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Fuzzy τ*-Generalized Closed Sets in Fuzzy Topological Spaces

M.K. Mishra[1], D. Anandhi[2], M.Prabhavathy[3]
[1]Director R&D, [2,3]Asst. Prof, E.G.S. Pillay Arts and Science College Nagapattinam

***Abstract:** In this paper, we introduce a new class of sets called fuzzyτ*-generalized closed sets and fuzzy τ*-generalized open sets in fuzzy topological spaces and explore some of their properties.*
***Keywords:** fuzzy closed set fuzzy open set, fuzzyτ*-g-closed set, fuzzy τ*-g-open set.*

I. INTRODUCTION

Let X be a non-empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha\in\Lambda\}$ of fuzzy sets of X is defined by to be the mapping sup A_α (resp. inf A_α) . A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \leq B(x)$ for each $x\in X$. A fuzzy point x_β in X is a fuzzy set defined by $x_\beta (y) = \beta$ for y=x and x(y) =0 for $y \neq x$, $\beta\in[0,1]$ and $y \in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi –coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x\in X$ such that $A(x) + B(x) > 1$.$A \leq B$ if and only if $\rceil(A_qB^c)$.A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if 0,1 belongs to τ and τ is closed with respect to arbitrary union and finite intersection .The members of τ are called fuzzy open sets and their complement are fuzzy closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy closed super sets of A and the interior of A (denoted by int(A))is the union of all fuzzy open subsets of A.

II. PRELIMINARIES

We recall the following definitions:

A. *Definition 2.1:* A subset A of a fuzzy topological space (X, τ) is called;

1) Fuzzy Generalized closed (briefly fuzzy g-closed) if $cl(A) \subseteq G$ whenever $A \subseteq G$ and G is fuzzy open in X.
2) Fuzzy Semi-generalized closed (briefly fuzzy sg-closed) if $scl(A) \subseteq G$ whenever $A \subseteq G$ and G is fuzzy semiopen in X.
3) Fuzzy Generalized semi closed (briefly fuzzy gs-closed) if $scl(A) \subseteq G$ whenever $A \subseteq G$ and G is fuzzy open in X.
4) Fuzzyα-closed[8] if $cl(int(cl(A))) \subseteq A$.
5) Fuzzyα-generalized closed (briefly fuzzy αg-closed) if $cl\alpha(A)\subseteq G$ whenever $A \subseteq G$ and G is fuzzy open in X.
6) Fuzzy Generalized α-closed (briefly fuzzy gα-closed) if spcl $(A) \subseteq G$ whenever $A \subseteq G$ and G is fuzzy open in X.
7) Fuzzy Generalized semi-preclosed (briefly fuzzy gsp-closed) if $scl(A) \subseteq G$ whenever $A \subseteq G$ and G is fuzzy open in X.
8) Fuzzy Strongly generalized closed (briefly fuzzy strongly g-closed)if $cl(A) \subseteq G$ whenever $A \subseteq G$ and G is fuzzy g-open in X.
9) Fuzzy Preclosed if $cl(int(A)) \subseteq A$.
10) Fuzzy Semi-closed if $int(cl(A)) \subseteq A$.
11) Fuzzy Semi-preclosed (briefly fuzzy sp-closed) if $int(cl(int(A))) \subseteq A$.

The complements of the above mentioned sets are called their respective fuzzy open sets.

B. *Definition 2.2:*For the subset A of a fuzzy topological X, the fuzzy generalized closure operator cl*[5] is defined by the intersection of all fuzzy g-closed sets containing A.

C. *Definition 2.3:*For the subset A of a fuzzy topological X, the topology τ* is defined by $\tau^* = \{G : cl^*(G^c) = G^c \}$

D. *Definition 2.4:*For the fuzzy subset A of a fuzzy topological X,

1) Thefuzzy semi-closure of A (briefly scl(A)) is defined as the intersection of all fuzzy semi-closed sets containing A.
2) The fuzzy semi-preclosure of A (briefly fuzzy spcl(A)) is defined as the intersection of all fuzzy semi-preclosed sets containing A.
3) The fuzzy α-closure of A (briefly $cl_\alpha(A)$) is defined as the intersection of all fuzzy α-closed sets containing A.

III. FUZZY T*-GENERALIZED CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

In this section, we introduce the concept of fuzzy τ*-generalized closed sets in fuzzy topological spaces.

A. *Definition 3.1.*A fuzzy subset A of a fuzzy topological space X is called fuzzy τ*-generalized closed set (briefly fuzzy τ*-g-closed) if cl*(A)⊆ G whenever A ⊆ G and G is fuzzy τ*-open. The complement of fuzzy τ*-generalized closed set is called the fuzzy τ*-generalized open set (briefly fuzzy τ*-g-open)
.

B. *Theorem 3.1.*Every fuzzy closed set in X is fuzzy τ*-g-closed.
1) *Proof.* Let A be a fuzzy closed set. Let A ⊆ G. Since A is fuzzy closed, cl(A) = A ⊆ G. But cl*(A) ⊆ cl(A). Thus, we havecl*(A) ⊆ G whenever A ⊆ G and G is fuzzy τ*-open. Therefore A is fuzzy τ*-g-closed.

C. *Theorem 3.2.*Every fuzzy τ*-closed set in X is fuzzy τ*-g-closed.
1) *Proof.* Let A be a fuzzy τ*-closed set. Let A ⊆ G where G is fuzzy τ*-open. Since A is fuzzy τ*-closed, cl*(A) = A ⊆G. Thus, we have cl*(A) ⊆ G whenever A ⊆ G and G is fuzzy τ*-open. Therefore A is fuzzy τ*-g-closed.

D. *Theorem 3.3.*Every fuzzy g-closed set in X is a fuzzy τ*-g-closed set but not conversely.
1) *Proof :*Let A be a fuzzy g-closed set. Assume that A ⊆ G, G is fuzzy τ*-open in X. Then cl(A) ⊆ G, since A is fuzzy g-closed. But cl*(A) ⊆ cl(A). Therefore cl*(A) ⊆ G. Hence A is fuzzy τ*-g-closed. The converse of the above theorem need not be true .
2) *Remark 3.1.:*The following example shows that fuzzy τ*-g-closed sets are independent from fuzzy sp-closed set, fuzzy sg-closed set, fuzzy α-closed set, fuzzy preclosed set fuzzy gs-closed set, fuzzy gsp-closed set, fuzzy αg-closed set and fuzzy gα-closed set.

E. *Theorem 3.4.:*For any two fuzzy sets A and B, **c**l*(A ∪ B) = cl*(A) ∪ cl*(B)
1) *Proof :*Since A ⊆ A ∪ B , we have cl*(A) ⊆ cl*(A ∪ B) and since B ⊆ A ∪ B , we have cl*(B) ⊆ cl*(A ∪ B).Therefore cl*(A) ∪ cl*(B) ⊆ cl*(A ∪ B). Also , cl*(A) and cl*(B) are the fuzzy closed sets Therefore cl*(A) ∪ cl*(B) is also a fuzzy closed set. Again, A ⊆ cl*(A) and B ⊆ cl*(B) implies A∪B ⊆ cl*(A) ∪ cl*(B).Thus, cl*(A) ∪ cl*(B) is a closed set containing A ∪ B. Since cl*(A ∪ B) is the fuzzy smallest closed set containing A∪B we have cl*(A ∪ B) ⊆ cl*(A) ∪ cl*(B). Thus, cl*(A ∪B) = cl*(A) ∪ cl*(B

F. *Theorem 3.5.:*Union of two fuzzy τ* g-closed sets in X is a fuzzy τ*- g-closed set in X.
1) *Proof :*Let A and B be two fuzzy τ* g-closed sets. Let A ∪ B ⊆ G, where G is fuzzy τ*-open. Since A and B are fuzzy τ*-g-closed sets, cl*(A) ∪ cl*(B) ⊆ G. But cl*(A) ∪ cl*(B) = **c**l*(A ∪ B). Therefore **c**l*(A ∪ B) ⊆ G. Hence A ∪ B is a fuzzy τ*- g-closed set.

G. *Theorem 3.6.:*A subset A of X is fuzzy τ*-g-closed if and only if cl*(A) – A contains no non-empty fuzzy τ* -closed set in X.
1) *Proof:* Let A be a fuzzy τ*-g-closed set. Suppose that F is a nonempty fuzzy τ*-closed subset of cl*(A) – A.Now F ⊆ cl*(A) – A. Then F ⊆ cl*(A) ∩ Ac, since cl*(A) – A = cl*(A) ∩ A^c. Therefore F ⊆ cl*(A) and F ⊆ A^c. Since Fc is a fuzzy τ*-open set and A is a fuzzy τ*-g-closed, cl*(A) ⊆ F^c. That is F ⊆ $[cl*(A)]^c$. Hence F ⊆ cl*(A) ∩ $[cl*(A)]^c$ = φ. That is F = φ, a contradiction. Thus cl*(A) – A contain no non-empty fuzzy τ*-closed set in X. Conversely, assume that cl*(A) – A contains no nonempty fuzzy τ*-closed set. Let A ⊆ G, G is fuzzy τ*-open. Suppose that cl*(A) is not contained in G, then cl*(A) ∩ G^c is a non-empty fuzzy τ*-closed set of cl*(A) – A which is a contradiction. Therefore cl*(A) ⊆ G and hence A is fuzzy τ*-g-closed.
2) *Corollary 3.1.*A subset A of X is fuzzy τ* g-closed if and only if cl*(A) – A contain no non-empty fuzzy closed set in X.
3) *Proof:* Easy
4) *Corollary 3.2:*.A subset A of X is fuzzy τ*-g-closed if and only if cl*(A) – A contain no non-empty fuzzy open set in X.
5) *Proof:* The proof follows from the Theorem 3.10 and the fact that every open set is fuzzy τ*-open set in X.

H. *Theorem 3.7.*If a subset A of X is fuzzy τ*-g-closed and A ⊆ B ⊆ cl*(A), then B is fuzzy τ*-g-closed set in X.

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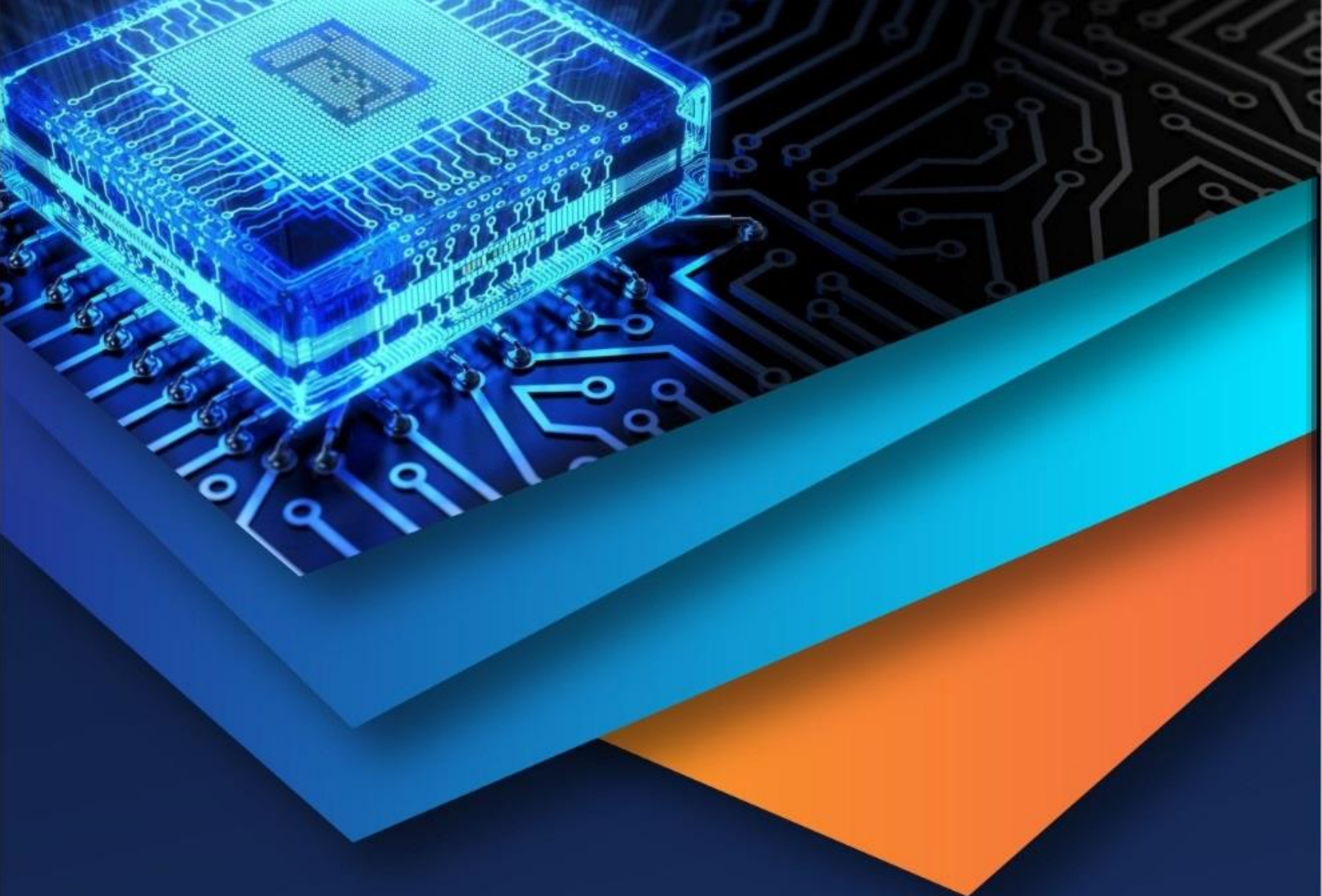
1) *Proof:* A be a fuzzy τ*-g-closed set such that A ⊆ B ⊆ cl*(A). Let U be a fuzzy τ*-open set of X such that B ⊆ U. Since A is fuzzy τ*-g-closed, we have cl*(A) ⊆ U. Now cl*(A) ⊆ cl*(B) ⊆ cl*[cl*(A)] = cl*(A) ⊆ U. That is cl*(B) ⊆ U, U is fuzzy τ*-open. Therefore B is fuzzy τ*-g-closed set in X.The converse of the above theorem need not be true .

REFERENCES

[1]. A.S Mashhour., Abd. El-Monsef. M. E. and El-Deeb S.N., On pre continuous mappings and weak pre-continuous mappings, Proc Math, Phys. Soc. Egypt 53(1982), 47–53.
[2]. B. Ghosh, Semi-continuous and semi-closed mappings and semi-connectedness in fuzzy setting,Fuzzy Sets and Systems 35(3) (1990), 345–355.
[3]. C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968), 182–190.
[4]. C.W. Baker on preserving g-super closed sets Kyungpook Math. J. 36(1996), 195-199.
[5]. G. Balasubramanian and V. Chandrasekar, Totally fuzzy semi continuous functions, Bull. Calcutta Math. Soc. 92(4) (2000), 305–312.
[6]. K. K. Azad, On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl. 82(1) (1981), 14–32.
[7]. K. M. Abd El-Hakeim, Generalized semi-continuous mappings in fuzzy topological spaces, J. Fuzzy Math. 7(3) (1999), 577–589.
[8]. L. A. Zadeh, Fuzzy sets Information and Control 8 (1965), 338–353.
[9]. M.K. Mishra et all on “Fuzzy super continuity” International Review in Fuzzy Mathematics July –December2012.
[10]. M.K. Mishra M. Shukla M. Fuzzy Regular Generalized Super Closed Set” International Journal of Scientific and Research December issue July December 2012.
[11]. M.K. Mishra, et all on “ Fuzzy super closed set” International Journal International Journal of Mathematics and applied Statistics.
[12]. N Levine.., generalized super closed sets in topology, Rend. Circ. Mat. Palermo 19(1970), 89–96.
[13]. N. Nagaveni., Studies on Generalizations of Homeomorphisms in Topological Fuzzy space s, Ph.D. Thesis, Bharathiar University, Coimbatore, 1999 .
[14]. N.Levine., Semi- super open sets and semi-continuity in topological fuzzy space s, Amer. Math. Monthly,70(1963), 36–41 .
[15]. N.Palaniappan., and Rao. K. C., Regular generalized super closed sets, Kyungpook Math. J. 33(1993), 211–219.
[16]. P. M. Pu and Y. M. Liu Fuzzy topology I Neighborhood structure of a fuzzy point and More-Smith Convergence. J. Math. Anal. Appl. 76(1980) ,571-594.
[17]. P. M. Pu, and Y. M. Liu, Fuzzy topology. I. Neighborhood structure of a fuzzy point and Moore-Smith convergence, J. Math. Anal. Appl. 76(2) (1980), 571–599.
[18]. Park. J. K. and Park. J.H., mildly generalized super closed sets, almost normal and mildly normal fuzzy space s, Chaos, Solutions and Fractals 20(2004), 1103–1111.
[19]. R. K. Saraf and M. Khanna, On gs-closed sets in fuzzy topology, J. Indian Acad. Math. 25(1),(2003), 133–143.
[20]. R. K. Saraf, and M. Khanna, Fuzzy generalized semipreclosed sets, J. Tripura Math. Soc.3(2001) 59–68.
[21]. R. K. Saraf, M. Caldas and S. Mishra, Results via Fg_-closed sets and Fg-closed sets, Pre print.
[22]. Syed Ali Fathima. S and Mariasingam. M, On fuzzy regular generalized super closed sets in topological fuzzy space s, International journal of mathematical archive-2(11), 2011, 2497 – 2502.
[23]. T. H. Yalva¸c, Semi-interior and semi closure of a fuzzy set, J. Math. Anal. Appl. 132(2) (1988),356–364.
[24]. Veera Kumar M.K.R.S., Between g* super closed sets and g-super closed sets, Mem.Fac.Sci.Kochi Univ. Ser .App .Math .,21 (2000),1-19.



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