iJRASET
International Journal For Research in Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 Issue: II Month of publication: February 2018

DOI:

Closed Form Wave Solutions to the Nonlinear Partial Differential Equations via the Rational (G'/G)-Expansion Method

Md. Tarikul Islam[1], Md. Arshad Ali[2], Md. Rakibul Hasan[3]
[1, 3] Department of Mathematics, Hajee Mohammad Danesh Science and Technology University, Dinajpur, Bangladesh
[2]Department of Computer Science and Engineering, Hajee Mohammad Danesh Science and Technology University, Dinajpur, Bangladesh

Abstract*: In this article, we search the closed form traveling wave solutions of nonlinear partial differential equations such as the compound KdV-Burgers equation and the compound KdV equation by the rational (G'/G) -expansion method. The considered equations are converted into ordinary differential equations by a suitable composite transformation and then the method is applied to investigate the solutions. The suggested method provides three types exact traveling wave solutions; namely the hyperbolic function solution, the trigonometric function solution and the rational function solution which are new and more general than the existing results in the literature. This method is more reliable and efficient to construct new and general exact solutions.

Keywords: Rational (G'/G) -expansion method, Compound KdV-Burgers equation, Compound KdV equation, Exact solution.

I. INTRODUCTION

Nonlinear partial differential equations (NPDEs) are widely used to describe complex phenomena in various fields including either the scientific works or engineering fields, such as fluid mechanics, chemical physics, chemichel kinematics, plasma physics, elastic media, optical fibers, solid state physics, biology, atmospheric and oceanic phenomena and so on. The investigation of the traveling wave solutions of some nonlinear partial differential equations (NPDEs) derived from such fields plays an important role. Certain special form solutions to NPDEs may depend only on a single combination of variables such as solitons. A soliton is a self-reinforcing solitary wave, a wave packet or pulse that upholds its profile while it travels at constant speed. In the past years, a good number of researchers use various methods for finding explicit solutions of NPDEs. To investigate the special solutions of NPDEs, many powerful and direct methods have been established and developed [1-21]. The Chinese Mathematician Wang et al. [22] first proposed (G'/G) -expansion method by which the traveling wave solutions of the nonlinear partial differential equations (NPDEs) are obtained. Making use of this method, some useful equations are also studied in Refs. [23-27]. In the recent years, further researchers have modified (G'/G) -expansion method variously and found more new and general traveling wave solutions [28-35].In this article, we employ the rational (G'/G) -expansion method to examine the closed form traveling wave solutions for the compound KdV-Burgers equation and the compound KdV equation and obtain many new and more general the closed form traveling wave solutions.

II. DESCRIPTION OF THE METHOD

Consider the following nonlinear partial differential equation in two independent variables x and t :

$$P(u,u_x,u_t,u_{xx},u_{tt},u_{tx},...)=0\,, \tag{1}$$

where $u=u(x,t)$ is an unknown function, P is a polynomial in $u=u(x,t)$ and its various partial derivatives. The followings are the main steps of the method.

Step 1. Use the traveling wave transformation:

$$u=u(x,t),\ \xi = x \pm vt\,, \tag{2}$$

where v is the traveling wave speed. With the help of Eq. (2), the Eq. (1) becomes an ordinary differential equation as

$$Q(u,u',u'',u''',...)=0 \tag{3}$$

containing $u(\xi)$ and its various derivatives. The prime denotes the order of derivative with respect to ξ .

Step 2. For convenience, integrate Eq. (3) one or more times and integral constant can be set to zero.

Step 3. Suppose that the solution of Eq. (3) can be expressed in (G'/G) as follows:

$$u(\xi)=\frac{\sum_{i=0}^{n} a_i (G'/G)^i}{\sum_{i=0}^{n} b_i (G'/G)^i}, \tag{4}$$

where a_n and b_n are non-zero real constants to be determined later and $G=G(\xi)$ satisfies the following second order liner ODE:

$$G''(\xi)+\lambda G'(\xi)+\mu G(\xi)=0, \tag{5}$$

where λ and μ are real constants. Eq. (5) can be rearranged into

$$\frac{d}{d\xi}(G'/G)=-(G'/G)^2-\lambda(G'/G)-\mu . \tag{6}$$

Eq. (5) (or equivalent to Eq. (6)) possesses the following general solutions:

$$\left(\frac{G'}{G}\right)=\begin{cases} -\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{A\sinh((\sqrt{\lambda^2-4\mu}/2)\xi)+B\cosh((\sqrt{\lambda^2-4\mu}/2)\xi)}{A\cosh((\sqrt{\lambda^2-4\mu}/2)\xi)+B\sinh((\sqrt{\lambda^2-4\mu}/2)\xi)}\right), & \lambda^2-4\mu>0, \\ -\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-A\sin((\sqrt{4\mu-\lambda^2}/2)\xi)+B\cos((\sqrt{4\mu-\lambda^2}/2)\xi)}{A\cos((\sqrt{4\mu-\lambda^2}/2)\xi)+B\sin((\sqrt{4\mu-\lambda^2}/2)\xi)}\right), & \lambda^2-4\mu<0, \\ -\frac{\lambda}{2}+\frac{B}{A+B\xi}, & \lambda^2-4\mu=0, \end{cases} \tag{7}$$

where A and B are arbitrary constants.

Step 4: To determine the positive integer n , substitute (4) along with (5) into Eq. (3) and take the homogeneous balance between the highest order derivatives and the highest order nonlinear terms appearing in (3). If the degree of $u(\xi)$ is $\deg[u(\xi)]=n$, therefore, the degree of the other expressions will be as follows:

$$\deg[\frac{d^m u(\xi)}{d\xi^m}]=n+m,\ \deg[u^m(\frac{d^l u(\xi)}{d\xi^l})^p]=mn+p(n+l).$$

Step 5: Use Eq. (4) along with Eq. (5) into Eq. (3) with the value of n obtained in step 4. This substitution forms a polynomial of (G'/G). Equating the coefficients of (G'/G) and set to zero. This procedure yields a system of algebraic equations which can be solved for getting a_i, b_i, λ, μ and v and the value of the other needful parameters.

Step 6: We substitute the values of a_i, b_i, λ, μ and v together with the solutions given in Eq. (7) into Eq. (4). This completes the determination of the solutions to the nonlinear evolution equation (1).

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 6 Issue II, February 2018- Available at www.ijraset.com

III. APPLICATION OF THE METHOD

In this section, we apply the rational (G'/G)-expansion method to construct the closed form traveling wave solutions of the compound KdV-Burgers equation and the compound KdV equation.

A. The Compound KdV-Burgers Equation

Consider the Compound KdV-Burgers Equation

$$u_t + puu_x + qu^2u_x + ru_{xx} - su_{xxx} = 0, \tag{8}$$

where p, q, r and s are arbitrary constants.

The traveling wave transformation $u = u(x,t),\ \xi = x - vt$ reduce Eq. (8) to the ODE

$$-vu' + puu' + qu^2u' + ru'' - su''' = 0, \tag{9}$$

which under the integration becomes

$$-vu + \frac{p}{2}u^2 + \frac{q}{3}u^3 + ru' - su'' + c = 0, \tag{10}$$

with integral constant c. Balancing the terms u^3 and u'' in Eq. (10), we obtain $n = 1$. Then the solution Eq. (4) takes the form

$$u(\xi) = \frac{a_0 + a_1(G'/G)}{b_0 + b_1(G'/G)}, \tag{11}$$

Substituting Eq. (11) into Eq. (10), the left hand side of Eq. (10) becomes a polynomial in (G'/G). Setting each coefficient of this polynomial to zero, we obtain an over determined set of algebraic equations (for simplicity, we will omit them to display) for a_0, b_0, a_1, b_1, v and c. Solving this set of equations by using the symbolic computation software, such as Maple, we obtain the following set of solutions:

$$a_0 = \pm\frac{\sqrt{6}b_1 s\mu}{\sqrt{q}},\ a_1 = \pm\frac{b_1\sqrt{6q}(3s\lambda - r) - 3b_1 p\sqrt{s}}{6q\sqrt{s}},\ v = \frac{6qs^2\lambda^2 + 2qr^2 - 3p^2s - 24s^2q\mu}{12qs},$$

$$b_0 = 0, c = -\frac{pr^2}{12qs} + \frac{ps\mu}{q} + \frac{p^3}{24q^2} - \frac{ps\lambda^2}{4q} \mp \frac{rs\lambda^2}{\sqrt{6qs}} \pm \frac{2\sqrt{6}sr\mu}{3\sqrt{q}} \pm \frac{\sqrt{6}r^3}{54\sqrt{qss}}, \tag{12}$$

where b_1, λ and μ are all arbitrary constants.

Now, substituting Eq. (12) into solution Eq. (11), we obtain

$$u(\xi) = \pm\left(\frac{3s\lambda - r}{\sqrt{6qs}} - \frac{p}{2q}\right) \pm \frac{\sqrt{6}s\mu}{\sqrt{q}}(G'/G)^{-1}, \tag{13}$$

where $\xi = x - vt$; λ and μ are all arbitrary constants.

Substituting Eq. (6) into Eq. (13) and simplifying we have the following exact traveling wave solutions:

When $\lambda^2 - 4\mu > 0$, we obtain the hyperbolic function solution

$$u_1(\xi)=\pm\left(\frac{3s\lambda-r}{\sqrt{6qs}}-\frac{p}{2q}\right)\pm\frac{\sqrt{6}s\mu}{\sqrt{q}}\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{A\sinh((\sqrt{\lambda^2-4\mu}/2)\xi)+B\cosh((\sqrt{\lambda^2-4\mu}/2)\xi)}{A\cosh((\sqrt{\lambda^2-4\mu}/2)\xi)+B\sinh((\sqrt{\lambda^2-4\mu}/2)\xi)}\right)\right)^{-1} \tag{14}$$

Since A and B are arbitrary constants, we may choose $A=r_1\cosh\theta,\ B=r_1\sinh\theta$ and obtain the solutions

$$u_1(\xi)=\pm\left(\frac{3s\lambda-r}{\sqrt{6qs}}-\frac{p}{2q}\right)\pm\frac{\sqrt{6}s\mu}{\sqrt{q}}\left(-\frac{\lambda}{2}+\frac{r_1\sqrt{\lambda^2-4\mu}}{2}\tanh((\sqrt{\lambda^2-4\mu}/2)\xi+\theta)\right)^{-1} \tag{15}$$

where $r_1=\sqrt{A^2-B^2}$, $\theta=\tanh^{-1}(B/A)$, $\xi=x-\dfrac{6qs^2\lambda^2+2qr^2-3p^2s-24s^2q\mu}{12qs}t$; λ and μ are all arbitrary constants.

When $\lambda^2-4\mu<0$, we obtain the trigonometric function solution

$$u_2(\xi)=\pm\left(\frac{3s\lambda-r}{\sqrt{6qs}}-\frac{p}{2q}\right)\pm\frac{\sqrt{6}s\mu}{\sqrt{q}}\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-A\sin((\sqrt{4\mu-\lambda^2}/2)\xi)+B\cos((\sqrt{4\mu-\lambda^2}/2)\xi)}{A\cos((\sqrt{4\mu-\lambda^2}/2)\xi)+B\sin((\sqrt{4\mu-\lambda^2}/2)\xi)}\right)\right)^{-1} \tag{16}$$

Since A and B are arbitrary constants, ifwe choose $A=r_2\cos\phi,\ B=r_2\sin\phi$ then after simplification we obtain the solution

$$u_2(\xi)=\pm\left(\frac{3s\lambda-r}{\sqrt{6qs}}-\frac{p}{2q}\right)\pm\frac{\sqrt{6}s\mu}{\sqrt{q}}\left(-\frac{\lambda}{2}-\frac{r_2\sqrt{4\mu-\lambda^2}}{2}\tan((\sqrt{4\mu-\lambda^2}/2)\xi-\phi)\right)^{-1} \tag{17}$$

where $r_2=\sqrt{A^2-B^2}$, $\phi=\tan^{-1}(B/A)$, $\xi=x-\dfrac{6qs^2\lambda^2+2qr^2-3p^2s-24s^2q\mu}{12qs}t$; λ and μ are all arbitrary constants.

When $\lambda^2-4\mu=0$, we obtain rational solution

$$u_3(\xi)=\pm\left(\frac{3s\lambda-r}{\sqrt{6qs}}-\frac{p}{2q}\right)\pm\frac{\sqrt{6}s\mu}{\sqrt{q}}\left(-\frac{\lambda}{2}+\frac{B}{A+B\xi}\right)^{-1}, \tag{18}$$

where $\xi=x-\dfrac{6qs^2\lambda^2+2qr^2-3p^2s-24s^2q\mu}{12qs}t$; λ and μ are all arbitrary constants.

The above obtained solutions to the compound KdV-Burgers equation are new and more general than the existing results in the literature.

B. The Compound KdV Equation

Now we construct the traveling wave solutions of the Compound KdV Equation

$$u_t+puu_x+qu^2u_x-su_{xxx}=0, \tag{19}$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 6 Issue II, February 2018- Available at www.ijraset.com

where p, q and s are non-zero real constants.

Making use of the traveling wave transformation $u = u(x,t),\ \xi = x - vt$ reduces Eq. (19) into the ODE

$$-vu' + puu' + qu^2u' - su''' = 0, \tag{20}$$

which becomes

$$-vu + \frac{p}{2}u^2 + \frac{q}{3}u^3 - su'' + c = 0, \tag{21}$$

under the integration with integral constant c. Balancing the terms u^3 and u'' in Eq. (21), we obtain $n = 1$. Then the solution Eq. (4) takes the form

$$u(\xi) = \frac{a_0 + a_1(G'/G)}{b_0 + b_1(G'/G)}, \tag{22}$$

Substituting Eq. (22) into Eq. (21), the left hand side of Eq. (21) becomes a polynomial in (G'/G). Setting each coefficient of this polynomial to zero, we obtain an over determined set of algebraic equations (for simplicity, we will omit them to display) for a_0, b_0, a_1, b_1, v and c. Solving this set of equations by using the symbolic computation software, such as Maple 13, we obtain the following set of solutions:

Set 1 $a_0 = \frac{b_1}{4q}\{-\lambda p \pm \sqrt{6qs}(\lambda^2 - 4\mu)\}$, $a_1 = -\frac{pb_1}{q}$, $b_0 = \frac{b_1\lambda}{2}$, $v = \frac{2qs\lambda^2 - p^2 - 8qs\mu}{4q}$,

$$c = \frac{p}{24q^2}(p^2 - 6qs\lambda^2 + 24qs\mu), \tag{23}$$

where b_0, b_1, λ and μ are all arbitrary constants.

Set 2 $a_0 = \frac{1}{2q}\{-pb_0 \pm \sqrt{6qs}(2b_1\mu - b_0\lambda)\}$, $a_1 = \frac{1}{2q}\{pb_1 \pm \sqrt{6qs}(b_1\lambda - 2b_0)\}$,

$$v = \frac{1}{4q}(2qs\lambda^2 - p^2 - 8qs\mu), c = \frac{p}{24q^2}(p^2 - 6qs\lambda^2 + 24qs\mu), \tag{24}$$

where b_0, b_1, λ and μ are all arbitrary constants.

Set 3 $a_0 = -\frac{a_1 b_0}{b_1}$, $c = -\frac{a_1}{6b_1^3}(2qa_1^2 - 6vb_1^2 + 3pa_1b_1)$, (25)

where a_1, b_0, b_1, v, λ and μ are all arbitrary constants.
Now, substituting Eq. (23) into solution Eq. (22), we obtain

$$u(\xi) = \frac{-p\lambda \pm \sqrt{6qs}(\lambda^2 - 4\mu) - 4p(G'/G)}{4q\{\lambda + 2(G'/G)\}} \tag{26}$$

Using Eq. (6) into Eq. (26), we can obtain the following exact traveling wave solutions to Eq. (19).

When $\lambda^2 - 4\mu > 0$, we obtain the hyperbolic function solution

$$u_1(\xi) = \frac{1}{4q}\frac{-p\lambda \pm \sqrt{6qs}(\lambda^2-4\mu) - 4p\left(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{A\sinh((\sqrt{\lambda^2-4\mu}/2)\xi) + B\cosh((\sqrt{\lambda^2-4\mu}/2)\xi)}{A\cosh((\sqrt{\lambda^2-4\mu}/2)\xi) + B\sinh((\sqrt{\lambda^2-4\mu}/2)\xi)}\right)\right)}{\lambda + 2\left(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{A\sinh((\sqrt{\lambda^2-4\mu}/2)\xi) + B\cosh((\sqrt{\lambda^2-4\mu}/2)\xi)}{A\cosh((\sqrt{\lambda^2-4\mu}/2)\xi) + B\sinh((\sqrt{\lambda^2-4\mu}/2)\xi)}\right)\right)} \quad (27)$$

Since A and B are arbitrary constants. We might choose $A = r_1\cosh\theta,\ B = r_1\sinh\theta$ and obtain the solutions

$$u_1(\xi) = \frac{1}{4q}\frac{-p\lambda \pm \sqrt{6qs}(\lambda^2-4\mu) - 2p\{-\lambda + r_1\sqrt{\lambda^2-4\mu}\tanh((\sqrt{\lambda^2-4\mu}/2)\xi+\theta)\}}{r_1\sqrt{\lambda^2-4\mu}\tanh((\sqrt{\lambda^2-4\mu}/2)\xi+\theta)} \quad (28)$$

where $r_1 = \sqrt{A^2 - B^2}$, $\theta = \tanh^{-1}(B/A)$, $\xi = x - \frac{2qs\lambda^2 - p^2 - 8qs\mu}{4q}t$; λ and μ are all arbitrary constants.

When $\lambda^2 - 4\mu < 0$, we obtain the trigonometric function solution

$$u_2(\xi) = \frac{1}{4q}\frac{-p\lambda \pm \sqrt{6qs}(\lambda^2-4\mu) - 4p\left(-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-A\sinh((\sqrt{4\mu-\lambda^2}/2)\xi) + B\cosh((\sqrt{4\mu-\lambda^2}/2)\xi)}{A\cosh((\sqrt{4\mu-\lambda^2}/2)\xi) + B\sinh((\sqrt{4\mu-\lambda^2}/2)\xi)}\right)\right)}{\lambda + 2\left(-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-A\sinh((\sqrt{4\mu-\lambda^2}/2)\xi) + B\cosh((\sqrt{4\mu-\lambda^2}/2)\xi)}{A\cosh((\sqrt{4\mu-\lambda^2}/2)\xi) + B\sinh((\sqrt{4\mu-\lambda^2}/2)\xi)}\right)\right)} \quad (29)$$

Since A and B are arbitrary constants. We might choose $A = r_2\cos\phi,\ B = r_2\sin\phi$ and obtain the solutions

$$u_2(\xi) = \frac{1}{4q}\frac{-p\lambda \pm \sqrt{6qs}(\lambda^2-4\mu) - 2p\{-\lambda - r_2\sqrt{4\mu-\lambda^2}\tan((\sqrt{4\mu-\lambda^2}/2)\xi-\phi)\}}{-r_2\sqrt{\lambda^2-4\mu}\tan((\sqrt{4\mu-\lambda^2}/2)\xi-\phi)} \quad (30)$$

where $r_2 = \sqrt{A^2 - B^2}$, $\phi = \tan^{-1}(B/A)$, $\xi = x - \frac{2qs\lambda^2 - p^2 - 8qs\mu}{4q}t$; λ and μ are all arbitrary constants.

When $\lambda^2 - 4\mu = 0$, we obtain the rational function solution

$$u_3(\xi) = \frac{1}{4q} \times \frac{-p\lambda - 4p\left(-\frac{\lambda}{2} + \frac{B}{A+B\xi}\right)}{\lambda + 2\left(-\frac{\lambda}{2} + \frac{B}{A+B\xi}\right)}, \quad (31)$$

where $\xi = x - \frac{2qs\lambda^2 - p^2 - 8qs\mu}{4q}t$; λ and μ are all arbitrary constants.

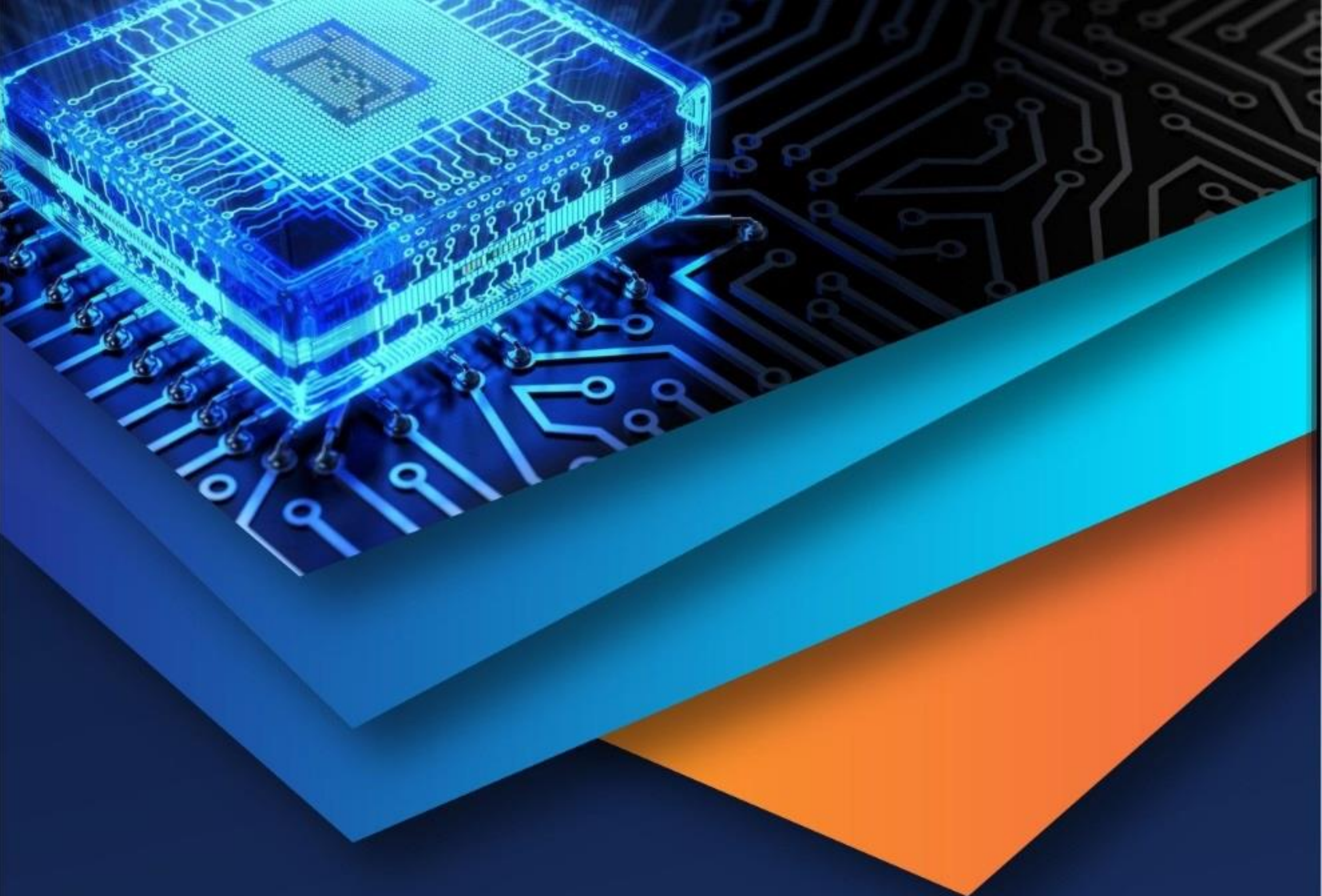
Using Eq.(6) into Eqs. (24) and (25) we also might construct much more new and general solutions to the compound KdV equation. For simplification and to avoid the annoyed of the readers these are not recorded here.

IV. CONCLUSION

In this article, the rational (G'/G) -expansion method has been applied to the compound KdV-Burgers equation and the compound KdV equation. Using this method, we have successfully found the traveling wave solutions in terms of hyperbolic, trigonometric and rational functions. On comparing the results throughout this article with those of [36], we see that, our results are more new and general. This shows that, the performance of this method is reliable, effective and giving more new and general solutions to many other nonlinear partial differential equations (NPDEs). Although the method is applied to only a small number (two) of nonlinear equations, it can be applied to many other equations.

REFERENCES

[1] M. Wadati, H. Shanuki and K. Konno, Prog. Theor. Phys. , **53**, 419, 1975
[2] M.J. Ablowitz and H. Segur, “Solitons and inverse scattering transform”, SIAM, Philadelphia, 1981
[3] C. Rogers and W.F. Shadwick, “Backlund transformations”, Academic Press, New York, 1982
[4] G.W. Bluman and Kumei, “Symmetries and differential equations”, Springer-Verlag, New York, 1989
[5] M.J. Ablowitz and P.A. Clarkson, “Solitons, nonlinear evolution equations and inverse scatteringtransform”, Cambridge Univ. Press, Cambridge, 1991
[6] V.B. Matveev and M.A. Salle, “Darboux transformation and solitons, Springer, Berlin, 1991
[7] G. Adomin, “Solving frontier problems of physics: The decomposition method”, Kluwer,Boston,1994
[8] M.L. Wang, “Exact solutions for a compound KdV- Burgers equation”, Phys. Lett. A, **213**, 279-287, 1996
[9] W. Malfliet and W. Hereman, “The tanh method I: exact solutions of nonlinear evolution and wave Equations”, Phys. Scr., **54**, 563-568, 1996
[10] C.T. Yan, “A simple transformation for nonlinear waves”, Phys. Lett. A, **224**, 77-84, 1996
[11] J.H. He, “The homotopy perturbation method for nonlinear ossilators with discontinuities”, Appl. Math. Comput., **151**, **287**-292, 2004
[12] Y. Yan, “An improved algebra method and its applications in nonlinear wave equations”, Chaos Solitons Fractals, **21**, 1013-1021, 2004
[13] R. Hirota, “The direct method in soliton theory”, Cambridge University Press, Cambridge, 200
[14] G.T. Liu and T.Y. Fan, “New applications of developed Jacobielliptic function expansion methods”, Phys. Lett. A, **345**, 161-166, 2005
[15] M.L. Wang and X.Z. Li, “Extended F-expansion method and periodic wave solutions for the generalized zakharov equations, Phys. Lett. A, **343**, 48- 54, 2005
[16] J.H. He and X.H. Wu, “Exp-function method for nonlinear wave equations”, Chaos Solitons Fractals, **30**, 700-708, 2006
[17] M.A. Abdou, “The extended F-expansion method and its applications for a class of nonlinear evolution equations”, Chaos Solitons Fractals, **31**, 95-104, 2007
[18] T. Ozisand I. Aslan, “Exact and explicit solutions to the (3+1)-dimensional JimboMiwa equation via Bexp-function method”, Phys. Lett. A, **372**, 7011-7015, 2008
[19] A. Yildirim and Z. Pinar, “Application of the exp-function method for solving nonlinear reaction-B diffusion equations arising in mathematical biology”, Computers and Mathematics with Applications **60**, 1873-1880, 2010
[20] M.A. Balci and A. Yildirim, “Analysis of Fractional Nonlinear Differential Equations Using the Homotopy Perturbation Method”, Z. Naturforsch. **66a**, 87-92, 2011
[21] M.L. Wang , “Solitary wave solutions for variant Boussinesq equations”, Phys. Lett. A, **199**169-172, 1995.
[22] M. Wang, X. Li and J. Zhang, “The (G'/G) -expansion method and traveling wave solutions of nonlinear evolution equations in mathematical physics”, Phys. Lett. A, 372, 417-423, 2008
[23] M.A. Akbar, N.H.M. Ali and E.M.E. Zayed, Commun. Theor. Phys. **57**,173, 2013
[24] M.A. Akbar, N.H.M. Ali and E.M.E. Zayed, “A generalized and improved (G'/G) -Expansion Method for nonlinear evolution equations”, Math. Prob. Engr. Article ID 459879 22 pages. doi: 10.1155/2012/459879, 2012
[25] M.A. Akbar, N.H.M. Ali and S.T. Mohyud-Din,“The alternative-expansion method with generalized riccati equation: Application to fifth order (1+1)-dimensional Caudrey-Dodd-Gibbon equation”,Int. J. Phys. Sci. **7,** 743-752, 2012
[26] M.A. Akbar, N.H.M. Ali and S.T. Mohyud-Din, “Some new exact traveling wave solutions to the (3+1)-dimensional kadomtsev-Petviashvili equation”, World Appl. Sci. J. **16**,1551-1558, 2012
[27] E.M.E. Zayed, “The (G'/G) -Expansion method and its applications to some nonlinear evolution equations in the mathematical physics”, J. Appl. Math. Comput. 30, 89-103, 2009
[28] J. Zhang, X. Wei and Y. Lu, “A generalized(G'/G)-expansion method and its application”, Phys. Lett. A **372**,3653-3658, 2008
[29] J. Zhang, F. Jiang and X. Zhao, “An improved (G'/G)-expansion method for solving nonlinear evolution equations”, Int. J, Com. Math. **87**,1716-1725, 2010
[30] E.M.E. Zayed,“New traveling wave solutions for higher dimensional nonlinear evolution equations using a generalized (G'/G)-expansion method”, J. Phys. A: Math. Theor. **42**,195202, 2009.
[31] E.M.E. Zayed, “The (G'/G)-expansion method combined with the riccati equation for finding exact
[32] solutions of nonlinear PDES”, J. Appl. Math. & Informatics **29**, 351-367, 2011.
[33] M.N. Alam, M.A. Akbar and S.T. Mohyud-Din, “A novel (G'/G)-expansion method and its
[34] application to the Boussinesq equation”, Chin. Phys. B, 2014.
[35] M.T. Islam, M.A. Akbar and A.K. Azad,“Traveling wave solutions to some nonlinear fractional partial differential equations through the rational (G′/G)-expansion Method”, J. Ocean. Eng. Sci., in press, 10.1016/j.joes. 2017.12.003, 2018



10.22214/IJRASET

INDEX COPERNICUS
45.98

IMPACT FACTOR:
7.129

TOGETHER WE REACH THE GOAL

IMPACT FACTOR:
7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY