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Fuzzy τ^* -Generalized Closed Sets in Fuzzy Topological Spaces

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Abstract: In this paper, we introduce a new class of sets called fuzzy τ^* -generalized closed sets and fuzzy τ^* -generalized open sets in fuzzy topological spaces and explore some of their properties.

Keywords: fuzzy closed set fuzzy open set, fuzzy τ^* -g-closed set, fuzzy τ^* -g-open set.

I. INTRODUCTION

Let X be a non-empty set and $I = [0,1]$. A fuzzy set on X is a mapping from X into I . The null fuzzy set 0 is the mapping from X into I which assumes only the value 0 and whole fuzzy set 1 is a mapping from X into I which takes the value 1 only. The union (resp. intersection) of a family $\{A_\alpha : \alpha \in \Lambda\}$ of fuzzy sets of X is defined by to be the mapping $\sup A_\alpha$ (resp. $\inf A_\alpha$). A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \leq B(x)$ for each $x \in X$. A fuzzy point x_β in X is a fuzzy set defined by $x_\beta(y) = \beta$ for $y = x$ and $x(y) = 0$ for $y \neq x$, $\beta \in [0,1]$ and $y \in X$. A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_\beta q A$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi-coincident with a fuzzy set B denoted by $A q B$ if and only if there exists a point $x \in X$ such that $A(x) + B(x) > 1$. $A \subseteq B$ if and only if $\neg(A_q B^c)$. A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if $0,1$ belongs to τ and τ is closed with respect to arbitrary union and finite intersection. The members of τ are called fuzzy open sets and their complement are fuzzy closed sets. For any fuzzy set A of X the closure of A (denoted by $cl(A)$) is the intersection of all the fuzzy closed super sets of A and the interior of A (denoted by $int(A)$) is the union of all fuzzy open subsets of A .

II. PRELIMINARIES

We recall the following definitions:

A. **Definition 2.1:** A subset A of a fuzzy topological space (X, τ) is called;

- 1) Fuzzy Generalized closed (briefly fuzzy g-closed) if $cl(A) \subseteq G$ whenever $A \subseteq G$ and G is fuzzy open in X .
- 2) Fuzzy Semi-generalized closed (briefly fuzzy sg-closed) if $scl(A) \subseteq G$ whenever $A \subseteq G$ and G is fuzzy semiopen in X .
- 3) Fuzzy Generalized semi closed (briefly fuzzy gs-closed) if $scl(A) \subseteq G$ whenever $A \subseteq G$ and G is fuzzy open in X .
- 4) Fuzzy α -closed [8] if $cl(int(cl(A))) \subseteq A$.
- 5) Fuzzy α -generalized closed (briefly fuzzy α g-closed) if $cl_\alpha(A) \subseteq G$ whenever $A \subseteq G$ and G is fuzzy open in X .
- 6) Fuzzy Generalized α -closed (briefly fuzzy α g-closed) if $spcl(A) \subseteq G$ whenever $A \subseteq G$ and G is fuzzy open in X .
- 7) Fuzzy Generalized semi-preclosed (briefly fuzzy gsp-closed) if $scl(A) \subseteq G$ whenever $A \subseteq G$ and G is fuzzy open in X .
- 8) Fuzzy Strongly generalized closed (briefly fuzzy strongly g-closed) if $cl(A) \subseteq G$ whenever $A \subseteq G$ and G is fuzzy g-open in X .
- 9) Fuzzy Preclosed if $cl(int(A)) \subseteq A$.
- 10) Fuzzy Semi-closed if $int(cl(A)) \subseteq A$.
- 11) Fuzzy Semi-preclosed (briefly fuzzy sp-closed) if $int(cl(int(A))) \subseteq A$.

The complements of the above mentioned sets are called their respective fuzzy open sets.

B. **Definition 2.2:** For the subset A of a fuzzy topological X , the fuzzy generalized closure operator $cl^*[5]$ is defined by the intersection of all fuzzy g-closed sets containing A .

C. **Definition 2.3:** For the subset A of a fuzzy topological X , the topology τ^* is defined by $\tau^* = \{G : cl^*(G^c) = G^c\}$

D. **Definition 2.4:** For the fuzzy subset A of a fuzzy topological X ,

- 1) The fuzzy semi-closure of A (briefly $scl(A)$) is defined as the intersection of all fuzzy semi-closed sets containing A .
- 2) The fuzzy semi-preclosure of A (briefly fuzzy $spcl(A)$) is defined as the intersection of all fuzzy semi-preclosed sets containing A .
- 3) The fuzzy α -closure of A (briefly $cl_\alpha(A)$) is defined as the intersection of all fuzzy α -closed sets containing A .

III. FUZZY τ^* -GENERALIZED CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

In this section, we introduce the concept of fuzzy τ^* -generalized closed sets in fuzzy topological spaces.

A. *Definition 3.1.* A fuzzy subset A of a fuzzy topological space X is called fuzzy τ^* -generalized closed set (briefly fuzzy τ^* -g-closed) if $\text{cl}^*(A) \subseteq G$ whenever $A \subseteq G$ and G is fuzzy τ^* -open. The complement of fuzzy τ^* -generalized closed set is called the fuzzy τ^* -generalized open set (briefly fuzzy τ^* -g-open).

B. *Theorem 3.1.* Every fuzzy closed set in X is fuzzy τ^* -g-closed.

1) *Proof.* Let A be a fuzzy closed set. Let $A \subseteq G$. Since A is fuzzy closed, $\text{cl}(A) = A \subseteq G$. But $\text{cl}^*(A) \subseteq \text{cl}(A)$. Thus, we have $\text{cl}^*(A) \subseteq G$ whenever $A \subseteq G$ and G is fuzzy τ^* -open. Therefore A is fuzzy τ^* -g-closed.

C. *Theorem 3.2.* Every fuzzy τ^* -closed set in X is fuzzy τ^* -g-closed.

1) *Proof.* Let A be a fuzzy τ^* -closed set. Let $A \subseteq G$ where G is fuzzy τ^* -open. Since A is fuzzy τ^* -closed, $\text{cl}^*(A) = A \subseteq G$. Thus, we have $\text{cl}^*(A) \subseteq G$ whenever $A \subseteq G$ and G is fuzzy τ^* -open. Therefore A is fuzzy τ^* -g-closed.

D. *Theorem 3.3.* Every fuzzy g-closed set in X is a fuzzy τ^* -g-closed set but not conversely.

1) *Proof.* Let A be a fuzzy g-closed set. Assume that $A \subseteq G$, G is fuzzy τ^* -open in X . Then $\text{cl}(A) \subseteq G$, since A is fuzzy g-closed. But $\text{cl}^*(A) \subseteq \text{cl}(A)$. Therefore $\text{cl}^*(A) \subseteq G$. Hence A is fuzzy τ^* -g-closed. The converse of the above theorem need not be true.

2) *Remark 3.1.* The following example shows that fuzzy τ^* -g-closed sets are independent from fuzzy sp-closed set, fuzzy sg-closed set, fuzzy α -closed set, fuzzy preclosed set, fuzzy gs-closed set, fuzzy gsp-closed set, fuzzy ag-closed set and fuzzy ga-closed set.

E. *Theorem 3.4.* For any two fuzzy sets A and B , $\text{cl}^*(A \cup B) = \text{cl}^*(A) \cup \text{cl}^*(B)$

1) *Proof.* Since $A \subseteq A \cup B$, we have $\text{cl}^*(A) \subseteq \text{cl}^*(A \cup B)$ and since $B \subseteq A \cup B$, we have $\text{cl}^*(B) \subseteq \text{cl}^*(A \cup B)$. Therefore $\text{cl}^*(A) \cup \text{cl}^*(B) \subseteq \text{cl}^*(A \cup B)$. Also, $\text{cl}^*(A)$ and $\text{cl}^*(B)$ are the fuzzy closed sets. Therefore $\text{cl}^*(A) \cup \text{cl}^*(B)$ is also a fuzzy closed set. Again, $A \subseteq \text{cl}^*(A)$ and $B \subseteq \text{cl}^*(B)$ implies $A \cup B \subseteq \text{cl}^*(A) \cup \text{cl}^*(B)$. Thus, $\text{cl}^*(A) \cup \text{cl}^*(B)$ is a closed set containing $A \cup B$. Since $\text{cl}^*(A \cup B)$ is the fuzzy smallest closed set containing $A \cup B$ we have $\text{cl}^*(A \cup B) \subseteq \text{cl}^*(A) \cup \text{cl}^*(B)$. Thus, $\text{cl}^*(A \cup B) = \text{cl}^*(A) \cup \text{cl}^*(B)$.

F. *Theorem 3.5.* Union of two fuzzy τ^* g-closed sets in X is a fuzzy τ^* -g-closed set in X .

1) *Proof.* Let A and B be two fuzzy τ^* g-closed sets. Let $A \cup B \subseteq G$, where G is fuzzy τ^* -open. Since A and B are fuzzy τ^* -g-closed sets, $\text{cl}^*(A) \subseteq G$ and $\text{cl}^*(B) \subseteq G$. But $\text{cl}^*(A) \cup \text{cl}^*(B) = \text{cl}^*(A \cup B)$. Therefore $\text{cl}^*(A \cup B) \subseteq G$. Hence $A \cup B$ is a fuzzy τ^* -g-closed set.

G. *Theorem 3.6.* A subset A of X is fuzzy τ^* -g-closed if and only if $\text{cl}^*(A) - A$ contains no non-empty fuzzy τ^* -closed set in X .

1) *Proof.* Let A be a fuzzy τ^* -g-closed set. Suppose that F is a nonempty fuzzy τ^* -closed subset of $\text{cl}^*(A) - A$. Now $F \subseteq \text{cl}^*(A) - A$. Then $F \subseteq \text{cl}^*(A) \cap A^c$, since $\text{cl}^*(A) - A = \text{cl}^*(A) \cap A^c$. Therefore $F \subseteq \text{cl}^*(A)$ and $F \subseteq A^c$. Since F is a fuzzy τ^* -closed set and A is a fuzzy τ^* -g-closed, $\text{cl}^*(A) \subseteq F^c$. That is $F \subseteq [\text{cl}^*(A)]^c$. Hence $F \subseteq \text{cl}^*(A) \cap [\text{cl}^*(A)]^c = \emptyset$. That is $F = \emptyset$, a contradiction. Thus $\text{cl}^*(A) - A$ contains no non-empty fuzzy τ^* -closed set in X . Conversely, assume that $\text{cl}^*(A) - A$ contains no nonempty fuzzy τ^* -closed set. Let $A \subseteq G$, G is fuzzy τ^* -open. Suppose that $\text{cl}^*(A)$ is not contained in G , then $\text{cl}^*(A) \cap G^c$ is a non-empty fuzzy τ^* -closed set of $\text{cl}^*(A) - A$ which is a contradiction. Therefore $\text{cl}^*(A) \subseteq G$ and hence A is fuzzy τ^* -g-closed.

2) *Corollary 3.1.* A subset A of X is fuzzy τ^* g-closed if and only if $\text{cl}^*(A) - A$ contains no non-empty fuzzy closed set in X .

3) *Proof:* Easy

4) *Corollary 3.2.* A subset A of X is fuzzy τ^* -g-closed if and only if $\text{cl}^*(A) - A$ contains no non-empty fuzzy open set in X .

5) *Proof:* The proof follows from the Theorem 3.10 and the fact that every open set is fuzzy τ^* -open set in X .

H. *Theorem 3.7.* If a subset A of X is fuzzy τ^* -g-closed and $A \subseteq B \subseteq \text{cl}^*(A)$, then B is fuzzy τ^* -g-closed set in X .

- 1) *Proof:* A be a fuzzy τ^* -g-closed set such that $A \subseteq B \subseteq cl^*(A)$. Let U be a fuzzy τ^* -open set of X such that $B \subseteq U$. Since A is fuzzy τ^* -g-closed, we have $cl^*(A) \subseteq U$. Now $cl^*(A) \subseteq cl^*(B) \subseteq cl^*[cl^*(A)] = cl^*(A) \subseteq U$. That is $cl^*(B) \subseteq U$, U is fuzzy τ^* -open. Therefore B is fuzzy τ^* -g-closed set in X. The converse of the above theorem need not be true .

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