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# Reliability Analysis of A Single Pile–Non Probabilistic Approach

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***Abstract: Reliability analysis of structures has become important with the present-day emphasis on performance-based design because in structural engineering, uncertainties arise from different sources. Although probability theory has been traditionally used to represent all types of uncertainties, it may not be proper to use probability theory to represent uncertainties in the presence of limited knowledge. In the present study, an attempt has been made to perform reliability analysis in the context of fuzzy set theory and possibility theory. Two different methods of fuzzy reliability analysis, proposed in the literature, are identified. The performances of these methods are studied with the help of reliability analysis of a single axial pile considering different formats of limit state function. The effect of the variation in uncertainty of the input fuzzy variables on the reliability is studied. It is noted that the results obtained using both the methods of fuzzy reliability analysis are comparable. Since both the methods give same results, any of the methods can be used depending upon their applicability.***
***Keywords: fuzzy reliability, possibilistic reliability, failure possibility, reliability analysis.***

## I. INTRODUCTION

The evaluation of safety of structures is a task of much importance since the performance of the structure is assessed by its safety, service ability and economy. The safety of a structure depends on resistance, R of the structure and the action, S (load/ load effect) on the structure. The resistance or response of the structure depends on physical properties of material and geometrical properties of the structure and the action on the structure is a function of various types of loads like dead load, live load, wind load, etc. There will be inherent variations in the physical properties of materials, loads, occurrence of natural phenomena, etc. Thus the information about input variables is never certain, precise and complete. In the presence of these uncertainties, ensuring absolute safety of a structure is near to impossible. However, in the majority of texts and courses, the uncertainty is relegated to a minor position and the reason for this negligence ranges from the point it is easier to the fact that it is somewhat difficult to embrace uncertainty easily and directly. Till 1960, no serious attempt was made to consider explicitly the randomness of variables in the analysis, design and evaluation of safety; even though it was known that the above parameters are random. Later, engineers and research workers started realizing the need for the evaluation of safety taking into account the uncertainty in variables.

## II. UNCERTAINTIES IN STRUCTURAL ENGINEERING

There are ranges of uncertainties that arise in structural engineering and in the case of real structures, uncertainties may be due to intrinsic variability of mechanical and physical properties as well as to the lack of knowledge when information are scarce and/or subject to some errors, for instance related to the test procedure. Uncertainties can be broadly classified into two categories namely aleatory and epistemic uncertainties [12], which must be treated in different ways. Depending on the type of uncertainty an appropriate theory should be used to model and process the same in engineering decision making. Although probability theory has been traditionally used to represent both types of uncertainties, various researchers have pointed out that it may not be proper to use probability theory to represent epistemic uncertainty in the presence of limited knowledge. A number of alternative theories, such as fuzzy set theory, evidence theory, convex modelling, and interval analysis, for modelling uncertainties have been proposed by various researchers. It is to be noted that selection of a particular uncertainty handling method depends upon the type and source of uncertainty.

## III. CONCEPT OF RELIABILITY

### *A. Definition of reliability*

The concept of reliability has been applied to many fields and has been interpreted in many ways. The most common definition is that "reliability is the probability of an item performing its intended function over a given period of time under operating conditions encountered" [9].

Reliability is always estimated corresponding to a performance or limit state function. Commonly used limit state functions can be broadly divided into two groups; the serviceability and the strength limit states. The probabilistic approach describes the resistances and the loads as random because their values are not perfectly known. Statistics is used to obtain, from the available set of data or measurements, parameters which define the occurrence properties of variables which are random in nature; probability converts these information to occurrence functions (probability density functions-PDFs and cumulative density functions-CDFs) and defines the general framework for reliability analysis, whose main objective is to obtain failure probabilities of the structural system response, which are compared with limit values to assess the reliability of the structure. Greater the importance of the structure to be designed, greater the demand of assuring small probability of failure.

## IV. FUZZY METHOD OF RELIABILITY ANALYSIS

An attempt has been made to perform reliability analysis in the framework of fuzzy set and possibility theory. Fuzzy numbers are used to define an equivalence class of probability distributions compatible with available data and corresponding upper and lower cumulative density functions and to consider the variables involved in the civil engineering problems as fuzzy numbers. The method to carry out fuzzy reliability analysis mainly based on the formulations proposed by Berbara Ferracuti and Isaac Elishak off [1]. The methodology is explained below;

Consider basic problem of strength of material in which a structural component subjected to load (action),$P_S$ and the strength (resistance) of material of the component is $P_R$. For safety of the component, load$P_S$ on the component should be less than strength $P_R$. In the simplest approximation it is considered the load as a fuzzy number $P\hat{s}$ through membership function $\mu_S$, while the strength $P_R$ may still be considered as a deterministic quantity $P_r$. Necessity and possibility of an event that the fuzzy stress satisfies inequality $P\hat{s} \le P_r$ represent the lower and upper bounds of the probability of the event [8]. Since the probability that the action less than the resistance R is reliability, we get,

$$N(P\hat{s} \le P_r) \le R_0 \le \Pi(P\hat{s} \le P_r) \quad \ldots(1)$$

In other words, possibility and necessity of fulfilling the inequality $P\hat{s} \le P_r$ are upper and lower bounds of reliability respectively, i.e.,

$$R_* = N(P\hat{s} \le P_r)$$
$$R^* = \Pi(P\hat{s} \le P_r) \quad \ldots(2)$$

Fig. 1 depicts the necessity and possibility functions that satisfy the inequality $P\hat{s} \le P_r$ for a triangular fuzzy load, at $P = P_r$, necessity and possibility represent the lower and upper bounds of reliability. In other words, the upper and lower bound of reliability are the value of the possibility and necessity function at z = 0. The upper bound of reliability is not of much importance. The lower bound of reliability may be called as fuzzy reliability.

The possibility of failure then can be obtained as,

$$\Pi_f = \Pi(P\hat{s} > P_r)$$

$$\Pi_f = 1 - N(P\hat{s} \le P_r)$$

$$\Pi_f = 1 - R_* \quad \ldots(3)$$

The procedure for fuzzy set theory based reliability analysis can be represented by flowchart shown in Fig. 2.

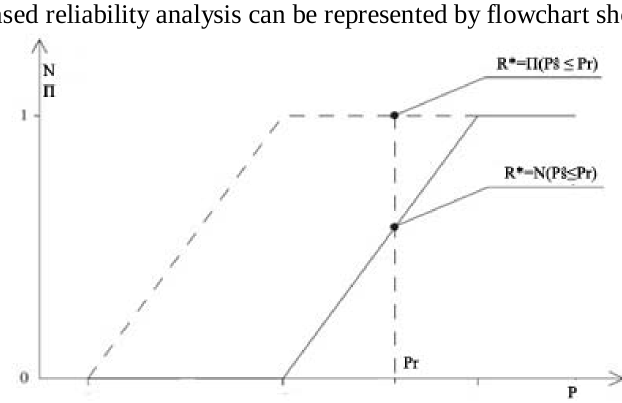


Fig. 1. Possibility and necessity functions that satisfy the inequality Σ ≤ σ for triangular fuzzy stress.

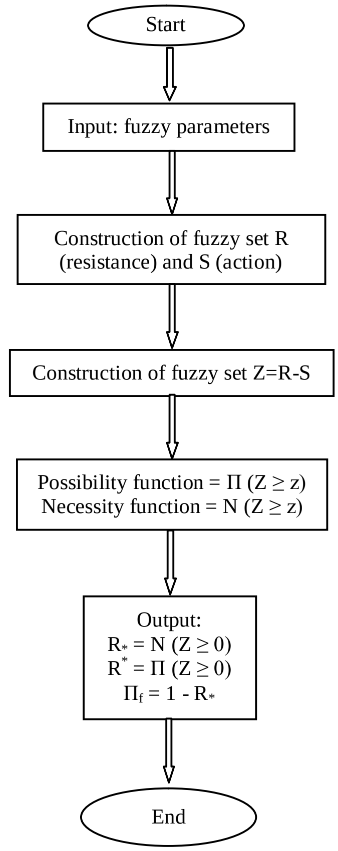


Fig. 2. Flowchart for fuzzy method of reliability analysis

## V. POSSIBILITY METHOD OF RELIABILITY ANALYSIS

The method of possibilistic reliability analysis mainly based on the methodology proposed by C. Cremona and Y. Gao [3]. This method is an original alternative to the probabilistic reliability theory, keeping the same features regarding some theoretical concepts like design points, failure probability, reliability indeces.

Let us consider a limit state function given by Equation (4), composed of non-interactive L-R fuzzy variables $X_i$. Like in probabilistic reliability theory, in this approach also the initial space composed of fuzzy numbers $X_i$ is transformed into a particular space composed of special class of non-interactive fuzzy numbers expressed as L-R fuzzy intervals, generally Gaussian fuzzy numbers.

$$g(X_1, X_2, \dots, X_n) = 0 \qquad \dots (4)$$

The possibilistic approach will try to evaluate safety in terms of possibility of failure, $\Pi_f$ which is defined as the possibility that the value of limit state equation less than or equal to zero. In other words, failure possibility is equal to the degree of possibility of the limit state function g({X}) at the value equal to zero. Mathematically, it is expressed as,

$$\Pi_f = \Pi(g(\{X\}) \leq 0) \qquad \dots (5)$$

Now, it is necessary to transform the set of fuzzy variables, {X} into set of fuzzy Gaussian numbers, {U} using τ-transform and the τ-transform must verify the following property;

$$\pi_X(x) = \pi_U\big(\tau_X(x)\big) \qquad \dots (6)$$

The τ-transform is invertible. The transformation for each variable becomes,

$$\left.\begin{aligned} \tau_{xi}(x_i) &= u_i = (L^*)^{-1}\big(L(x_i)\big) && \text{if } x_i < m_{xi} \\ \tau_{xi}(x_i) &= u_i = 0 && \text{if } m_{xi} \leq x_i \leq n_{xi} \\ \tau_{xi}(x_i) &= u_i = (R^*)^{-1}\big(R(x_i)\big) && \text{if } x_i > n_{xi} \end{aligned}\right\} \qquad \dots (7)$$

The inverse transformation can be written as follows;

$$\left.\begin{aligned} \tau_{xi}^{-1}(u_i) &= x_i = m_{xi} - \alpha_{Lxi} L_i^{-1}\left(L^*(u_i)\right) && \text{if } u_i < 0 \\ \tau_{xi}^{-1}(u_i) &= x_i = x_i^* && \text{if } u_i = 0 \\ \tau_{xi}^{-1}(u_i) &= x_i = n_{xi} + \alpha_{Rxi} R_i^{-1}\left(R^*(u_i)\right) && \text{if } u_i > 0 \end{aligned}\right\} \quad \dots (8)$$

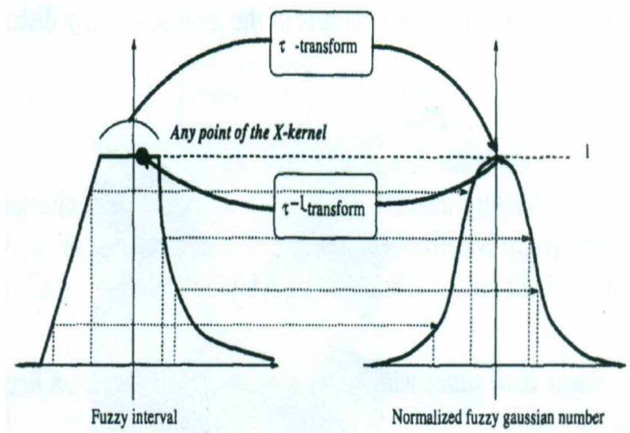


Fig. 3. τ-transform of fuzzy interval into normalized fuzzy Gaussian numbers

Once all the variables are transformed, the limit state function can be expressed in terms of fuzzy Gaussian numbers;

$$g(\{X\}) = \left(g\left(\tau^{-1}(\{U\})\right)\right) = g_U(\{U\}) \quad \dots (9)$$

With the help of rule of signs, choosing appropriate branch of each variable in the construction of limit state function $g_U$ and using inverse transformation given in Equation (8) in Equation (9), the limit state function can be written as;

$g_U(U)=g(\dots, m_{1i}-\alpha_{1Li}L_{1i}^{-1}(L^*(U)),\dots, n_{2j}+\alpha_{2Rj}R_{2j}^{-}(R^*(U)),\dots))$

Since $R^*(-U) = L^*(U)$ for fuzzy Gaussian numbers, we can write

$g_U(U)=g(\dots,m_{1i}-\alpha_{1Li}L_{1i}^{-1}(L^*(U)),\dots, n_{2j}+\alpha_{2Rj}R_{2j}^{-1}(L^*(U)),\dots)$

$g_V(V) = g(\dots, m_{1i}-\alpha_{1Li}\ L_{1i}^{-1}(V),\dots, n_{2j} + \alpha_{2Rj}\ R_{2j}^{-1}(V),\dots)$

The possibility of failure,

$\Pi_f = \Pi(g_U(\{U\}) \le 0)$

max (V) with $g_V(V) = 0$ and $V < 1$

which gives $\Pi_f = V$

The above solution scheme is made assuming the failure possibility less than one. Therefore, it is necessary to check initially if the failure possibility is smaller than one. For determining whether the failure possibility is less than one, it is sufficient to check

$$g(\dots, m_{xi}, \dots, n_{xj}, \dots) \le 0 \quad \dots (10)$$

If the condition given in Equation (10) satisfies, possibility of failure is equal to one; otherwise failure possibility is less than one. The possibilistic reliability index denoted by λ is the solution of the minimization problem expressed as follows;

$\lambda = \min(\|U_\infty\|)$ according to: $g_U(U) = 0$, and $|U_i| = |U_j|, \forall i,j.$

$\lambda = (L^*)^{-1}(\Pi_f) = \sqrt{-\ln(\Pi_f)}$ .

It should be noted that λ takes its values in $R^+$. A possibility of failure equal to one corresponds to a reliability index of zero. Once we get the failure possibility, possibilistic reliability may be obtained as,

$R_0 = 1 - \Pi_f$

The procedure for fuzzy reliability analysis by possibilistic reliability theory can be represented by flowchart shown in Fig. 4.

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## VI. COMPARISION BETWEEN TWO METHODS OF RELIABILITY ANALYSIS

Comparison between the two methods for reliability analysis with their advantages and limitations are given in Table I. The two methods discussed are capable of estimating reliability of any component and can be used with their own applicability since both the methods have their own advantages and limitations. The possibility of failure and fuzzy reliability index can be obtained in the possibilistic method. The bounds of reliability can be obtained in the fuzzy set theory method. The computations involved in this method are comparatively simpler, and gives better results.

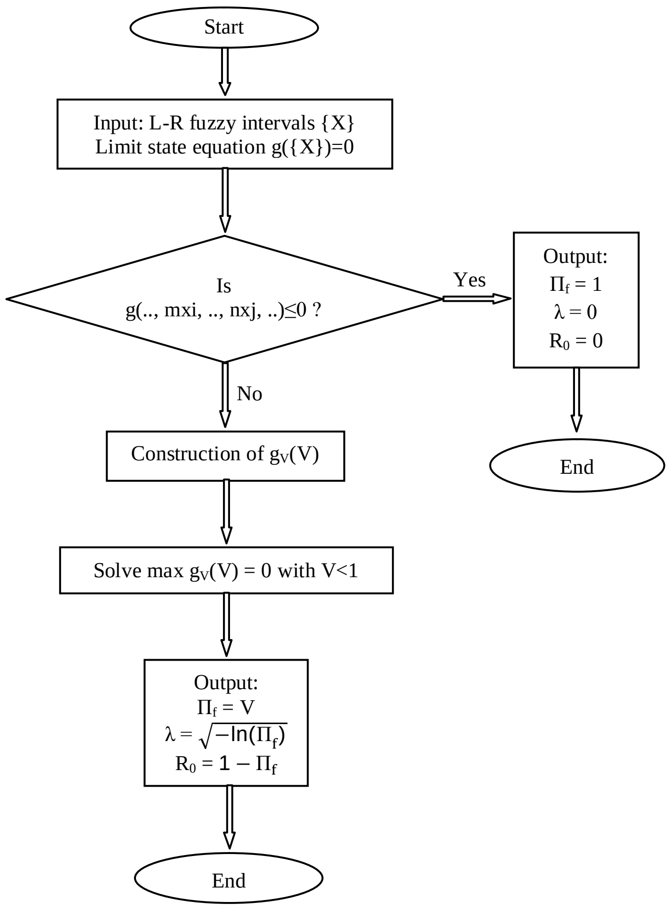


Fig. 4. Flowchart for possibilistic method of reliability analysis

### A. *Effect of variation of uncertainty on reliability*

Variation in reliability when the fuzziness in the variables varied is studied. The fuzzy reliability and possibilistic reliability are estimated by varying the support of the fuzzy set for resistance (R) keeping the action on the structure deterministic. The plot of reliability values against variation in resistance is given in fig. 5.

The performances of the two methods for reliability analysis are studied in the next section through the reliability analysis of an axially loaded single pile.

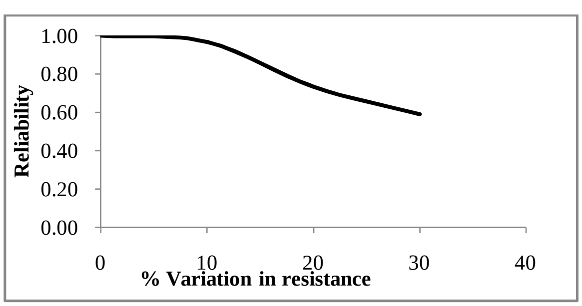


Fig. 5. Plot of reliability values against variation in resistance

 

TABLE I.
COMPARISON OF TWO METHODS OF FUZZY RELIABILITY ANALYSIS

| Sl. no. | Aspect considered | Fuzzy set theory method | Possibilistic method |
|---|---|---|---|
| 1 | Possibility of failure | $\Pi_f = 1 - R_*$ | $\Pi_f = \Pi(Z \leq 0)$ |
| 2 | Reliability | Reliability bounds;<br>$R_* = N(Z \geq 0)$<br>$R^* = \Pi(Z \geq 0)$ | Reliability;<br>$R_0 = 1 - \Pi_f$<br>Possibilistic reliability index;<br>$\lambda = \sqrt{-\ln \Pi_f}$ |
| 3 | Advantages | ▪ Bounds of reliability are defined.<br>▪ The method is simpler and computations involved are easier.<br>▪ The method can also be adopted when solution is not available in explicit/closed form to compute the parameters considered in performance function.<br>▪ The method can also be adopted when the membership function of fuzzy parameters are not defined by any function. | ▪ The features are similar to probabilistic one and much easier in implementations.<br>▪ The method involves less computation.<br>▪ The safety index – possibilistic reliability index can be obtained.<br>▪ The possibilistic reliability index is an invariant. |
| 4 | Limitations | ▪ Even though the computations are easier, it may become time consuming if more number of variables involved in limit state function because more computation will be involved.<br>▪ The safety index cannot be obtained in this method. | ▪ Applicable only for non-interactive fuzzy variables.<br>▪ Computations may be lesser but solving the equation $g_V(V)=0$ may become difficult if limit state function is non-linear with more number of variables.<br>▪ The results will be less accurate if closed form solution is not available in finding the parameters considered in the limit state equation.<br>▪ The method cannot be adopted when the membership function for input variables is not defined.<br>▪ If a variable appears more than once in limit state function, the possibility of failure will be over-estimated when the repeated variable is duplicated. |

## VII. ILLUSTRATION OF THE METHODS OF RELIABILITY ANALYSIS

Uncertainty associated with single pile-soil system is considered. Reliability analysis of an axially loaded single pile, with soil properties considered as fuzzy parameters, is performed using both the methods explained above. The possibilistic and fuzzy reliabilities against different limit states are determined.

### *A. Description of the problem*

A single pile-soil system as shown in Fig. 6, is considered for the analysis. The pile is made of concrete with circular cross section subjected to an axial compressive load at the pile head. The soil profile has two sand layers; upper layer consists of loose sand and lower layer is dense sand-silt stratum. he geometrical and material properties of the pile, load, soil properties and the variations in load, pile properties are adopted from [6] and the variations of soil properties are considered as per [5].

### *B. Pile properties and load data:*

The considered fuzzy variables in the analysis are given in Table 2 and deterministic parameters are given below;

 

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Length of the pile: L = 15 m.
Diameter of the pile: D = 30 cms.

*C. Soil properties*

The soil characteristics and its behaviour can be modelled using load transfer curves; q-z and f-z (or t-z) as shown in Fig. 7 where q is unit tip resistance mobilized at tip settlement z and f is unit frictional resistance mobilized at pile shaft settlement z. The variations of these properties are assumed to be constant in each layer. The COV of the central values, spread and range of COV of soil properties are given in Table III.

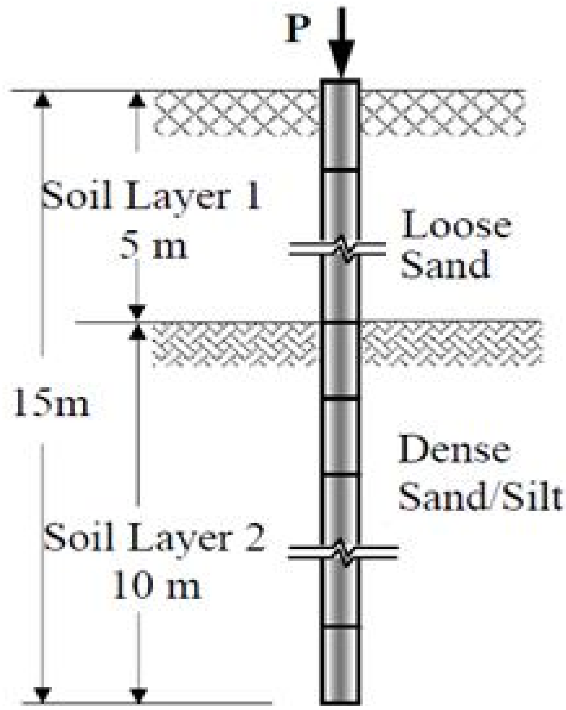


Fig. 6. Axially loaded single pile

From Fig. 7,
Critical pile tip movement $z_t$=3.0 cms.
Critical pile shaft movement $z_s$=0.25 cms in soil layers 1 and 2.
The q-z curve may be expressed as [10];

$$q = \begin{cases} q_{max}\left(\frac{z}{z_t}\right)^{1/3} & \text{for } z \le z_t \\ q_{max} & \text{for } z > z_t \end{cases} \quad \text{... (11)}$$

The t-z curve for the given problem, as can be seen in Fig. 7, is linear and for calculation purpose, it may be expressed as [10];

$$f = \begin{cases} f_{max}\left(\frac{z}{z_s}\right) & \text{for } z \le z_s \\ f_{max} & \text{for } z > z_s \end{cases} \quad \text{... (12)}$$

where,
f = unit shaft resistance at any pile shaft movement, z.
$f_{max}$ = unit shaft resistance mobilized at critical pile shaft movement, $z_{s.}$
q = unit base resistance for any pile tip movement, z.
$q_{max}$ = unit base resistance for critical pile tip movement, $z_{t.}$.

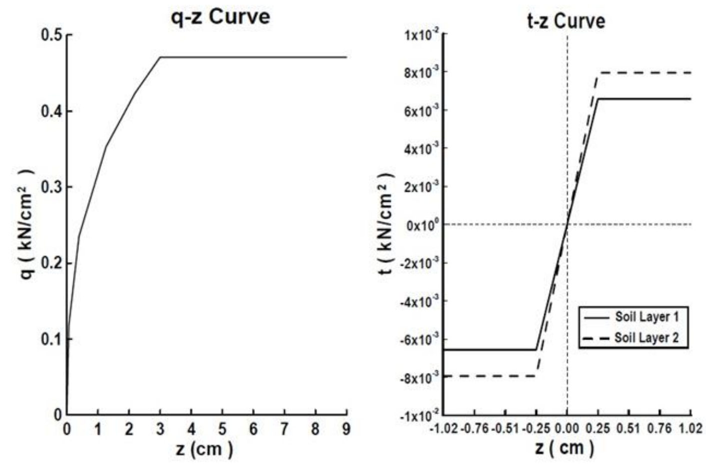


Fig. 7. q-z curve for layer 2 and t-z curves for layer 1 and 2

TABLE II.
PILE PROPERTIES AND LOAD DATA

| Parameters involved | Central value ($m_x$) | COV ($\delta_x$) | Variation ($s_x$) | Range ($\alpha_x$=3* $s_x$) | Type of variable |
|---|---|---|---|---|---|
| Young's modulus of concrete (E, kN/cm$^2$) | 2462 | 6.0 | 147.72 | 443.16 | Triangular fuzzy set |
| Compressive strength of pile ($\sigma_{ck}$, kN/cm$^2$) | 2.746 | 10.0 | 0.2746 | 0.8238 | Triangular fuzzy set |
| Cross section area of the pile (A, cm$^2$) | 706.90 | 5.0 | 35.345 | 106.035 | Triangular fuzzy set |
| Axial load at the pile head (P, kN) | 800 | 15.0 | 120 | 360 | Triangular fuzzy set |

### *D. Limit state functions*

A pile-soil system can fail by excessive vertical movement of pile shaft, representing serviceability failure and also it can fail in strength. Each limit state needs to be considered separately. In the present study, three limit state functions for an axially loaded pile-soil system are considered;

### *E. Pile strength limit state*

The strength parameters involved in pile compressive strength limit state are compressive strength of the concrete, $\sigma_{ck}$ and cross sectional area, A of the pile and the axial compressive load, P is the action on the pile.
The limit state function is given by Equation (17) and all the three variables are considered as triangular fuzzy numbers.

$$g_\sigma = \sigma_{ck} - \frac{P}{A} \qquad \text{... (13)}$$

### *F. Excessive settlement limit state*

The limit state function in excessive vertical displacement at top of the pile is given by,

$$g_z = z_{allow} - z \qquad \text{... (14)}$$

Where $z_{allow}$ is the specified vertical displacement at top of the pile which is assumed to be 1.00 cm and z is the settlement at the top of the pile under applied load.

### *G. Soil resistance limit state*

The soil resistance limit state function is given by Equation (15).

$$g_Q = (A_b q_{max} + A_s f_{max}) - (Q_p + Q_s) \qquad \text{... (15)}$$

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$q_{max}$ is unit base resistance mobilized at critical pile tip movement.
$f_{max}$ is unit shaft resistance mobilized at critical pile shaft movement.
$A_b$ is area of the pile base.
$A_s$ is circumferential area of the pile.
$Q_p$ and $Q_s$ are tip and friction reaction forces of the pile due to applied load.

TABLE III.
DATA OF SOIL PROPERTIES

| Soil properties | $m_x$ | $\delta_x$ | $\delta(m_x)$ | $\delta(s_x)$ | $\alpha_x$ | Type of variable |
|---|---|---|---|---|---|---|
| Unit tip resistance ($q_{max}$, kN/cm$^2$) | 0.4707 | 20 | 9 | 3 | 0.3078 | Trapezoidal fuzzy set |
| Unit frictional resistance in layer 1 ($f_{max}$, kN/cm$^2$) | $6.571\times10^{-3}$ | 15 | 9 | 3 | $3.223\times10^{-3}$ | Trapezoidal fuzzy set |
| Unit frictional resistance in layer 2 ($f_{max}$, kN/cm$^2$) | $7.944\times10^{-3}$ | 15 | 9 | 3 | $3.897\times10^{-3}$ | Trapezoidal fuzzy set |

For the reliability analysis in settlement and soil resistance limit states, the sectional area, material properties of pile and load are considered to be deterministic since they are not of much importance compared to the soil properties.
Prior to reliability analysis, the pile analysis is carried out to get the fuzzy sets for pile head settlement, reactive forces developed due to applied axial load, P=800 kN using load-transfer method[7]. Then fuzzy reliability analysis is performed using the two methods explained in the previous sections.

### *H. Results obtained for pile analysis using load-transfer method*

The fuzzy sets obtained from load-transfer method for settlement z and reactive forces $Q_p$ and $Q_s$ are shown in Fig. 8, 9 and 10.

### *I. Results of reliability analysis of pile using possibilistic and fuzzy method*

The results of reliability analysis of the single pile subjected to axial compressive load in three limit states considered are given in Table IV, it is noted that the reliabilities estimated using both the methods are same for all the three limit states considered. Reliability of pile against settlement limit state is 1.00 and also the safety index, λ is high, suggesting that the pile is safe against failure due to vertical settlement criteria. Reliability of the pile against pile strength limit state is also high compared to the reliability against soil resistance limit state.

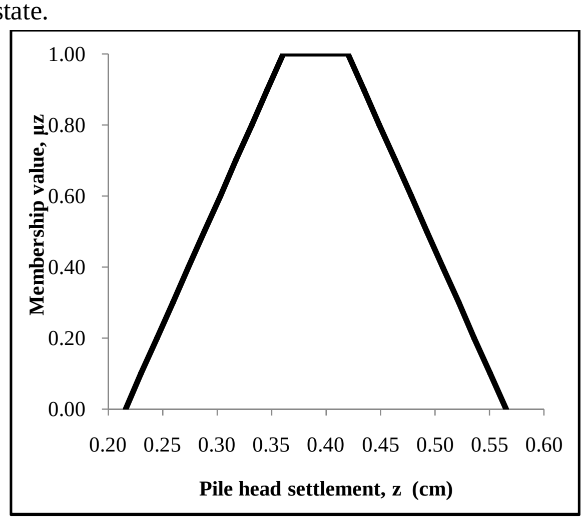


Fig. 8. Fuzzy set of pile settlement, z at top

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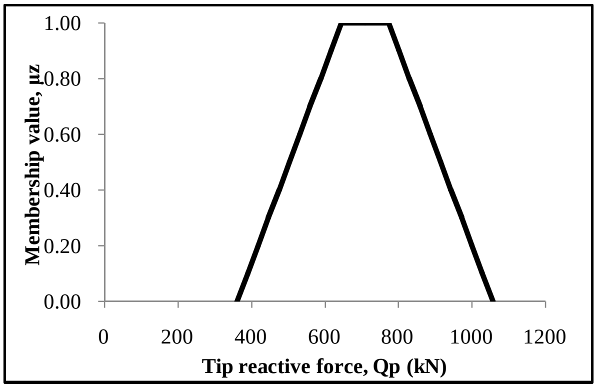


Fig. 9. Fuzzy set of reactive force, Qp at pile tip

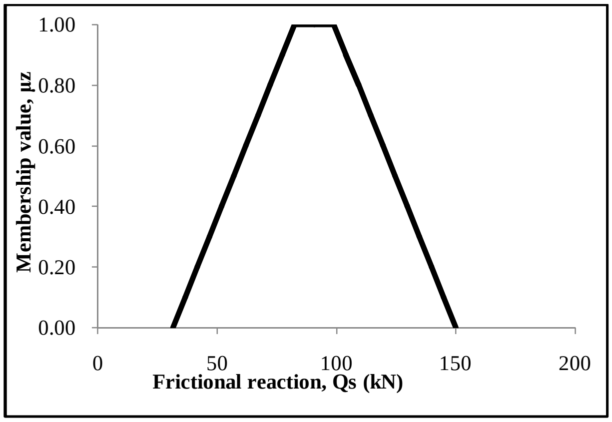


Fig. 10. Fuzzy set of frictional reaction, Qs

TABLE IV.
RESULTS OF FUZZY RELIABILITY ANALYSIS OF AXIALLY LOADED SINGLE PILE

| | Results of Fuzzy set theory method | | | Results of Possibilistic method | | |
|---|---|---|---|---|---|---|
| Limit state | $R^*$ | $R_*$ | $\Pi_f$ | $\Pi_f$ | $R_0$ | $\lambda$ |
| Pile strength | 1.00 | 0.995 | 0.00478 | 0.005 | 0.995 | 2.314 |
| Settlement | 1.00 | 1.000 | 0.00000 | 0.000 | 1.000 | Indeterminate |
| Soil resistance | 1.00 | 0.416 | 0.58396 | 0.584 | 0.416 | 0.733 |

## VIII. CONCLUDING SUMMARY

An attempt has been made to perform reliability analyses in the frameworks of possibility- and fuzzy set- theory. The reliability analyses based on the two methods are illustrated with a numerical example of axially loaded single pile. From the results obtained, it is noted that the reliability values obtained from both the methods are more or less the same, for all the three limit states considered. However, the computations involved in fuzzy set theory method are comparatively simpler. Also, by using this method, the bounds of reliability can be obtained. . Hence, the fuzzy set theory method is recommended for carrying out fuzzy reliability analysis of structural components.

## IX. ACKNOWLEDGEMENT

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