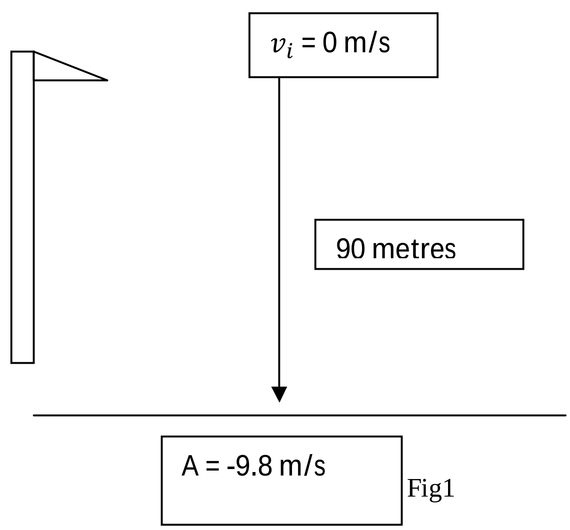

Fig1

v_i = 0 m/s
d = -90 m
a = -9.8 m/s^2

To find: t = ?

The next step is to find an apt equation to find the unknown variable(t). We know the value of the variables d, v_i, a and the unknown variable t. So let us choose an equation involving these variables.

$$d = v_i t + ½ at^2$$

The next step is to substitute the known variables and to find the value of the unknown variable.

-90 m = (0 m/s) t + ½ (-9.8 m/s^2) (t^2)

t = 4.29 secs

Therefore, the time taken by the bomb to explode on hitting the ground is 4.29 seconds.

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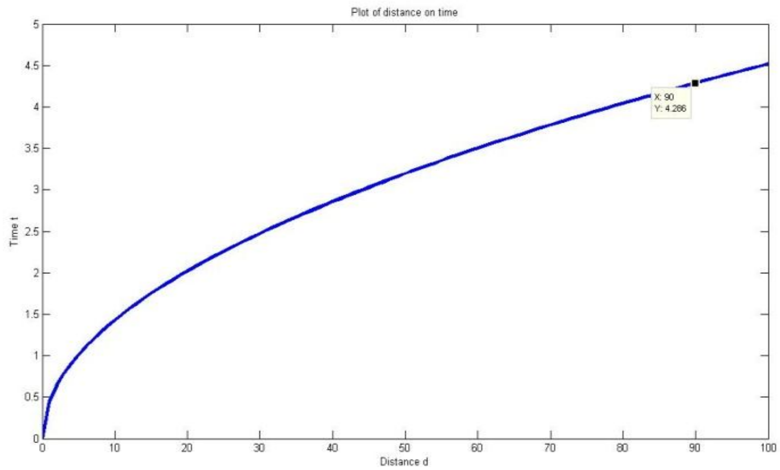

Fig.2The path for different height is given in Fig.2.

*B.*A basketball player throws the ball vertically upward with an initial velocity of 32.6 m/s. Find the maximum height to which the ball is thrown.

1) *Solution:* The first step of the problem is forming a diagram of the situation. In the second step, we identify the known variables. The known variables from the problem are initial velocity. The initial velocity is 32.6 m/s(The positive sign is used since the initial velocity of the basketball is an upward velocity). The final velocity must be 0 since it attains the peak of the trajectory. The acceleration of the basketball is -9.8 m/s^2 since the value of acceleration for any kinematic equation is -9.8 m/s^2. The next step is to identify the unknown variable.

Given: v_i = 32.6 m/s

v_f = 0 m/s

a = -9.8 m/s

2) *Diagram*

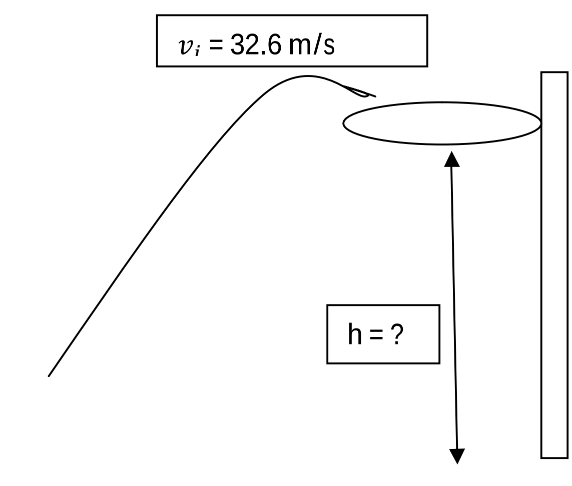

Fig.3

he next step is to find an apt equation to find the unknown variable displacement(The maximum height to which the ball is thrown). We know the values of u, a, v. So let us choose an equation involving these variables.

$$v^2 = u^2 + 2as$$

$$(0 \text{ m/s})^2 = (32.6 \text{ m/s})^2 + 2(-9.8 \text{ m/s})\ d$$

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d = 1062.76 ^{2} ^{2} / 19.6 m/ ^{2}
d = 54.22 m

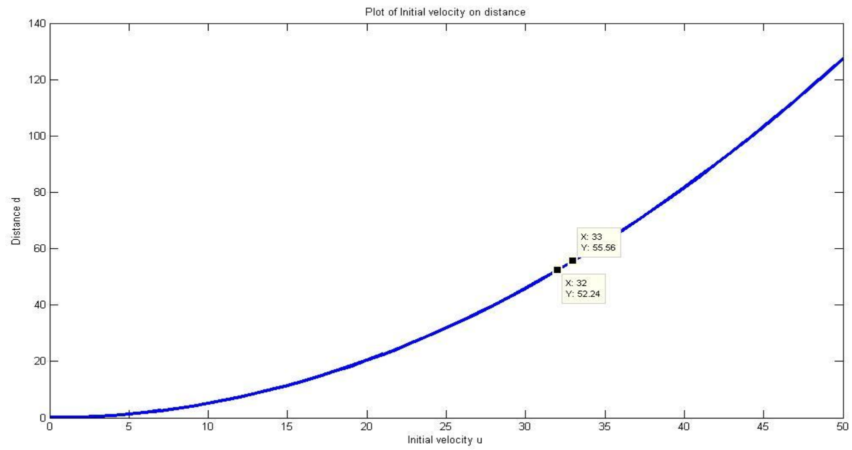

Fig.4

The maximum height to which the ball is thrown is 54.22 metres. Fig.4 shows the graph between initial velocity and maximum height. A rock is dropped off a cliff that is 110 m tall. What is the time duration taken by the rock to hit the ground? What is the final velocity of the rock?

3) Solution: The first step of the problem is to draw a diagram of the situation. In the second step, we identify the known variables. The known variables from the problem is displacement. The displacement is -110 m(The negative sign is used since the rock is falling downward). The initial velocity must be 0 since it is not moving initially and it is dropped from a person's hand. The acceleration of the rock is -9.8 m/s^2 since the value of acceleration for any kinematic equation is -9.8 m/s^2. The next step is to identify the unknown variable. It may be observed that initial velocity is increased, maximum height is decreased.

Diagram:

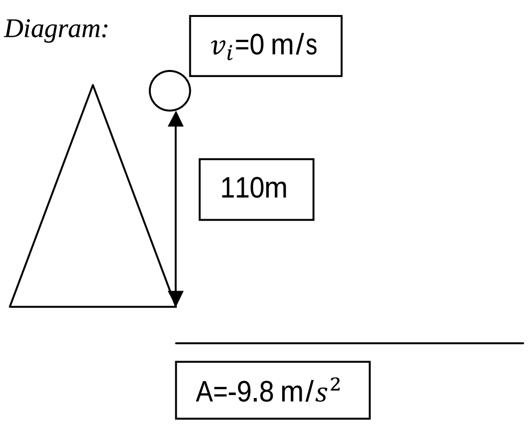

Fig.5

Given

d=110m

v_i=0 m/s

a=-9.8 m/s^2

To find:

a) t= ?

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b) v_f= ? The next step is to find an apt equation for (a) to find the unknown variable time(The time taken by the rock to hit the ground). We know the values of d, v_i, a and the unknown variable t. So let us choose an equation involving these variables.

$$d = v_i t + ½ at^2$$

The next step is to substitute the known variables and to find the value of the unknown variable.

$$-110 \text{ m} = (0 \text{ m/s})\ t + ½\ (-9.8 \text{ m}/s^2)\ (t^2)$$
$$-110 \text{ m} = 0 – (4.9 \text{ m}/s^2)\ (t^2)$$
$$-110 \text{ m} / - 4.9 \text{ m}/s^2 = t^2$$
$$22.4489\ s^2 = t^2$$
$$t = 4.74 \text{ secs}$$

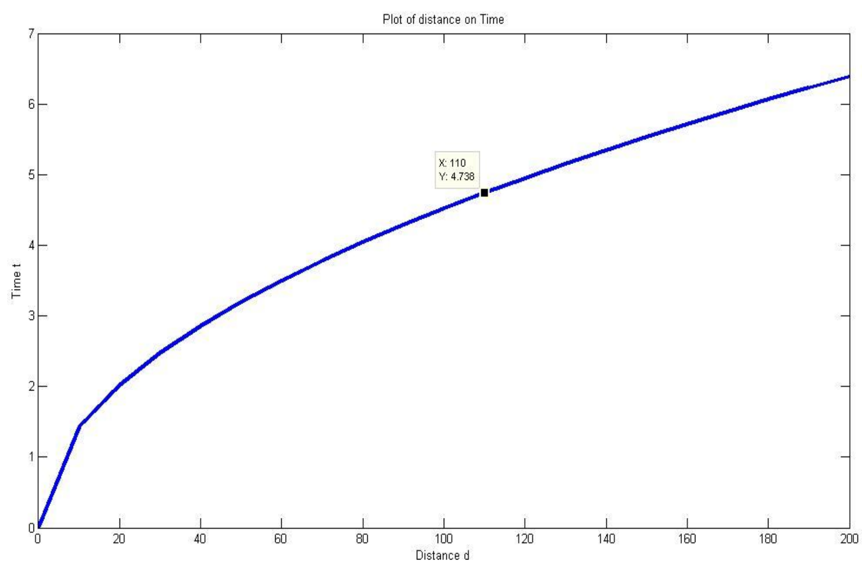

Fig.6

The next step is to find an apt equation for (b) to find the unknown variable final velocity (The velocity of the rock when it hits the ground). We know the values of t, v_i, a and the unknown variable v_f. So let us choose an equation involving these variables.

$$v_f = v_i + at$$
$$v_f = 0\text{m/s} + (-9.8 \text{ m}/s^2).4.74 \text{ sec}$$
$$v_f = -46.452 \text{ m/s}$$

The time taken by the rock to hit the ground is 4.74 seconds and the final velocity of the rock is -46.452 m/s.

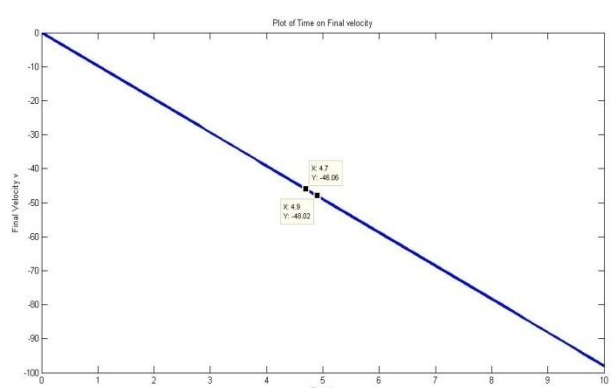

Fig.7

IV. CONCLUSION

The kinematic equations are used to study the motion of a free falling object. A free falling object will fall under the influence of gravity. There are two main characteristics of free falling objects (i.e.,) they do not encounter air resistance and all object accelerate at a rate of 9.8 m/s^2. Several conditions are discussed to study the motion of a free falling object and some examples are given to illustrate the theory. The actual solutions of the problems are compared with the diagrammatic pictures got by MATLAB.

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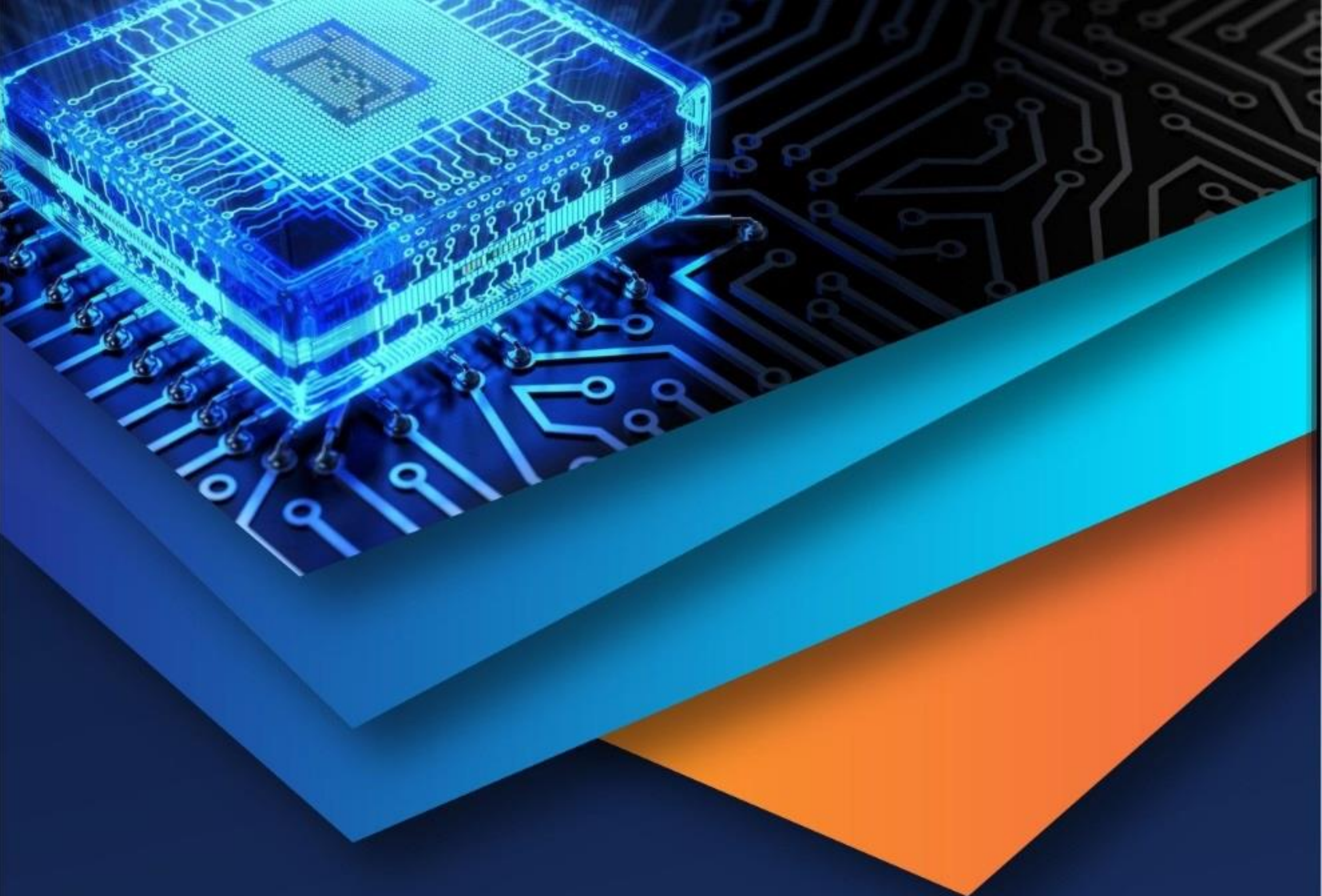

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