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A Study on Domination Number in Jahangir Graph

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Abstract: Domination of a graph is widely applied in data analyzing, networking, decision making technique, Jahangir graph applied built in constructing uses many different model shapes . Here, I discussed about the Domination, Split domination, Non split domination and Total domination properties in Jahangir Graph. In particular its applied in $J_{2,8}$ and the results are discussed.

Key Words: Domination, Split Domination, Non-Split Domination, Inverse domination, Jahangir graph

I. INTRODUCTION

The concept of domination in graphs began in the middle of the 17th century and the Queen problem, where the question was how many queens on a chess board were required so that no square on the chessboard was unavailable to queen move. The Question was First stated mathematically by de Jaenisch in 1862[6].The problem was formalized in work by Berge and ore in 1958[8],with one of the first treatments by berge in 1962.

The Mathematical study of domination theory in graphs started around 1960, Its roots go back to 1862.when C.F De Jaenush studied the problem of determining the minimum number of queens necessary to cover an nxn chess board in such way that every square is attached by one of the queens.

This problem were studied in detail by two brothers A.M Yaglom and I.M Yaglom around 1964.They have derived solutions of some of these kinds of problems for rooks, knights, kings and bishops. C. Berge wrote a book on graph theory in which be defined the concept of the domination number in 1958.He called this number the co-efficient of external stability. Actually the names, dominating set and domination number published in 1962. He used the Notation d(G) for the domination number of a graph. The Notation γ (G) was first used by E.J. Cockayne and S.T Headtinenu for the domination number for a graph which subsequently became the accepted notation.

A. Domination Number

The Domination Number γ (G) is the minimum size of a Dominating set of vertices in G(the cardinality of minimum dominating set.

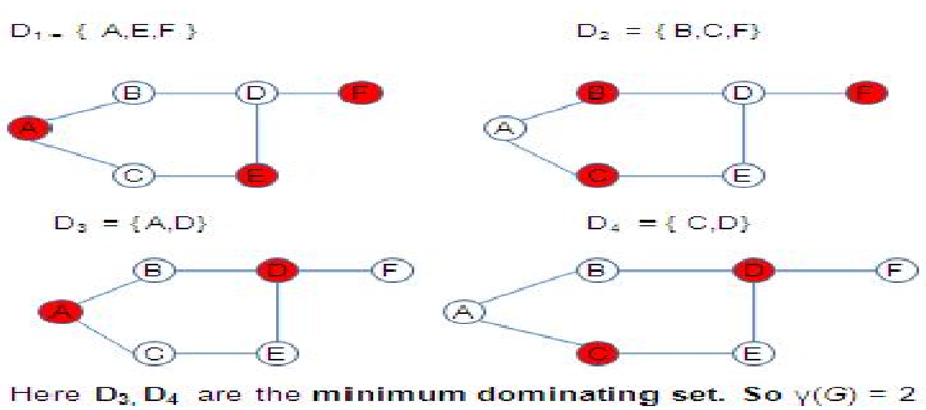

FIG 1

Here D2 are the minimum dominating set so
γ (G)=2 But D1 not a minimum dominating set.

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B. Dominating Set

A Dominating set for a graph G=(V,E) is a subset D of V such that every vertex not in D.(Every vertex V-D) is a joined to at least one member of D by some edge. ie) A set D of vertices in a graph G is called a dominating set of G if every vertex in V-D is adjacent to some vertex in D.

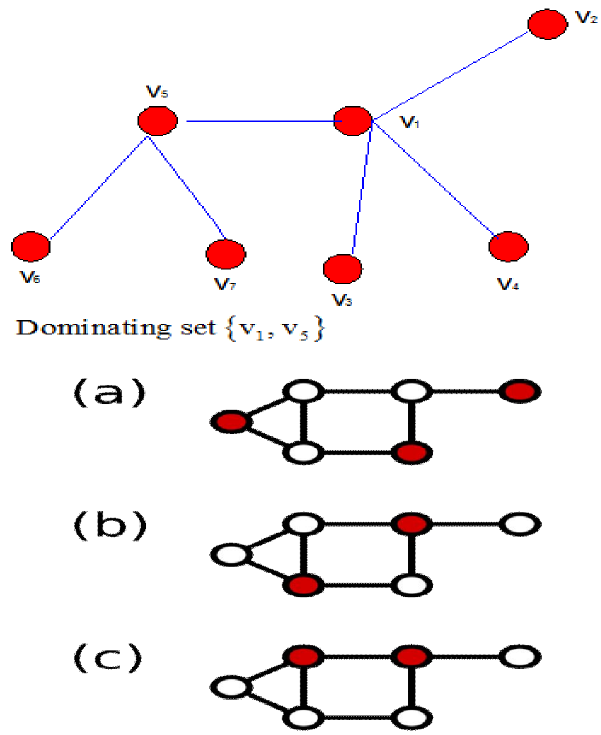

FIG 2

C. Minimum Dominating Set

If D Conist of minimum Number of vertices. A Dominating set D is Minimum Dominating set.

D. Minimal Dominating Set

A dominating set D of the graph G is said to be a minimal dominating set if for every vertex V in D, D-V is not a dominating set, then Minimal Dominating set has no proper subset of D is a Dominating set.

1) *Theorem 1.1:* A Dominating Set D is a Minimal dominating set if and only if for each vertex V (E, D) one of the following conditions. V is an isolated vertex of D, then there exist a vertex u$\in$ V-D, N(u) $\bigcap$ D={V}
2) *Theorem 1.2[18:* Let G be a graph without isolated vertices. If D is a minimal dominating set, Then V-D is a Dominating Set.
3) *Proof:* Suppose v is isolate vertex in G

   D is a γ set of G

   Then D-V is a dominating set of G-V

   γ (G-V)$\leq$ $|$D-V$|$

   $\leq |\mathbf{D}| = \gamma$ (G)

   γ (G-V) = γ (G)

   Hence V-D is a dominating set.

4) *Result 1.3:* Domination Number Satisfies

   n/1+Δ $\leq \gamma \leq$ n

n= │V│the vertex count of a graph

Δ is maximum vertex degree.

5) *Result1.4 [18]:* If G is a graph with no isolated vertices, then

γ (G)≤ P/2

6) *Result 1.5 [18]:* If G(V,E) is a simple graph then 2│q│ ≤ │P^2│-│P│

II. SPLIT DOMINATING SET

A Graph G=(V,E), A Dominating Set D of G is a split Dominating Set if the induced sub graph<V-D> is Disconnected. Diagram

A. Split Domination Number

A Graph G=(V,E). The split domination Number is the minimum cardinality of the split dominating set.

It is denoted by γ_s (G)

γ_s (G)=2

[4] γ_s (G)= Min { S is a split dominating set} is the split domination number.

B. Result 2.4[19]

γ (Cp)=[P/3] if P≥4

γ (Wp)=3 if P≥5

γ (Km,n)= M if 2 ≤ m≤ n

C. Result 2.5

For any graph G, γ_s(G)≤ α_0(G), (G) is the vertex covering number of G

D. Result 2.6

γ_S (G)≤ δ (G) Where δ (G) minimum degree of G

E. Result 2.7

Let G be a graph such that both G and its complements G bar are connected, γ_S (G)+ γ_s (G)bar P≤ (P-3)

III. NON SPLIT DOMINATING SET[20]

A dominating set D of G is a Non split dominating set if the induced sub graph<V-D> is connected. Diagram

A. Non Split Domination Number

A graph G=V,E. The non split domination number denoted by γ_{ns} (G).It is the minimum cardinality of a No split Dominating set.

For any connected graph G, γ_{ns} (G) ≤ P-1, further quality holds if and only if G is a star.

IV. CONNECTED DOMINATING SET

A graph (G=V,E) be a graph a connecting set D of G is a connected dominating set if the induced sub graph <D> is connected.

A. Connected Domination Number

A graph (G=V,E) the connected domination number γ_c(G) is the minimum cardinality of a connected dominating set.

B. Result 4.3

If diagram (G)=5 then γ_s (G) ≥ γ_{ns} (G) Bar

C. Result 4.3

For any graph G , γ_{ns} (G) $\leq$ p - ω (G)+1.

ω (G) is the clique number of G.

V. TOTAL DOMINATING SET

A Subject S $\subseteq$ V is a dominating set of G if every vertex V-S has a neighbor in S and is a total dominating set(TDS).

A. Properties

1) If every vertex in V has a neighbor in S.
2) The total domination number γt(G) is the minimum cardinality of a dominating set of G.
3) A Total dominating set of G with Minimum cardinality is called a γt(G) Set.
4) Total domination was introduced by Cockayne and Hedetniemi(2).

B. Total Domination Number

The total domination Number γt and γt- set of G are defined similarily to γ(G) and γ-set
γt- any Non-trivial connected graph.
For any Non-Trivial connected graph G, γt (G) $\leq$ 3

C. Property 5.2 [21]

For G=(V,E) and For all u,v £ V if u and v are adjacent they Dominate each other the natleast.
dom(u,v)=1

VI. JAHANGIR GRAPH

A. Defintion 6.1

Jahangir graph ($J_{m,n}$)m- Cycle , n- Vertices. Jahangir graph $J_{m,n}$ for n $\geq$3 is a graph on nm+1 vertices. ie) a graph consisting of a cycle $C_{m,n}$ with one additional vertex which is adjacent to n vertices of $C_{m,n}$ at distance m to each other on $C_{m,n}$[1].

B. Examples

Jahangir graph $J_{2,8}$.The Figure $J_{2,8}$ Appears on Jahangir Tomb in His mausoleum. It lies in 5 kilometers North west of Lahore Pakistan across the River Ravi[1]

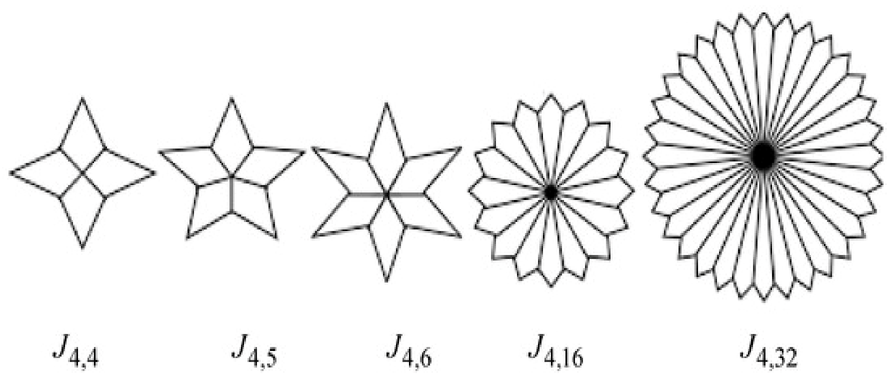

FIG 3

Wiener index and Hosoyoa polynomial of $J_{2,m}$ $J_{3,m}$ and$J_{4,m}$ are computed in **[22],[23]**

VII. DOMINATION NUMBER IN JAHANGIR GRAPH

Jahangir Graph statisfies Domination Number in the following conditions.

A. Result 7.1

n/1+$\Delta \leq \gamma \leq$ n

We know,

Δ is maximum vertex degree For example, choose $J_{2,8}$

γ (J2,8)=6

Δ(G)=8

So, we get 17/1+8 $\leq \gamma \leq$ 17

B. *Theorem 7.2*

Let G be a Jahangir graph $J_{m,n}$ with no isolated vertices, then γ $(J_{m,n}) \leq$ P/2

Proof Let G be a Jahangir graph $J_{m,n}$ with no isolated vertices. We prove that

Now, analyzing the above property in Jahangir graph $J_{m,n}$ with no isolated vertices.

Let γ (G) be a Domination Number where G is a Jahangir graph $J_{m,n}$ and so,

γ $(J_{m,n})$ has m cycle and n vertices By the definition of Cycle

A Cycle graph or Circular graph that consist of a single cycle (or) some number of vertices connected in a closed chain

Now we take $J_{m,n}$ this mean $J_{m,n}$

m=2, n=8

γ $(J_{m,n}) \leq$ P/2, where P is a path2$\leq$ 8

C. *Theorem 7.3*

If $J_{m,n}$ is a simple graph then 2$|q| \leq |p^2|-|p|$ Proof

$J_{m,n}$ is a simple graph

By the definition of Simple graph :A Simple graph contains no loops or multiple edges Jahangir graph is a simple graph and it means no loops or multiple edge

Here for $J_{2,8}$, we get $|24| \leq |16^2|-|16|$

D. *Theorem-7.4*

For any connected graph $J_{m,n}$ without pendant vertex

γ $(J_{m,n}) \leq \delta$ $(J_{m,n})$

Proof

Let G be a Jahangir graph $J_{m,n}$ without pendant vertex.

By the Defintion of Pendant Vertex

A Vertex V of G is said to be pendant vertex if and only if it has degree 1.

So, if Jahangir graph does not have a pendant vertex,

Then minimum degree of δ $(J_{2,8})$ =1

Hence, we get,

γ $(J_{2,8}) \leq \delta$ $(J_{2,8})$

i.e, 6 $\leq$ 1

E. *Theorem 7.5*

Let G be Jahangir graph $J_{m,n,}$ $\gamma_s(J_{m,n}) \leq \alpha_o(J_{m,n})_o(J_{m,n})$ is the vertex covering number of G. Proof

G be a Jahangir graph $J_{m,n}$We need to prove that

$\gamma_s(J_{m,n}) \leq \alpha_o(J_{m,n})$

α_0(G) – Vertex covering Number of G

$\alpha_0(J_{m,n})$- Vertex covering Number of $J_{m,n}$

Let G=(V,E) be a graph, A subset K of V is called a vertex covering if every edge of G is incident with or covered by a vertex in K.

$\gamma_s(J_{2,8}) \leq \alpha_0(J_{2,8})$

We say split dominating set is n-m

Then $\alpha_o = 2$

$\gamma_s(5) \leq \alpha_0(2)$,

i.e., $5 \leq 2$.

F. *Theorem 7.6*

$\gamma_s(J_{m,n}) \leq \delta(j_{m,n})$ Proof

We prove that,

$\gamma_s(J_{m,n}) \leq \delta(j_{m,n})$

Minimum degree

$\gamma_s(J_{m,n}) \leq \delta(j_{m,n})$

Now we take m=2cycle;n=8 vertices

$\gamma_s(J_{2,8}) \leq \delta(j_{2,8})$

Minimum degree of its vertices $\delta(G)$

Now $\delta(j_{m,n})$ is the minimum degree of its verticlesso

$\delta(j_{2,8})$

Next

$\gamma_s(J_{2,8}) \leq \delta(j_{2,8})$

$5 \leq 1$

G. *Result 7.7*

Now, Change in Jahangir graph

$\gamma_s(C_p) = [P/3]$ if $p \geq 4$ is a Jahangir graph .

$\gamma_s(W_p) = 3$ if $p \geq 5$ so J(1,10) is a Jahangir graph

$\gamma_s(K_{m,n}) = m$ if $2 \leq m \leq n$

The Jahangir graph not a complete graph, so this is contradiction

Jahangir graph of non split is a contradiction

Hence the proof

$\gamma_{ns}(J_{m,n}) \leq P-\omega(J_{m,n})+1$

Where $w(J_{m,n})$ is the Clique number of $J_{m,n}$

Proof γ_{ns} is a connected

We prove that

$\gamma_{ns}(J_{m,n}) \leq P-\omega(J_{m,n})+1$

By the Definition of Clique Number

The Clique Number of a graph G, denoted w(G) is the number of vertices in a maximum clique of G. Equivalenty, it is the size of a largest clique or maximal clique Of G.

By the clique number contradiction in Jahangir graph.

Hence the proof.

VIII. CONCLUSION

Finally, on discussing domination, split, non-split and total domination in Jahangir graph, the concept of dominations satisfied in $J_{2,8}$.But in split domination satisfies only for wheel type Jahangir graph. It does not satisfies $J_{2,8}$since it is not a complete graph after splitting. If we can form any Networking Data in the Jahangir graph Model with complete property we can apply all the domination properties to reduce the dimensions.

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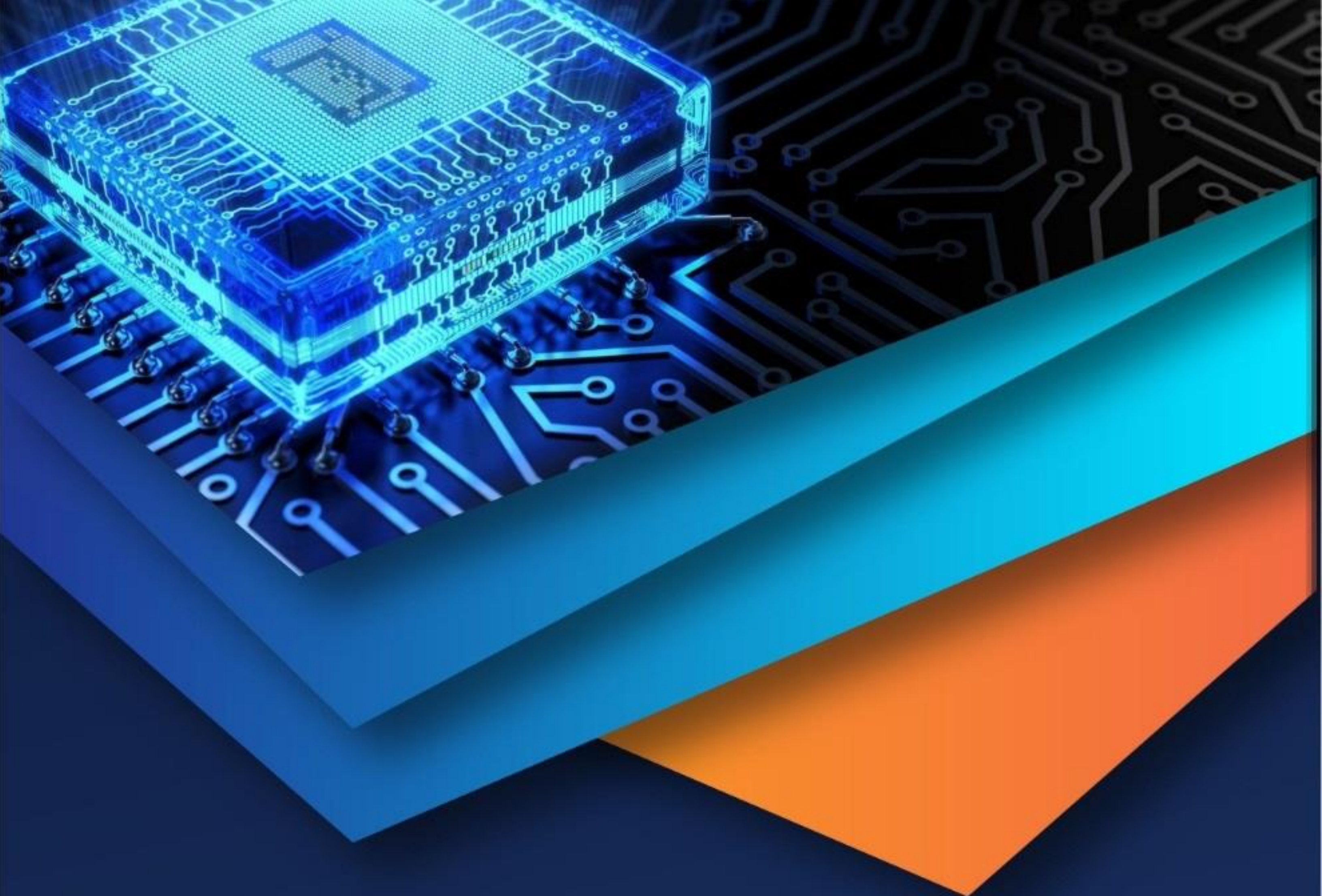

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