iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 Issue: II Month of publication: February 2018

DOI:

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 6 Issue II, February 2018- Available at www.ijraset.com

A Comparison of Fractal Dimension Algorithms by Hurst Exponent using Gold Price Time Series

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Abstract*: Fractal approach is a suitable method for the analysis of the complex time series. The Fractal dimension is the measure of complexity and the chaotic behaviours of the fractal time series. A variety of algorithms are existing for the computation of Fractal dimension. In this study, the most common methods of estimating Fractal dimension of financial time series are analyzed and compared. The analysis is performed over the gold price data. Fractal dimension through the Hurst exponent is estimated to provide effective and efficient understanding of the characteristics of the financial time series. In this paper the Fractal dimension is calculated from Hurst exponent by the methods the Rescaled range analysis(R/S analysis) and the Detrended fluctuation analysis (DFA). The Time series is analyzed and the comparison between the Fractal dimension algorithms by the Rescaled range analysis and the Detrended fluctuation analysis is established. The obtained results show that the long memory phenomenon is present and the time series is persistent.
***Keywords*: Hurst exponent; Detrended fluctuation analysis; Fractal Dimension.**

I. INTRODUCTION

Fractal is a geometric shape that is self similar in nature and has fractional dimension. The term Fractal was coined from the Latin word 'Fractus' by Benoit Mandelbrot in 1975. Fractal objects are represented by means of the Geometric patterns with non-integer dimensions [14]. The Fractals have a large degree of self similarity within themselves is described by Higuchi.T [1]. The complexity of stock exchanges Financial Time Series was analysed through Higuchi's Fractal dimension by T.G.Grace Elizabeth Rani and G.Jayalalitha [13]. The Fractal analysis is an unified technique, widely applies the Rescaled range analysis (R/S analysis) and the Fractal analysis from different approaches to assess the Fractal characteristics of the time series due to its stability is given by Hurst H.E [2]. For the investigation of Fractal time series the R/S analysis method was proposed by Mandelbrot [3]. It is based on the analysis of accumulated magnitude deviation of observations series and standard deviation. The Rescaled range analysis is the vital tool in Fractal data modelling. The Rescaled range (R/S) value is a statistical measure of the irregularity of a time series introduced by the British hydrologist Harold Edwin Hurst. Its aim is to suggest an assessment of how the apparent variability of a time series changes with the length of the time-period being considered is given by Naiman [4], Liashenko and Kravets [5]. The Rescaled range (R/S value) is attained from dividing the range of the values existing in a part of the time series by the standard deviation of the values over the alike part of the time series. Using the R/S analysis, the characteristics of the financial time series of the gold price rate is analyzed by Naiman [4]. The Detrended fluctuation Analysis (DFA) is a technique for measuring the same power law scaling observed through Rescaled Range analysis. Like R/S analysis, a synthetic walk is created, however a detrending operation is performed where the polynomial is locally fit to walk within each segment to identify the trend, and then that trend is subsequently removed is given by Qian and Rasheed [6]. Detrended fluctuation Analysis (DFA) is typically described as to enable the exact estimation of the power law (fractal) scaling (Hurst exponent) of a time series. DFA performs well, compared to heuristic techniques such as R/S analysis. Many researchers have applied the original methodology and similar techniques with the Detrended fluctuation Analysis due to its purported ability to logical nonstationaries is done by Marton and Barassai [7], Golinska [8], Bryce and Sprague [9]. The Hurst Exponent is a measure that quantifies the smoothness of fractal time series based on the asymptotic behavior of the rescaled range of the process. If $0 < H < 0.5$, we observe short memory phenomenon, i.e., it indicates ant persistent behavior of the series. If $0.5 < H < 1$ then the series is said to have long memory phenomenon, i.e., it indicates persistent behavior of the time series. If the Hurst exponent value becomes 0.5, it is the indication that there exists fractal Brownian motion or Random Walk in the financial time series. The limit of the trend rises in anticipation of H-value attains its maximum value to one. Most financial time series are persistent with the Hurst exponent which is greater than 0.5. The fractal dimension (FD) of the time series using the Hurst exponent (H) is calculated by the relationship formula $FD=2-H$ is applied by Golinska [8], Bryce and Sprague [9], Mali and Mukhopadhyay [10]. The two most renowned methods for the computation of the Fractal dimension for the financial time series have been applied to the analysis of the historical data of the gold price rate. However the analysis focuses on the financial time series of gold price rate, it can be exploited to any time series. In this research paper the Section II describes the Rescaled range analysis and the Detrended Fluctuation analysis algorithm for calculating the Hurst exponent and the Fractal Dimension in detail. The Section III expresses the experimentation of the algorithms operated on the Gold rate time series and it presents the discussion based on the results acquired by applying the algorithms on the time series. Finally, this paper is concluded in Section IV.

II. MATHEMATICAL ANALYSIS

A. The Rescaled Range Analysis (or) (R/S) Analysis

The Rescaled range analysis is commonly applied to test the Fractal characteristics of the time series. It is a non parametric statistical method applied in the fractal analysis. The algorithm is constructed as follows[6]:
The time series $X=\{x_1,x_2,x_3,\ldots,x_N\}$ is considered as the set of observation and n is the length of the time series.

Calculate the mean of the time series, $$M = \frac{1}{N}\sum_{t=1}^{N} X_i \qquad (2.1)$$ Create a mean -adjusted series, which is differ from the mean, (m): $Y_t=X_t-M$ for $t=1,2,\ldots,N$

Calculate the cumulative deviations Series Z: $$Z_t = \sum_{i=1}^{t} Y_i \qquad \text{for } t = 1, 2, \ldots, N \qquad (2.2)$$ (iv)

Create a range series R: $$R(N) = \max(Z_1, Z_2,\ldots, Z_N) - \min(Z_1, Z_2,\ldots, Z_N) \qquad (2.3)$$

Calculate the standard deviation of the series:
$$S(N) = \sqrt{\frac{1}{N}\sum_{i-1}^{N} (X_i - M)^2} \qquad (2.4)$$ Calculate the rescaled range series (R/S) = $\frac{R(N)}{S(N)}$

Finally the Hurst exponent is obtained by the formula
$$H = \frac{\log(R/S)}{\log(N)} \qquad (2.5)$$

Where N is the period of the sample and R/S is the standard deviation ratio. This formula of Hurst exponent shows that the increasing scale reduces the standard deviation and reduces the number of observations influence on its growth. The Hurst exponent inflates its value significantly. It will lead to erroneous conclusions about the persistency of random series [4].
Rescaled range is the “measure characterizing the divergence of time series defined as the range of the mean-centered values for a given duration divided by the standard deviation for that duration”.
By Power law, The Hurst exponent is defined as:
$$E\left(\frac{R(N)}{S(N)}\right) = k\, N^H$$, where k is a constant depends on the time series.

The Hurst exponent is estimated by plotting the values of log (N) versus log (R/S) and we obtain the Hurst exponent as the slope of the attained curve. The Hurst exponent is directly related to the fractal dimension, which measures the smoothness of the time series based on the asymptotic behavior of the process. The relationship between the fractal dimension D and the Hurst exponent H is given by:
$$D = 2 - H \quad where \quad 0 \le H \le 1.$$

B. Detrended Fluctuation Analysis

Detrended fluctuation analysis (DFA) was first proposed by Peng et al in 1994 [11]. Comparing to the R/S analysis expressed above, the DFA focuses on fluctuations around trend and the R/S analysis focuses on the range of the time series. Nowadays Detrended fluctuation analysis has become a standard tool in time series analysis. In time series analysis the detrended fluctuation analysis is used for determining the statistical self-affinity of a data series. The Hurst exponent of the time series is calculated using the Detrended fluctuation analysis. The starting steps of the procedure are the same as the one of R/S analysis as the whole series is

divided into non-overlapping periods of the same length which is again set on the same basis as in the mentioned procedure and thus the series profile is constructed. The algorithm for calculating Detrended fluctuation analysis consists the following algorithm:
Consider X={x_1, x_2, x_3,…, x_N} be the time series of length n.

1) Calculate the mean value of the analyzed time series,

$$M = \frac{1}{N}\sum_{t=1}^{n} X_i \tag{3.1}$$

2) By the cumulative sum, determine the integrated series

$$y(k) = \sum_{i=1}^{k} [X_i - M], \text{ where } \quad k = 1,2,\ldots, N. \tag{3.2}$$

Where X_i is the i^{th} data of the time series.

3) Divide the series *y(k)* into N_r=int (*N/r*) non-overlapping segments of equal length r. The value of *r* can be chosen depending upon the length of the time series. The local trend for each of the N_r segments is represented by the least square fit of the segments. The y coordinate of the straight line segments is denoted by $y_n(k)$. Next the integrated time series y(k) is detrended by subtracting the local trend, $y_n(k)$ in each interval [12].

The root mean square fluctuation of the integrated and detrended time series is calculated by

$$F(r) = \sqrt{\frac{1}{N}\sum_{k=1}^{N} [y(k) - y_r(k)]^2}. \tag{3.3}$$

The computation is repeated for all N_r non-overlapping segments of the time series to characterize between the average fluctuation F(r) and the segment size r. Usually, F(r) increases with the segment size r. A linear relationship on a log-log plot indices the presence of the power law scaling. By these conditions, the fluctuations can be characterized by a scaling exponent, the slope of the line relates log F(r) to log r. The log-log plot of the values of log (N) versus log (DFA) gives the estimation of the Hurst exponent as the slope of the attained curve. The Fractal dimension measures the smoothness of the time series based on the asymptotic behaviour of the process and $D = 2 - H \quad where \quad 0 \le H \le 1,$ is the relation between the fractal dimension (D) and the Hurst exponent (H).

III.EXPERIMENTS AND DISCUSSION

A. Rescaled Range Analysis

The time series data composed in this study consists of 1024 data values of the Gold price per gram in Indian rupees (INR) from the period July 2010 to March 2017 which was obtained from World Gold Council. The algorithms described above were implemented in MATLAB and are tested on the financial time series of the Gold price rate. The graphical representation of the time series considered for the estimation of the Hurst exponent and Fractal dimension is given in the Figure 1.

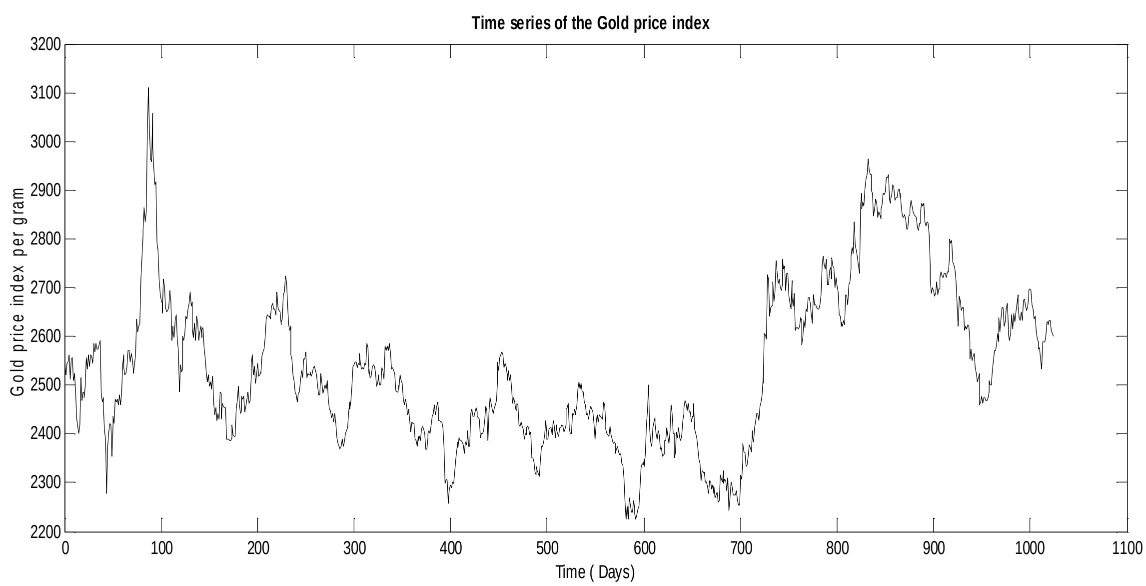

Figure 1: The time series of the Gold price index

The data collected over 1024 time intervals with various sample size and the rescaled range analysis values are listed in Table 1. The experimental data of Gold price rate is divided into different groups of data with sample size 1024, 512, 256, 128, 64, 32 and 16 using the formula N/2 where N represents the size of the sample data. The Hurst exponent values for the various sample size are calculated by executing the equation (2.5) in the MATLAB software.

Table1- Rescaled Range Analysis values in Hurst estimation:

Sample size(N)	$\log_2(N)$	$\log_2(R/S)$	$H = \frac{\log_2(R/S)}{\log_2(N)}$
1024	10	8.5135	0.8514
512	9	7.1164	0.7907
256	8	6.1954	0.7744
128	7	5.5149	0.7878
64	6	4.3868	0.7311
32	5	3.6569	0.7314
16	4	2.7486	0.6872

From the rescaled range analysis values the log-log plot between the log (N) and log (R/S) is given in Figure 2 as regression curve. The slope of the regression curve gives the estimate of the Hurst exponent to be H=0.7629. Hence the Fractal dimension is attained by the relation D=2-H as D=1.2371. Since the Hurst exponent H is near 0.76, indicates that there would be probability of 76% that the price of the Gold would rise during the period.

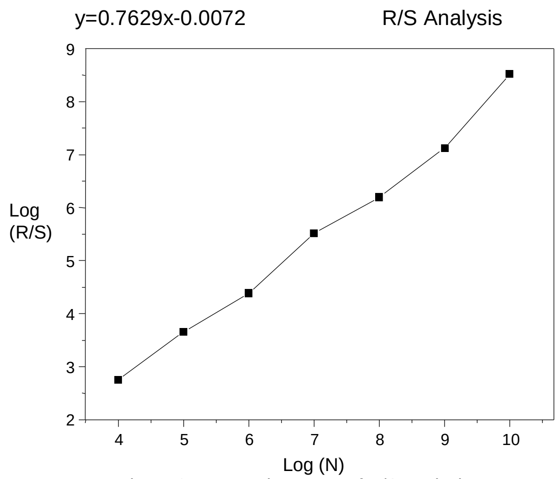

Figure 2: Regression curve of R/S analysis

B. Detrended Fluctuation Analysis

Detrended fluctuation analysis algorithm is applied on the considered experimental data 1024 values of the gold price rate time series and the calculations are executed in the MATLAB software. The different values of Detrended fluctuation analysis(DFA) for the various data sets consists of 1024,512,256,128 and 64 data are calculated by the DFA algorithm and The correlation between the values of log DFA and log N is represented by the curves given in the Figure 3. Considering the entire experimental data, from the Detrended fluctuation analysis the log-log plot between the log (N) and log (DFA) is given in Figure 4 as regression curve. The Hurst exponent value for the experimental data is calculated by executing equation (3.3) in the MATLAB software. The slope of the regression curve gives the estimate of the Hurst exponent to be H=0.7840. Hence the Fractal dimension is attained by the relation D=2-H as D=1.2160. Since the Hurst exponent H is near 0.78, which indicates that there would be probability of 78% that the price of the Gold would rise during the period.

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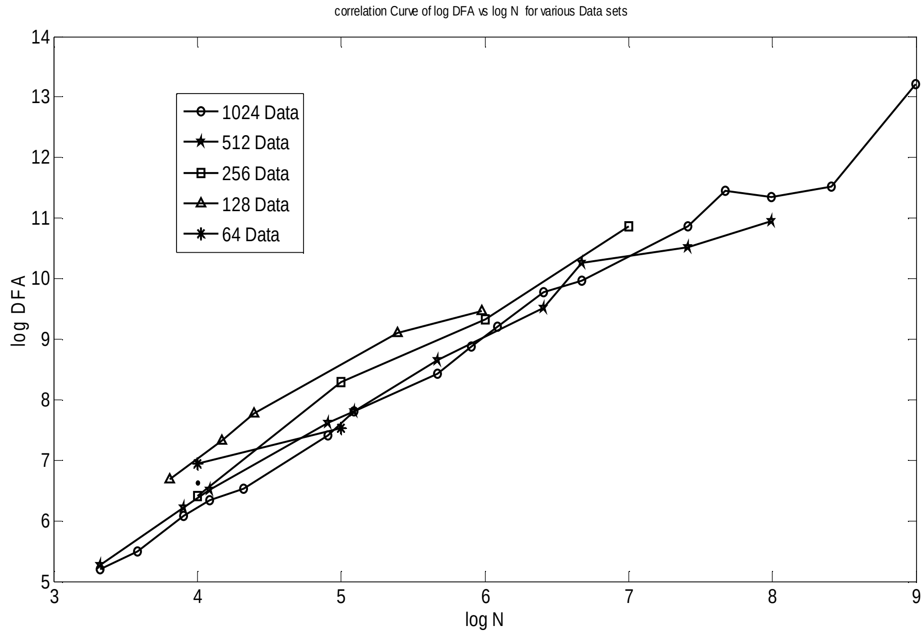

Figure 3: correlation Curve of log DFA vs log N for various Data sets

From the Detrended fluctuation analysis the log-log plot between the log (N) and log (DFA) for the whole experimental data is given in Figure 4 as regression curve. The slope of the line represents the value of the Hurst exponent.

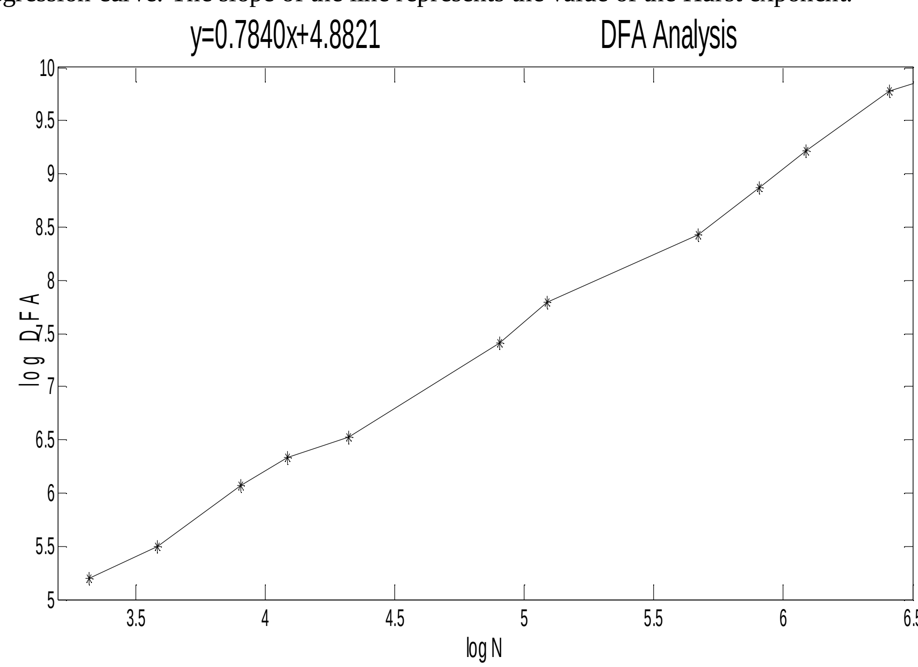

Figure 4: Regression curve of DFA

The Hurst exponent of the time series from the both algorithms show that, $0.5 < H < 1$ then series is said to have long memory phenomenon, i.e., it indicates persistent behaviour of the time series. This leads to the conclusion that the Gold rate time series is not random. If the Hurst exponent approaches 1, then it will reflect on the rise in the persistent behaviour of the time series. Comparing the above considered algorithms, the Hurst exponent calculated by the Detrended fluctuation analysis algorithm is closer to 1. Also by comparing the Fractal dimension of the two algorithms, the complexity of the time series is smaller in the Detrended fluctuation analysis algorithm compared to Rescaled range analysis algorithm. It shows that the Detrended fluctuation analysis algorithm is the most efficient than the Rescaled range analysis algorithm.

IV. CONCLUSIONS

In this paper, the Hurst exponent and the Fractal dimension for all 1024 data of the gold price rate from July 2010 to March 2017 are analyzed. We could find that the periods with large Hurst exponent can be predicted more accurately. This suggests that the Gold markets have the persistent behavior. The results show that the Fractal dimension by the Hurst exponent through the Detrended fluctuation analysis is the most consistent method compared to the Hurst exponent through the Rescaled range analysis in the analysis of the financial time series.

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