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# MHD Boundary Layer Flow over a Nonlinear Permeable Stretching Sheet in a Nanofluid with Convective Boundary Condition

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**Abstract:** *The present work analyses, the magneto hydrodynamic boundary layer flow, heat transfer over a stretching sheet in a nanofluid with convective boundary condition. Similarity transformation is used to convert the governing BVP in the form of partial differential equations. The nonlinear problem is solved using Runge-kutta shooting method. The effects of various embedded parameters on fluid velocity, temperature and particle concentration profiles have been shown graphically, and the results are compared with already published work.*

**Keywords:** *Nanofluids; Boundary layer; runge-Kutta Shooting*

**Method:** *Convective boundary condition; Similarity transformation*

## I. INTRODUCTION

Modern nanotechnology provides new opportunities to process and produce materials with average crystallite sizes below 50 nm. Nano fluids can be considered to be the next generation heat transfer fluids because they offer exciting new possibilities to enhance heat transfer performance compared to pure liquids. They are expected to have superior properties compared to conventional heat transfer fluids, as well as fluids containing micro-sized metallic particles. Also, nanofluids can improve abrasion-related properties as compared to the conventional solid/fluid mixtures. The development of nanofluids is still hindered by several factors such as the lack of agreement between results, poor characterization of suspensions, and the lack of theoretical understanding of the mechanisms. Suspended nanoparticles in various base fluids. A Nanofluid is a fluid containing nanometer sized particles, called Nanoparticles. These fluids are engineered colloidal suspension of nanoparticles in a base fluid. The nanoparticles used in nanofluids are typically made of metals, oxides, carbides, or carbon nanotubes. Common base fluids include water, ethylene Glycol and oil. Nanofluids have novel properties that make them potentially useful in many applications in heat transfer, including microelectronics, fuel cells, pharmaceutical processes, and hybrid-powered engine, engine cooling/vehicle thermal management, domestic refrigerator, chiller, heat exchanger, in grinding, machining and in boiler gas temperature reduction. They demonstrate enhanced thermal conductivity and the convective heat transfer coefficient compared to the base fluid. Knowledge of the rheological behavior of nanofluids is found to be very vital in deciding their suitability for convective heat transfer applications. In the present world of fast technology, the cooling of electronic devices is one of the prominent industrial requirements, but the low thermal conductivity of classical heat transfer fluid such as water, oil and ethylene glycol, is the primary limitation. This leads to the creation of innovative technique in which the nanoscale size (1–100 nm) solid particles are suspended into classical heat transfer fluid in order to change the thermo physical properties of host fluid, which enhance the heat transfer significantly. This colloidal suspension was first identified as 'nanofluid' by Stephen U.S. Choi [1] in 1995 at the Argonne National Laboratory. The recent development of heat transfer nanofluids and their mathematical modeling [2006] play a significant role in various industries. These fluids have numerous applications like cooling of electronics, transportation (engine cooling/vehicle thermal management), manufacturing, heat exchanger, nuclear systems cooling, biomedicine etc. [2009,2011]. After the pioneering work by Sakiadis [1961], a large amount of literature is available on boundary layer flow of Newtonian and non-Newtonian fluids over linear and nonlinear stretching surface. The problem of natural convection in a regular fluid past a vertical plate is a classical problem first studied theoretically by E. Pohlhausen in contribution to an experimental study by Schmidt and Beckmann [1930]. In the past few years, convective heat transfer in nanofluids has become a topic of major current interest. Recently Khan and Pop [2010] used the model of Kuznetsov and Nield [2010] to study the boundary layer flow of a nanofluid past a stretching sheet with a constant surface temperature. Makinde, and Aziz [2011] considered to study the effect of a convective boundary condition on boundary layer flow, heat and mass transfer and nanoparticle fraction over a stretching surface in a nanofluid. The transformed non-linear ordinary differential equations governing the flow are solved numerically by the Runge-Kutta Fourth order method.

The solution of boundary layer equation for a power law fluid in MHD was obtained by Helmy[1994]. Chiam[1995] investigated hydromagnetic flow over a surface stretching with power law velocity using shooting method. Ishaketal[2008] investigated MHD flow and heat transfer adjacent to a stretching vertical sheet. Nourazaretal[2011] investigated MHD forced convective flow of nanofluid over a horizontal stretching sheet with variable magnetic field with the effect of viscous dissipation. the numerical solution of unsteady MHD flow of nanofluid on the rotating stretching sheet. Hamad [2011] obtained an analytical solution by considering the effect of magnetic field for electrical conducting nanofluid flow over a linearly stretching sheet. Rana et al.[2011] investigated the numerical solution of unsteady MHD flow of nanofluid on the rotating stretching sheet.

Wang and Mujumdar [2008], Kakaç and Pramuanjaroenkij [2009], Chandrasekar et al. [2012] and Wu and Zhao [2013]. The effects of nanofluids could be considered in different ways such as dynamic effects which include the effects of Brownian motion and thermo phoresis diffusion [2013,2013,2014], and the static part of Maxwell’s theory [2013,2013,2013,2013]. Recently, many researchers, using similarity solution, have examined the boundary layer flow, heat and mass transfer of nanofluids over stretching sheets. Khan and Pop [2010] have analyzed the boundary-layer flow of a nanofluid past a stretching sheet using a model in which the Brownian motion and thermo phoresis effects were taken into account. They reduced the whole governing partial differential equations into a set of nonlinear ordinary differential equations and solved them numerically. In addition, the set of ordinary differential equations which was obtained by Khan and Pop [2011] has been solved by Hassani et al. [2011] using homotopy analysis method. After that, many researchers, using similarity solution approach, have extended the heat transfer of nanofluids over stretching sheets and examined the other effects such as the chemical reaction and heat radiation [2011], convective boundary condition [2012], nonlinear stretching velocity [2012], partial slip boundary condition [2012], magnetic nanofluid [2013], partial slip and convective boundary condition [2013], heat generation/absorption[2013], thermal and solutal slip [2013], nano non-Newtonian fluid[2013], and Oldroyd-B Nanofluid [2009]. At the present time, it is not clear when the boundary layer approximations are adequate for analysis of flow and heat transfer of nanofluids over a stretching sheet in the case of flow and heat transfer of nanofluids. As mentioned, the enhancement of the thermal conductivity of nanofluids is the most outstanding thermo-physical properties of nanofluids. In all of the previous studies [2010–2011], the effect of local volume fraction of nano particles on the thermal conductivity of the nanofluid was neglected. However, in the work of Buongiorno[2006], it has been reported that the local concentration of nanoparticles may significantly affect the local thermal conductivity of the nanofluids.

In this thesis, our main objective is to investigate the effect of a convective boundary condition boundary layer flow, heat transfer and nanoparticle fraction profiles over a stretching sheet in nanofluid. The governing boundary layer equations have been transformed to a two-point boundary value problem in similarity variables, and these have been solved numerically. The effects of embedded parameters on fluid velocity, temperature and particle concentration have been shown graphically. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a balance to the previous studies.

## II. CONVECTIVE TRANSPORT EQUATIONS

consider steady two-dimensional  $(x, y)$  boundary layer flow of a nanofluid past a stretching sheet with a linear velocity variation with the distance  $x$  i.e.  $u_w = cx^n$  where  $c$  is a real positive number, is stretching rate,  $n$  is a nonlinear stretching parameter, and  $x$  is the coordinate measured from the location, where the sheet velocity is zero

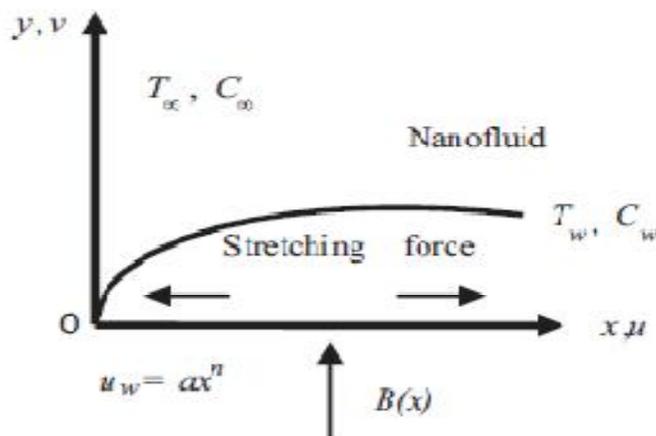


Fig A. Nano boundary layer flow over a nonlinear stretching sheet.

The sheet surface temperature  $T_w$ , to be determined later, is the result of a convective heating process which is characterized by temperature  $T_f$  and a heat transfer coefficient  $h$ . The nanoparticle volume fraction  $C$  at the wall is  $C_w$ , while at large values of  $y$ , the value is  $C_\infty$ . The Boungiorno model may be modified for this problem to give the following continuity, momentum, energy and volume fraction equations.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (5.1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left( \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma B_0^2 u}{\rho}, \quad (5.2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial y^2} \right) + \tau \left\{ D_B \left( \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} \right) + \frac{D_T}{T_\infty} \left[ \left( \frac{\partial T}{\partial y} \right)^2 \right] \right\}, \quad (5.3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left( \frac{\partial^2 C}{\partial y^2} \right) + \left( \frac{D_T}{T_\infty} \right) \left( \frac{\partial^2 T}{\partial y^2} \right), \quad (5.4)$$

where  $u$  and  $v$  are the velocity components along the  $x$  and  $y$  directions, respectively,  $p$  is the fluid pressure,  $\rho_f$  is the density of base fluid,  $\nu$  is the kinematic viscosity of the base fluid,  $\alpha$  is the thermal diffusivity of the base fluid,  $\tau = (\rho c)_p / (\rho c)_f$  is the ratio of nanoparticle heat capacity and the base fluid heat capacity,  $D_B$  is the Brownian diffusion coefficient,  $D_T$  is the thermophoretic diffusion coefficient and  $T$  is the local temperature. The subscript  $\infty$  denotes the values of at large values at large values of  $y$  where the fluid is quiescent. The boundary conditions may be written as

$$y = 0, u = ax^n, v = \pm v_w, -k \frac{\partial T}{\partial y} = h(T_f - T), C = C_w, \quad (5.5)$$

$$y \rightarrow \infty, u \rightarrow 0, \rightarrow T_\infty, C \rightarrow C_\infty, \quad (5.6)$$

We introduce the following dimensionless quantities

$$\eta = y \sqrt{\frac{a(n+1)}{2\nu}} x^{\frac{n-1}{2}}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad u = ax^n f'(\eta),$$

$$v = -\sqrt{\frac{av(n+1)}{2}} x^{\frac{n-1}{2}} \left\{ f + \frac{(n-1)}{n+1} \right\} \eta f', \quad (5.7)$$

$$\theta = \frac{T - T_\infty}{T_f - T_\infty},$$

Substituting (5.7) in (5.2), (5.3), (5.4) and (5.5) and (5.6), we obtain the following set of equations,

$$f''' + ff'' - f'^2 - \left(\frac{2n}{n+1}\right) f'^2 - Mf' = 0, \quad 5.8$$

$$\theta'' + Prf\theta' + PrNb\phi'\theta' + PrNt\theta'^2 = 0, \quad 5.9$$

$$\phi'' + Lef\phi' + \frac{Nt}{Nb}\theta'' = 0, \quad 5.10$$

subject to the following boundary conditions.

$$f(0) = f_w, f'(0) = 1, \theta'(0) = -Bi[1 - \theta(0)], \phi(0) = 1, \quad 5.11$$

$$f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0, \quad 5.12$$

where primes denote differentiation with respect to  $\eta$  and the five parameters appearing in Eqs. (5.9-5.12) are defined as follows.

$$Pr = \frac{\nu}{\alpha}, Le = \frac{\nu}{D_B}, Nb = \frac{(\rho c)_p D_B (C_w - C_\infty)}{(\rho c)_f \nu},$$

$$Nt = \frac{(\rho c)_p D_T (T_f - T_\infty)}{(\rho c)_f \nu T_\infty}, Bi = \frac{h(\nu/a)^{1/2}}{k}$$

$$M = \frac{2\sigma B_0^2}{a\rho(n+1)}, F_w = -\frac{v_w}{\sqrt{av}} \sqrt{\frac{2}{x^{n-1}(n+1)}}$$

5.13

With  $Nb = 0$  there is no thermal transport due to buoyancy effects created as a result of nanoparticle concentration gradients.

Here, we note that Eq. (5.8) with the corresponding boundary conditions on  $f$  provided by Eq. (5.11) has a closed form solution which is given by

$$f(\eta) = 1 - e^{-\eta}. \quad 5.14$$

In Eq. (6.14),  $Pr, Le, Nb, Nt$  and  $Bi$  denote the Prandtl number, the Lewis number, the Brownian motion parameter, the thermophoresis parameter and the Biot number respectively. The reduced Nusselt number  $Nur$  and the reduced Sherwood number  $Shr$  are obtained in terms of the dimensionless temperature at the surface,  $\theta'(0)$  and the dimensionless concentration at the sheet surface,  $\phi'(0)$ , respectively i.e.

$$Nur = Re_x^{-1/2} Nu = -\theta'(0), \quad 5.15$$

$$Shr = Re_x^{-1/2} Nu = -\phi'(0), \quad 5.16$$

where

$$Nu = \frac{q_w x}{k(T_w - T_\infty)}, Sh = \frac{q_m x}{D_B(\phi_w - \phi_\infty)}, Re_x = \frac{u_w(x)x}{\nu}, 5.17$$

where  $q_w$  is the surface (wall) heat flux and  $q_m$  is the surface (wall) mass flux.

Table 1:

Comparison of results for the reduced Nusselt number  $-\theta'(0)$  and the reduced Sherwood number  $\phi'(0)$  with Rana and Bhargava[2012] and F.Mahboobetal [ 2015 ] for  $M = f_w = 0$ .

nN <sub>t</sub> N <sub>b</sub> Rana and Bhargava[41]		F.Mahboobetal[52]		Present result				
$-\theta'(0)$	$\phi'(0)$	$-\theta'(0)$	$\phi'(0)$	$-\theta'(0)$	$\phi'(0)$			
0.2	0.1	0.5	0.5160	0.9062	0.5148	0.9014	0.5140	0.9012
	0.3		0.4553	0.8395	0.4520	0.8402	0.4502	0.8399
	0.5		0.3999	0.8048	0.3987	0.8059	0.3990	0.8398
3.0	0.1		0.4864	0.8445	0.4852	0.8447	0.4850	0.8450
	0.3		0.4282	0.7785	0.4271	0.7791	0.4270	0.7790
	0.5		0.3786	0.7379	0.3775	0.7390	0.3776	0.3780
10	0.1		0.4799	0.8323	0.4788	0.8325	0.4790	0.8330
	0.3		0.4227	0.7654	0.4216	0.7660	0.4220	0.7665

### III. RESULT AND DISCUSSION

Eqs. (5.8-5.10) subject to the boundary conditions, Eqs.(5.11) and (5.12), were solved numerically using Runge- kutta -Fehlberg fourth-fifth order method. As a further check on the accuracy of our numerical computations, Table 1 is the Comparison of results for the reduced Nusselt number  $-\theta'(0)$  and the reduced Sherwood number  $\phi'(0)$  with Rana and Bahargava[2012] and F.Mahboobetal [ 2015 ] for  $M= f_w =0$ , and are found to be in excellent agreement with our results.

We now turn our attention to the discussion of graphical results that provide additional insights into the problem under investigation.

#### A. Velocity Profiles

In Fig. 1, the velocity, profiles  $f'(\eta)$  are accessible for variation in Suction/Injection parameter  $f_w$ . With increasing values of the Suction/Injection parameter  $f_w$  the velocity  $f'(\eta)$ , in the boundary layer region decrease, whereas, due to the increase suction/injection parameter ( $f_w > 0$ ), the velocity profiles  $f'(\eta)$  displays an increasing trend.

Fig. 2 displays the effect of magnetic parameter  $M$  on velocity profile  $f'(\eta)$  and it is noticed that velocity profile decreases as  $M$  increases, this is due to the fact that Lorentz force tends to obstruct the flow velocity in the boundary layer region, resulting in thinning of momentum boundary layer thickness, which is consistent with the results of various published results so far.

It is observed from fig.3, that the effect of nonlinear stretching parameter  $n$  on dimensionless velocity profile  $f'$  is to decrease velocity slightly with increase of nonlinear stretching parameter  $n$ .

**B. Temperature profiles**

Fig. 4 shows the temperature distribution in the thermal boundary layer for different values of Brownian motion and the thermophoresis parameters. As both  $Nb$  and  $Nt$  increase, the boundary layer thickens, as noted earlier in discussing the tabular data, the surface temperature increases, and the curves become less steep indicating an attenuation of the reduced Nusselt number. As seen in Fig. 5, the effect of Lewis number on the temperature profiles is noticeable only in a region close to the sheet as the curves tend to merge at larger distances from the sheet. The Lewis number expresses the relative contribution of thermal diffusion rate to species diffusion rate in the boundary layer regime. An increase of Lewis number will reduce thermal boundary layer thickness and will be accompanied with a decrease in temperature. Larger  $Le$  will suppress concentration values, i.e. inhibit nanoparticle species diffusion. There will be much greater reduction in concentration boundary layer thickness than thermal boundary layer thickness over an increment in Lewis Number.

Fig. 6 illustrates the effect of Biot number on the thermal boundary layer. As expected, the stronger convection results in higher surface temperatures, causing the thermal effect to penetrate deeper into the quiescent fluid. The temperature profile depicted.

In Fig. 7 show that as the Prandtl number increases, the thickness of the thermal boundary layer decreases as the curve become increasingly steeper. As a consequence, the reduced Nusselt number, being proportional to the initial slope, increases. This pattern is reminiscent of the convective of the free convective boundary layer flow in a regular fluid[20]

Fig 8 shows that the effect of magnetic parameter  $M$  on the temperature profiles is noticeable only in region close to the sheet as the curves tend to merge at larger distances from the sheet.

In Fig 9, the temperature profiles  $\theta(\eta)$  are accessible for variation in Suction/Injection parameter  $f_w$  with increasing values of the Suction/Injection parameter  $f_w$ , the temperature profiles  $\theta(\eta)$  in the boundary layer decrease, whereas, due to the increase of suction/ injection parameter ( $f_w > 0$ ), temperature profiles displays an increasing trend.

**C. Concentration Profiles.**

The effect of  $Le$  on nanoparticle concentration profiles is shown in Fig. 11. Unlike the temperature profiles, the concentration profiles are only slightly affected by the strength of the Brownian motion and thermophoresis. A comparison of Fig. 5 and Fig. 11 shows that the Lewis number significantly affected the concentration distribution (Fig. 11), but has little influence on the temperature distribution (Fig. 5). For a base fluid of certain kinematic viscosity  $\nu$ , a higher Lewis number implies a lower Brownian diffusion coefficient  $D_b$  (see Eq.(5.13)) which must result in a shorter penetration depth for the concentration boundary layer. This is exactly what we see in Fig. 11. It was observed in Fig.12 that as the convective heating of the sheet is enhanced i.e.  $Bi$  increases, the thermal penetration depth increases. Because the concentration distribution is driven by the temperature field, one anticipates that a higher Biot number would promote a deeper penetration of the concentration. This anticipation is indeed realized in Fig. 12, which predict higher concentration at higher values of the Biot number. A comparison of Fig. 8 and fig. 13 shows that the Magnetic parameter significantly affected the concentration distribution (Fig.13), but has little influence on the temperature distribution (Fig.8).

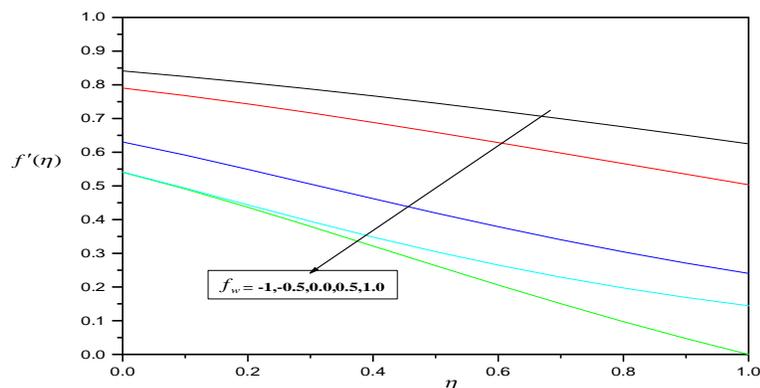


Fig. 1: Effect of  $f_w$  on velocity profiles when  $M = 1, n = 2, Nt = Nb = Bi = 0.1, Le = Pr = 5$ .

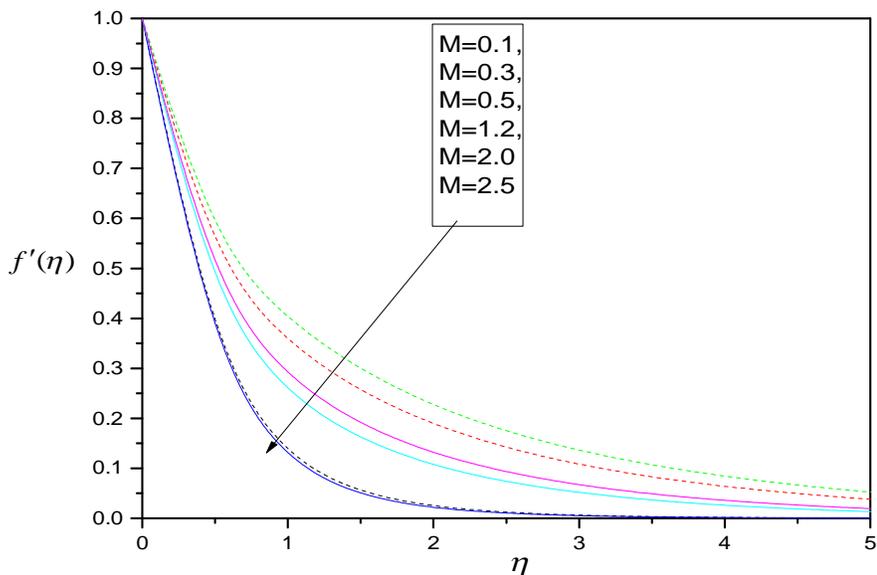


Fig.2: Effect of  $M$  on velocity profiles when  $f'(\eta)$   $Nt = Nb = Bi = 0.1, Le = Pr = 5, n = 2$ .

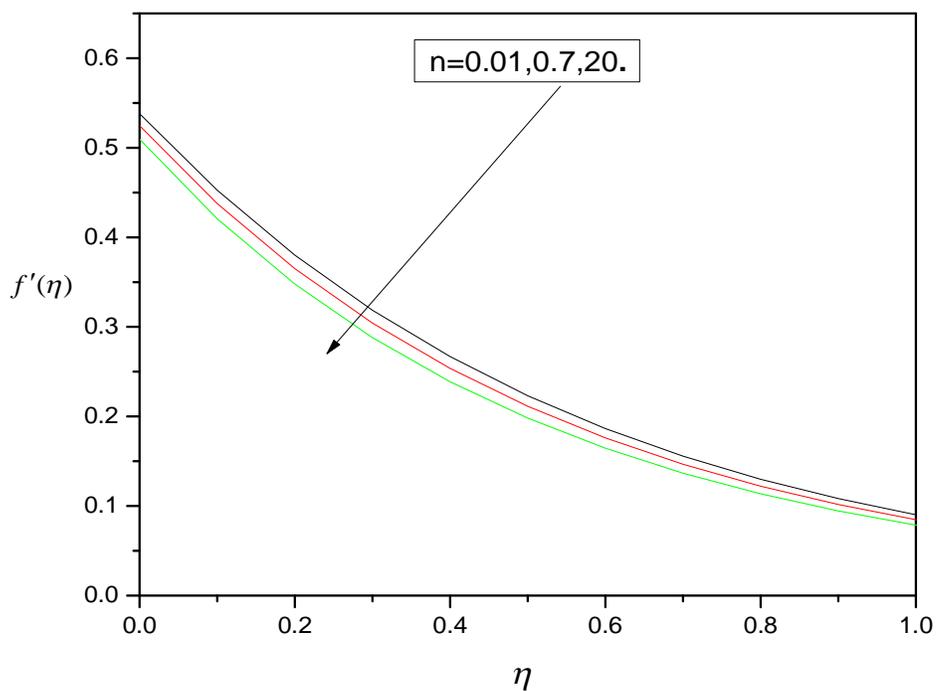


Fig. 3: Effect of nonlinear stretching parameter  $n$  on velocity profile  $f'(\eta)$  for various values of  $Nt = Nb = Bi = 0.1, Le = Pr = 5, M = 2$ .

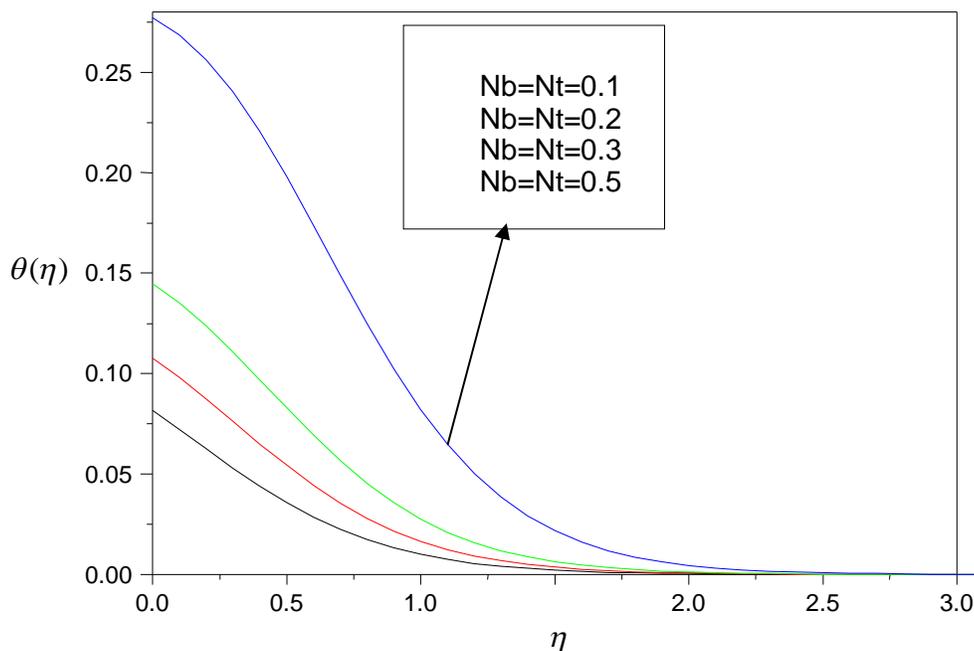


Fig. 4: Effect of  $Nt$  and  $Nb$  on temperature profiles when  $M=2, n=2, Le = 5, Pr = 5, Bi = 0.1$ .

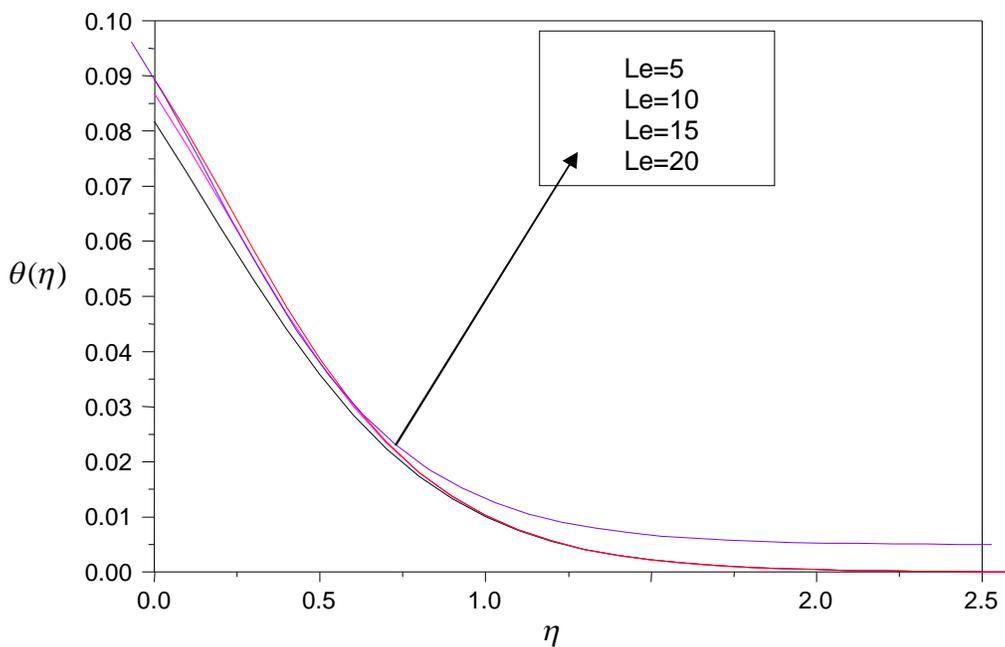


Fig. 5. Effect of  $Le$  on temperature profiles  $\theta(\eta)$  when  $M=2, Nt = Nb = 0.1, Pr = 5, Bi = 0.1$ .

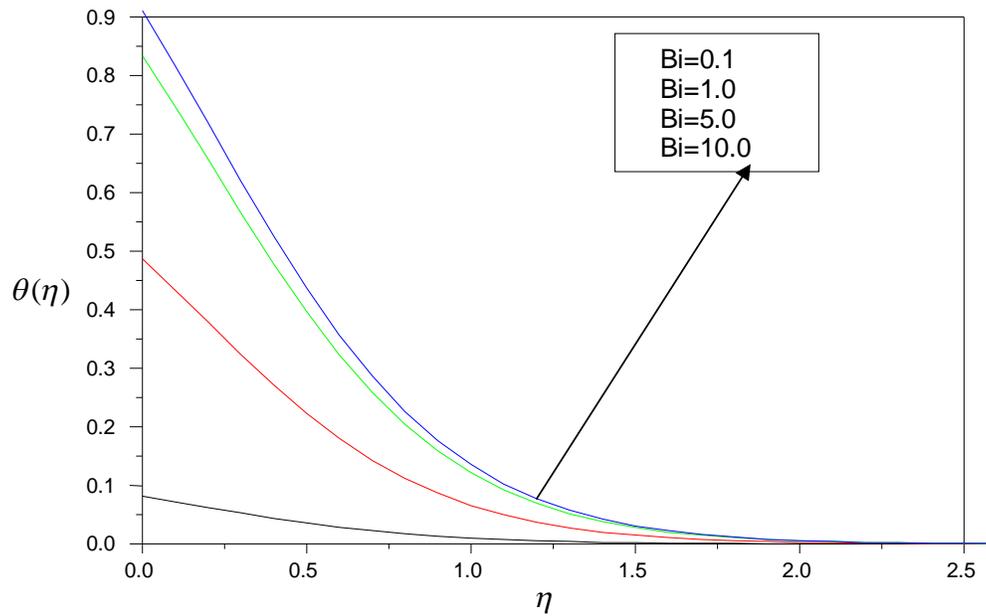


Fig. 6. Effect of  $Bi$  on temperature profiles  $\theta(\eta)$  when  $M=2, Nt = Nb = 0.1, Pr = Le = 5$ .

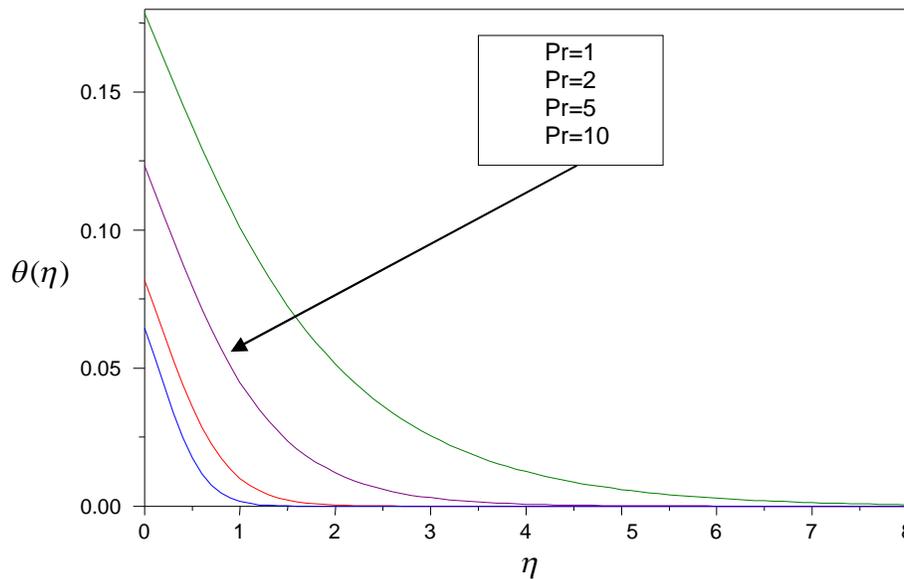


Fig. 7. Effect of  $Pr$  on temperature profiles when  $M=2, Nt = Nb = Bi = 0.1, Le = 5$ .

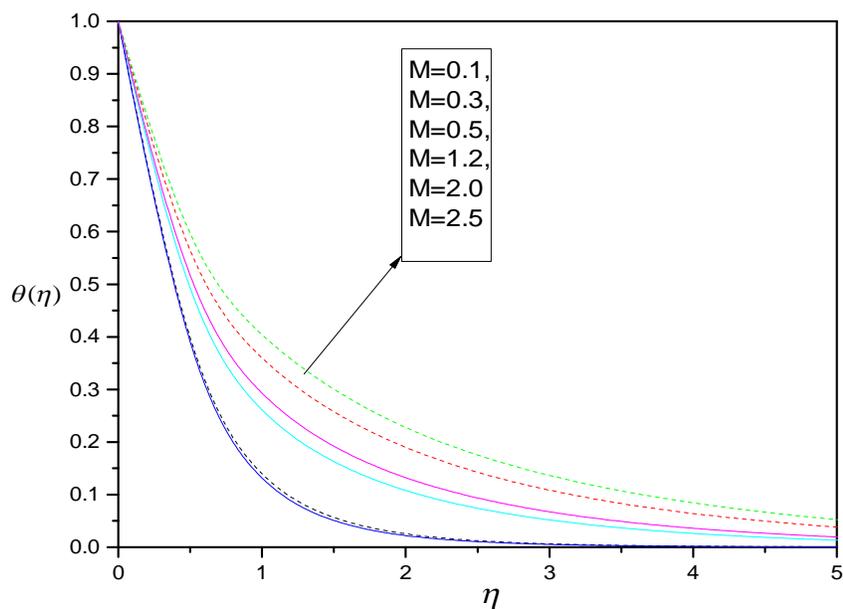


Fig. 8. Effect of  $M$  on temperature profiles when  $Nt = Nb = Bi = 0.1, Le = Pr = 5$ .

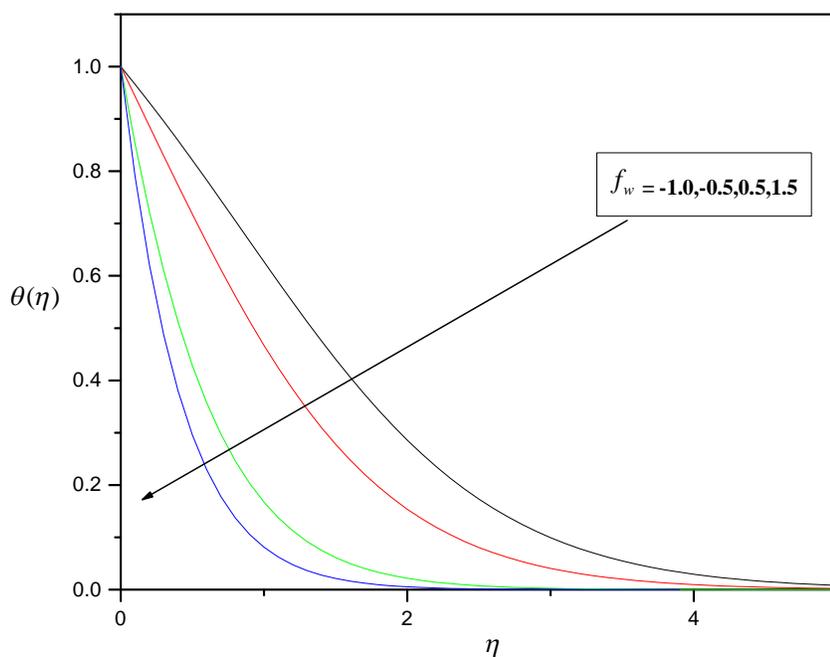


Fig 9. Effect of suction/injection parameter  $f_w$  on Temperature profiles  $\theta(\eta)$  for various values of  $Nt = Nb = Bi = 0.1, Le = Pr = 5, n = 2, M = 2$ .

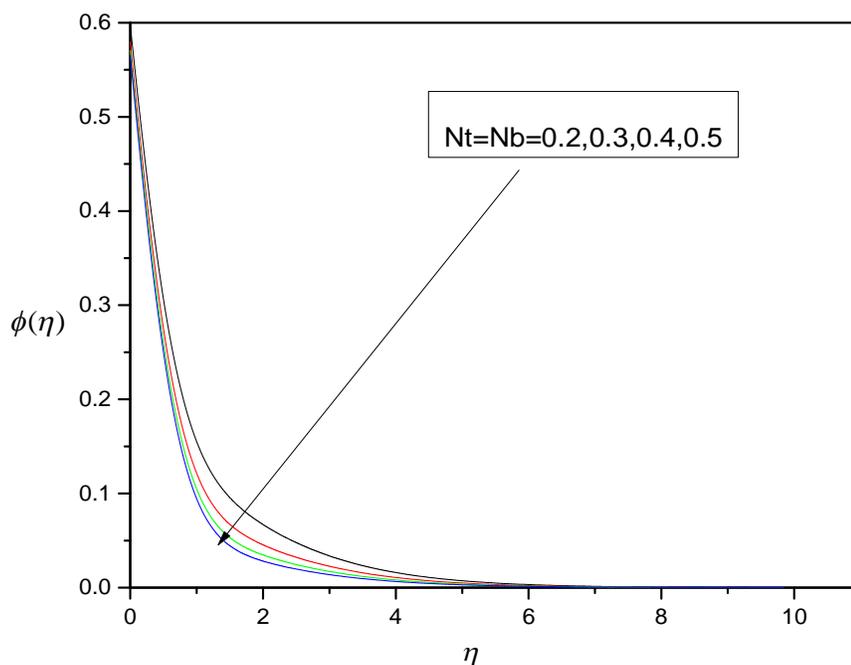


Fig. 10. Effect of  $Nt = Nb$  on concentration profile  $\phi(\eta)$  when  $Le = 5, Bi = 0.1, M = 1, n = 2$ .

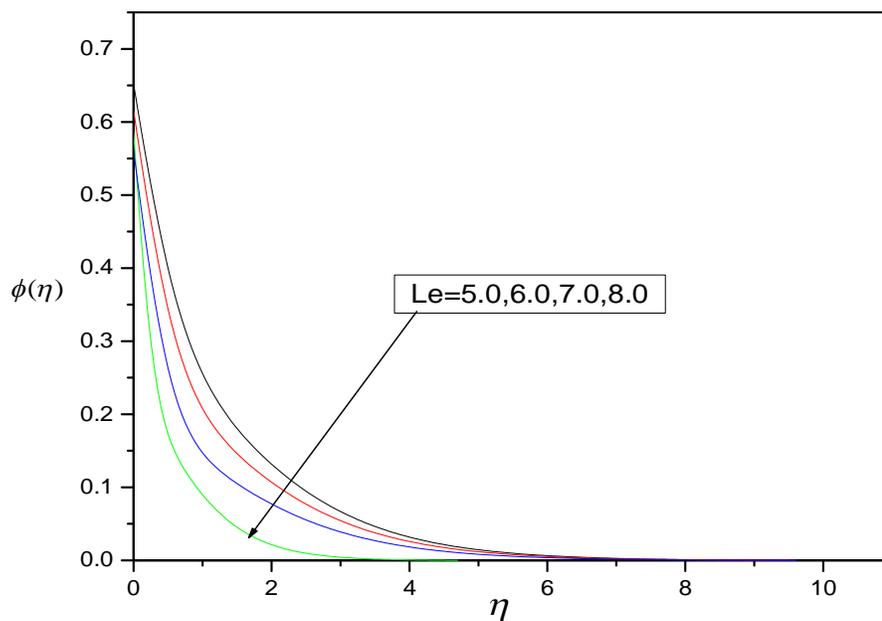


Fig. 11. Effect of concentration profiles  $\phi(\eta)$  when  $Nt = Nb = 0.1, Bi = 0.1, M = 1, n = 2$

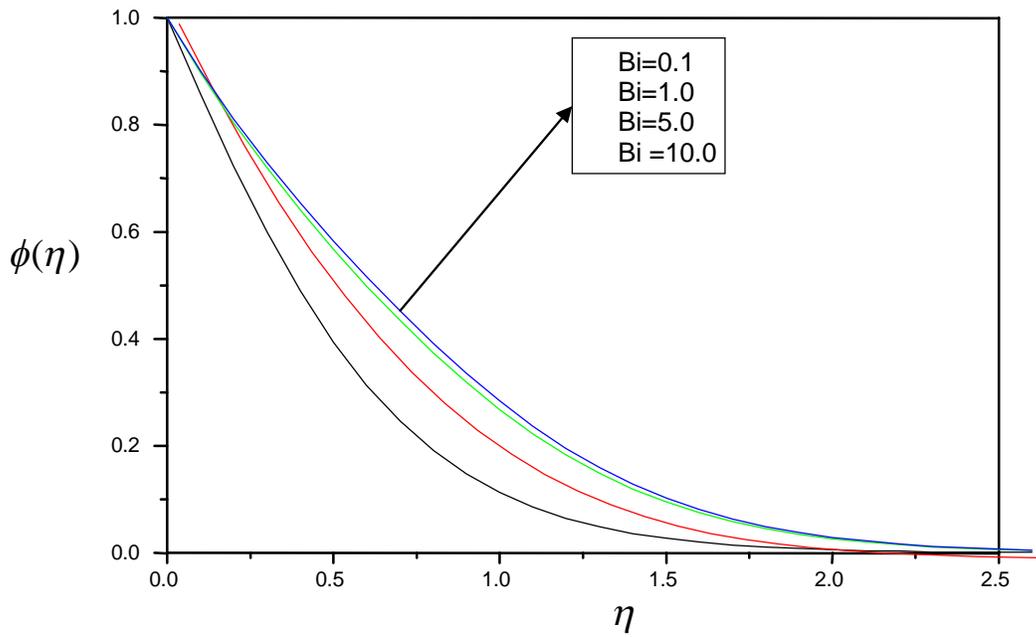


Fig. 12. Effect of  $Bi$  on concentration profiles  $\phi(\eta)$  when  $N_t = N_b = 0.1, Pr = Le = 5$ .

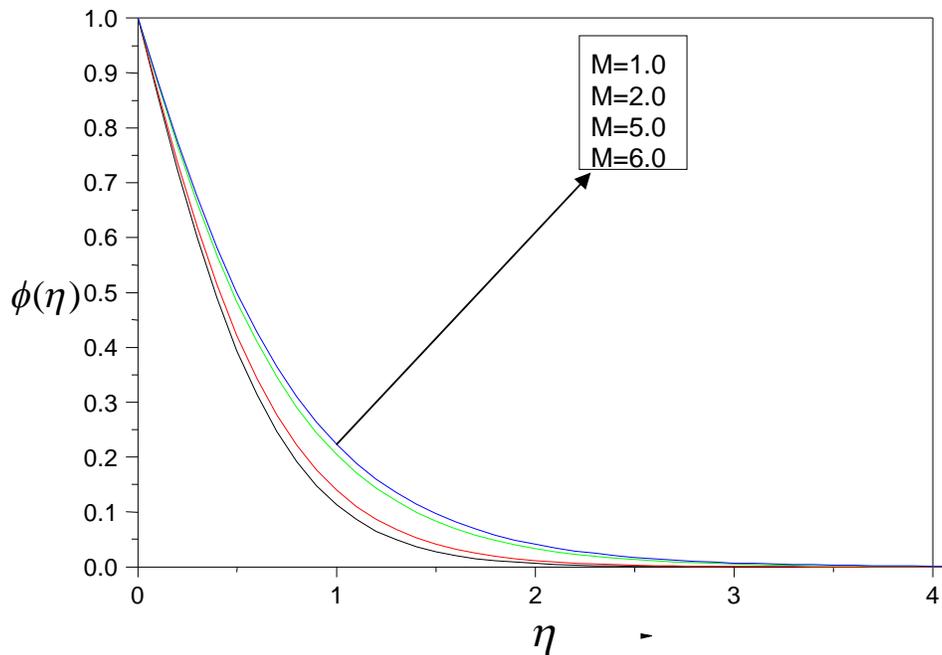


Fig. 13. Effect of  $M$  on concentration profiles  $\phi(\eta)$  when  $N_t = N_b = Bi = 0.1, Pr = Le = 5$ .

#### IV. CONCLUSION

A numerical study of the boundary layer flow in a nanofluid induced as a result of motion of a nonlinearly stretching sheet has been performed. The use of a convective heating boundary condition instead of a constant temperature or a constant heat flux makes this study more general novel. The following conclusions are derived

- A. The transport of momentum, energy and concentration of nanoparticles in the respective boundary layers depends on six parameters: Brownian motion parameter  $Nb$ , thermophoresis parameter  $Nt$ , Prandtl number  $Pr$ , Lewis number  $Le$ , convection Biot number  $Bi$  and Magnetic parameter  $M$ .
- B. For infinitely large Biot number characterizing the convective heating (which corresponds to the constant temperature boundary condition), the present results and those reported by Rana and Bhargava[2012] F. Mahboobetal[2015] match up to four places of decimal.
- C. For a fixed  $Pr$ ,  $Le$ ,  $EC$  and  $Bi$  the thermal boundary thickens and the local temperature rises as the Brownian motion and thermophoresis effects intensify. A similar effect on the thermal boundary is observed when  $Nb, Nt, Le$  and  $Bi$  are kept fixed and the Prandtl number  $Pr$  is increased or when  $Pr, Nb, Nt$  and  $Le$  are kept fixed and the Biot number is increased. However, when  $Pr, Nb, Nt$  and  $Bi$  are kept fixed, and the Lewis number is increased, the temperature distribution is affected only minimally.
- D. With the increase in  $Bi$ , the concentration layer thickens but the concentration layer becomes thinner as  $Le$  increases.
- E. For
- F. fixed  $Pr, Le$  and  $Bi$ , the reduced Nusselt number decreases but the reduced Sherwood number increases as Brownian motion and thermophoresis effects intensify.

##### 1) Nomenclature

$B_i$  Biot number

$a$  a positive constant associated with linear stretching

$D_B$  Brownian diffusion coefficient

$D_T$  Thermophoretic diffusion coefficient

$f(\eta)$  Dimensionless stream function

$g$  Gravitational acceleration

$h$  Convective heat transfer coefficient

$k$  Thermal conductivity of the nanofluid

$Le$  Lewis number

$Nb$  Brownian motion parameter

$Nt$  Thermophoresis parameter

$Nu$  Nusselt number

$Nur$  Reduced Nusselt number

$Pr$  Prandtl number

$p$  pressure

$q_m''$  Wall mass flux

$q_w''$  Wall heat flux

$Re_x$  Local Reynolds number

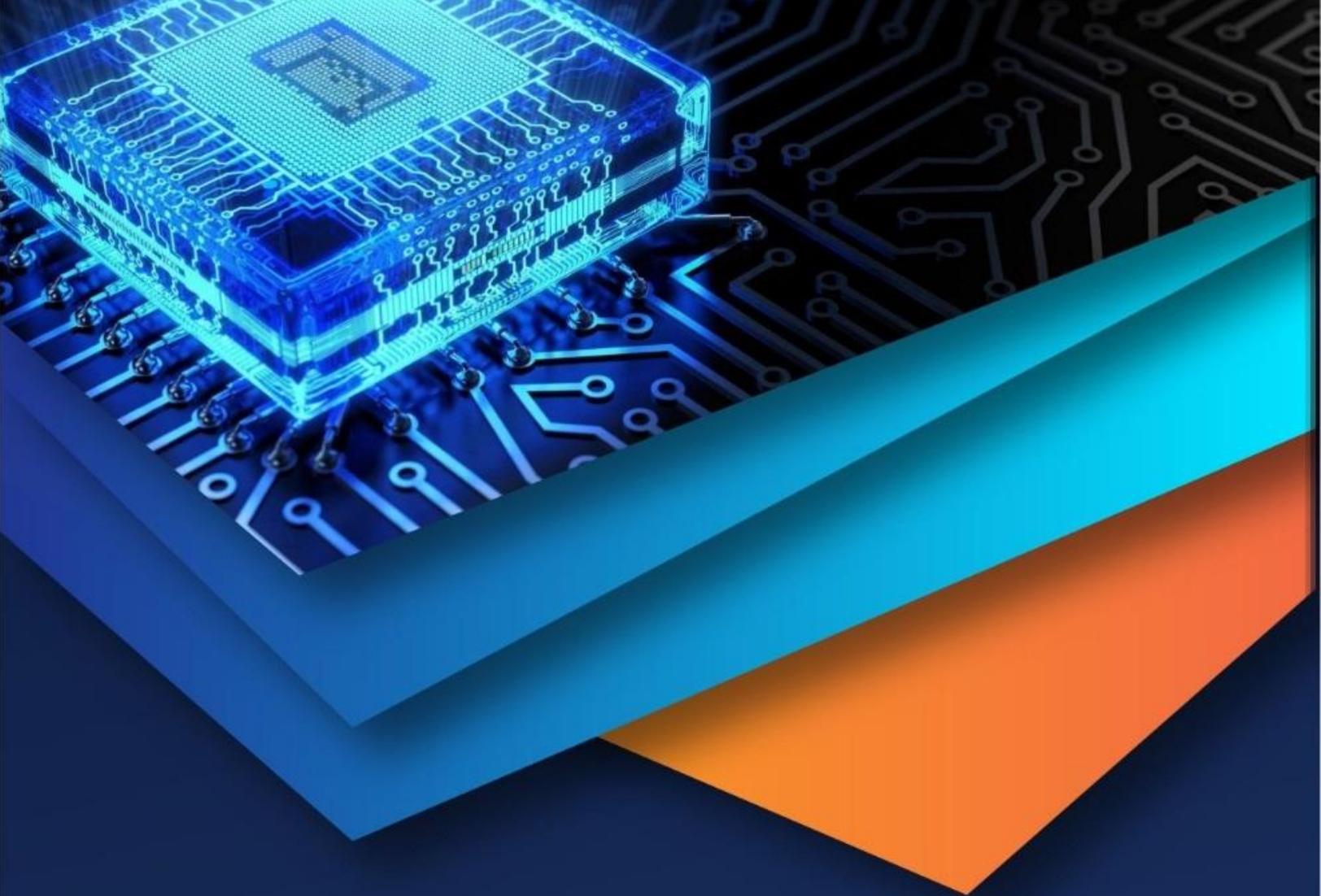
$Sh$  Sherwood number

$Shr$	Reduced Sherwood number
$M$	Magnetic number
$T$	Local fluid Temperature
$T_f$	Temperature of the hot fluid
$T_w$	Sheet surface (wall) temperature
$T_\infty$	Ambient temperature
$u, v$	Velocity components in x and y directions
$C$	nanoparticle volume fraction
$C_w$	Nanoparticle volume fraction at the wall
$C_\infty$	Nanoparticle volume fraction at large values of y(ambient)
EC	Eckert number
2) Greek symbol	
$\alpha$	Thermal diffusivity of the base fluid
$\eta$	Similarity variable
$\theta$	Dimensionless temperature
$\phi$	Dimensionless volume fraction
$\mu$	Absolute viscosity of the base fluid
$\nu$	Kinematic viscosity of the base fluid
$\rho_f$	Density of the base fluid
$\rho_p$	Nanoparticle mass density
$(\rho c)_f$	Heat capacity of the base fluid
$(\rho c)_p$	Heat capacity of the nanoparticle material
$\tau = (\rho c)_p / (\rho c)_f$	
$\psi$	Stream function

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