Structural Changes in Hot Rotating $^{100}\text{Sr}_{38}$ and $^{100}\text{Rb}_{37}$

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Abstract: This paper presents the nuclear structure studies of hot rotating $^{100}\text{Sr}_{38}$ and $^{100}\text{Rb}_{37}$. The statistical theory which incorporates deformation, collective and non-collective rotational degrees of freedom and shell effects is used to study the highly excited $^{100}\text{Sr}_{38}$ and $^{100}\text{Rb}_{37}$. The nuclei in these regions are $(A\sim 100)$ characterized by a small quadrupole deformation and soft to gamma deformation at low spins, it is confirmed by the present study. $^{100}\text{Sr}_{38}$ spherical nucleus at low temperature and angular momentum underwent a spherical to deformed shape transition at low temperature and angular momentum. $^{100}\text{Rb}_{37}$, deformed nucleus at low temperature and angular momentum underwent a deformed to spherical shape transition at low temperature and angular momentum.

Keywords: Statistical theory, shape transition

I. INTRODUCTION

The studies of nuclear structure at high excitation energies, the so called hot nuclei [1-6] identify today an important field of investigation in nuclear physics. During the last few decades considerable interests have been shown to the investigation of structural transitions as a function of both angular momentum and temperature of highly excited nuclei. Hot and rotating nuclei are expected to exhibit a rich variety of different shapes. A heavy ion fusion reaction produces a hot and rapidly rotating compound nucleus. Hot nuclei are formed in heavy ion fusion reactions where the relative kinetic energy of the colliding nuclei is converted into internal excitation energy and high angular momentum of the compound nuclei. After emission of neutrons has cooled down the system, the compound nucleus still keeps high angular moment and moderate heat energy. There is considerable interest in the study of the structure of nuclei in the mass region $A\sim 100$ due to the onset of deformation in the neutron rich nuclei [7-12]. These neutron rich nuclei are of special interest because they are just at the border between a rather spherical and a well deformed shape. Theoretically, the region has been studied using interacting boson model [7], Nilsson Strutinsky Cranking method [8], statistical theory[4,8] and Hartree-Fock-Bogolyubov [9]. Most of these methods correspond to cold nuclei and are applicable strictly speaking only to yrast spectroscopy. In these investigations of the nuclear shape at high excitation energies are absent, and the statistical properties of fast rotating nuclei have not been given proper consideration. In order to understand the equilibrium deformation in these hot and rotating $^{100}\text{Sr}_{38}$ and $^{100}\text{Rb}_{37}$ systems, a calculation was performed for estimating the equilibrium shape of a nucleus by minimizing the total free energy under the framework of temperature dependent rotating liquid drop model (RLDM) and cranked Nilsson oscillator model (CNM) of hot rotating nuclei for a given temperature and angular momentum [13-18].

II. THEORETICAL FORMALISM

The grand partition function for the hot rotating nuclei is given by

$$Q(\omega, \alpha_z, \alpha_n, \beta) = \sum_{e_z, e_n} \exp(e_z Z + \alpha_n N - \beta E_i(\omega))$$

The Lagrangian multipliers $\alpha_z$ and $\alpha_n$ conserve the proton and neutron numbers at a given temperature $T = 1/\beta$. The corresponding equations in terms of the single particle energies $e_i(\omega)$ are given by

$$\langle N \rangle = \sum_i n_i^p = \sum_i \left\lfloor 1 + \exp(\alpha_z + \beta e_i^p(\omega)) \right\rfloor^{-1}$$

$$\langle Z \rangle = \sum_i z_i^n = \sum_i \left\lfloor 1 + \exp(\alpha_n + \beta e_i^n(\omega)) \right\rfloor^{-1}$$

The total angular momentum is given by

$$M = \sum_i n_i^p m_i^p + \sum_i n_i^p m_i^z$$
The system of simultaneous non-linear Eqs. (2), (3) and (4) has to be solved to determine \( \alpha_s \) and \( \alpha_n \) for each value of \( \omega \) and \( T \). The total energy of the system is obtained as

\[
E(M, T) = \left[ \sum_i z_i^p c_i^p(\omega) + \hbar \omega \sum_i n_i^p m_i^p \right] + \left[ \sum_i n_i^p c_i^p(\omega) + \hbar \omega \sum_i n_i^p m_i^p \right]
\]

The excitation energy of the system is given by

\[
E^* = \sum_i n_i^p c_i^p(\omega) - \sum_i c_i^p(0) + \sum_i n_i^p c_i^n(\omega) - \sum_i c_i^n(0)
\]

The rotational energy \( E_{rot} \) is calculated using the following equation

\[
E_{rot}(M) = E(MT) - E(0T)
\]

The necessity of renormalizing the total energy does not arise here, since we are interested only in the energy difference between the excited and the ground states of the system and not in the actual magnitude of the energies. The entropy of the system is then obtained as

\[
S = \frac{\partial(T \ln Q)}{\partial T}
\]

The nucleon separation energy as a function of the angular momentum \( M \) and temperature \( T \) is calculated using the expressions given by

\[
S_x = TN \left[ \sum_i \left( 1 - n_i^p \right) n_i^p \right], \quad S_N = TN \left[ \sum_i \left( 1 - n_i^N \right) n_i^N \right]
\]

The single particle occupational probabilities \( n_i \) are given by eqn. (2) and eq. (3).

### III. RESULTS AND DISCUSSION

Numerical results are presented by performing systematic study of the \( ^{100}\text{Sr}_{58} \) and \( ^{100}\text{Rb}_{37} \) isotope within the framework of Statistical method. The angular momentum has been generated by adjusting the cranking frequency \( \omega \) for different deformations. The necessity arises due to the fact that calculations are to be performed for both non-collective and collective states of the nuclei. Calculations are carried out for the deformation parameter \( \epsilon = 0.0 \) to 1 with \( \Delta \epsilon = 0.1 \) and for \( \gamma = -120^\circ \) to \( \gamma = -180^\circ \). In our calculations the following convention is used: The deformation parameter = -180° corresponds to the oblate shape rotating about the symmetry axis while = -120° corresponds to the prolate shape rotating about an axis perpendicular to the symmetry axis. Calculations are carried out by minimizing the free energy for equilibrium deformation. The free energy of the system for each temperature and spin is minimised with respect to the deformation and shape parameter.

![FIG. 1a and 1bThe shape parameter β and γ as a function of M for different T for the nuclei Rb37](image-url)
The equilibrium deformation was obtained from the minimization of the free energy with respect to deformation parameter at different spins and temperature for $^{100}$Rb$_{37}$ is plotted in Figure 1a and 1b. It is clearly seen, the nucleus is oblate ($\gamma = 0.2$ and $\delta = -120^\circ$) and its shape changes with angular momentum (M), at $M = 6h$, a shape change from collective prolate to triaxial shape was observed. However, above a certain critical angular momentum (M= 26h for $T=0.5$MeV, M= 22h for $T=1.0$MeV, M= 28h for $T=1.5$MeV, M= 30h for $T=2.0$MeV, M= 32h for $T=2.5$MeV and M= 32h for $T=3.0$MeV) nucleus becomes collective oblate to non collective prolate. A significant effect of shape changes is observed at very low temperatures. A similar behaviour is exhibited by the nucleus for the temperature $T = 1.0$MeV. It is interesting to note that with increasing temperature the presence of triaxial shape vanish, the nuclei undergoes a shape transition from collective prolate to oblate shape. The effect caused by phase changes with $T$ is more clearly seen from the plot 1a. We observed that at particular temperature (at $T = 1.5$ MeV and at low angular momentum values) the nucleus undergoes a deformed to spherical shape transition, this may due to the shell effects which drive deformation disappear but not the ones providing the magic numbers in the spherical shape. The Fig. 2a and 2b shows the equilibrium shape of the system is determined by minimizing the free energy with respect to the deformation parameters $\delta$, $\gamma$ as a function of angular momentum for different temperature for the nuclei $^{38}$Sr. It is observed from Fig. 2a ($T = 0.5$MeV) that the nucleus is found to be spherical ($\delta = 0.0$ and $\gamma = -120^\circ$) for the angular-momentum range $M = 0$ to 28h, triaxial ($\delta = 0.1$ and $\gamma = -150^\circ$ for $M = 30$ and $\delta = 0.1$ and $\gamma = -160^\circ$ for $M = 36-42$h), finally reaches the oblate shape for $M = 54$ to 50h ($\epsilon = 0.3$and $\gamma = -180^\circ$). For $T = 1.0$MeV the nucleus remains at spherical shape with $\epsilon = 0.0$ and $\gamma = -120^\circ$ for the angular-momentum range $M = 0$ to 28h, triaxial ($\delta = 0.1$ and $\gamma = -150^\circ$ for $M = 36$ and $\delta = 0.1$ and $\gamma = -160^\circ$ for $M = 42$ h), finally reaches the oblate shape for $M = 54$ to 50h ($\epsilon = 0.3$ and $\gamma = -180^\circ$). Above $T = 1.0$ MeV the nuclei undergoes a shape transition from collective prolate to oblate shape ($M= 32$h for $T=1.5$MeV, $M= 36$h for $T=2.0$MeV, $M= 36$h for $T=2.5$MeV and $M= 36$h for $T=3.0$MeV) Fig. 3a and 3b represent the rotational energy for the hot rotating $^{38}$Sr and $^{100}$Rb, nuclei. The fluctuations in the curve at lower temperature indicating the structural changes which may be associated with shape transitions of the nucleus. As the angular momentum increases the rotational energy increases.
Fig. 4a and 4b represent the proton (S_z) and neutron (S_N) separation energy for the hot rotating $^{100}\text{Rb}_{37}$ nuclei. The fluctuations in the curve at lower temperature indicating the structural changes which may be associated with shape transitions of the nucleus. As the temperature increases, the proton and neutron separation energy also increases and it almost constant for all angular momentum (for particular temperature).

![Graph 1](image1)

**FIG. 4a and 4b** $S_z$ and $S_N$ as a function of $M$ for different $T$ the nuclei $^{100}\text{Rb}_{37}$

Fig. 5a and 5b represent the proton (S_z) and neutron (S_N) separation energy for the hot rotating $^{100}\text{Sr}_{38}$ nuclei. It is observed that the structural changes may cause the nuclei to change shape and change in the separation energy at low temperature. Similar to $\text{Rb}_{37}$, the temperature increases, the proton and neutron separation energy also increases and it almost constant for all angular momentum (for particular temperature).

![Graph 2](image2)

**FIG. 5a and 5b** $S_z$ and $S_N$ as a function of $M$ for different $T$ the nuclei $^{100}\text{Sr}_{38}$

IV. CONCLUSION

In this work we have presented the structural changes in hot rotating $^{100}\text{Sr}_{38}$ and $^{100}\text{Rb}_{37}$. A statistical theory incorporating deformation, collective and non-collective rotational degrees of freedom, temperature and angular momentum is used to study the highly excited fused compound system. The preliminary results show that $^{100}\text{Rb}_{37}$ found to be in deformed state at low temperature and angular momentum as the proton number is increased by one ($^{100}\text{Sr}_{38}$) the nucleus is found to be spherical in shape. At particular angular momentum both the nucleus had under goneanoblate to prolate shape transition.

REFERENCES


