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Geodetic Polynomial and Detour Geodetic Polynomial of Radial Graphs

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Abstract: Geodetic polynomials of paths, complete graphs and complete bipartite graphs, radial graphs are given. The detour geodetic polynomials of complete, complete bipartite and radial graphs are also obtained.
Keywords: Distance, Detour, Geodetic Polynomial, Radial Graphs.AMS Classification: 05C12, 05C60, 05C75

I. PRELIMINARIES

In this section we present some definitions and results which are essential in the study of succeeding sections.

A. Introduction

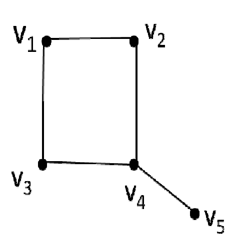
The standard distance d(u,v) between two vertices u and v in a connected graph G is the length of a shortest u-v path in G. Although this concept has been known for a very long time, it is only in recent decades that received considerable attention as a subject of its own. In 1990 Buckley and Harary wrote the book Distance in graphs. In 2004 Handbook of graph theory edited by Groes and Yellen[4] contains a section devoted exclusively to distance in graphs. A number of results on distance come from the fact that two vertices u and v are adjacent if and only if d(u,v)=1 and two distinct vertices u and v are non adjacent if and only if $d(u,v) \geq 2$. Hence any concept whose definition relies on the adjacency or non adjacency of two vertices in a graph can be restated in term of distance. In this paper we find the geodetic polynomials of some graphs, also we have found the detour geodetic polynomial of some graphs.

1) *Definition 1.1.1:* Let G(V,E) be a simple graph. Let u , v∈ V be any two vertices of G. A u-v path of length d(u, v) is called a u-v geodesic.

Let I[u,v] be the set of all vertices lying on some u-v geodesic of G. The set I(S) is defined by

$$I[S] = \bigcup_{u,v \in S} I[u,v]$$

2) *Definition 1.1.2:* A set S of vertices of G is called a geodetic set in G if I[S] = V[G] . A geodetic set of minimum cardinality is a *minimum geodetic set. The cardinalility of a minimum geodetic set in G is called geodetic number and is denoted by $\mathcal{G}(G)$.*
3) *Example1.1.3.* consider the following graph



Here $S_1=\{v_1, v_5\}$ and $S_2=\{v_1, v_2, v_3, v_5\}$ are geodetic sets.

Hence the Geodetic Number is 2

4) *Definition1.1.4*The eccentricity e(u) of a vertex u is the distance to a vertex farthest from u. The radius r(G) of G is defined by r(G) = min {e(u):u∈ v(G) }

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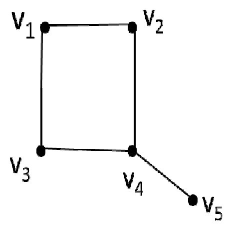
B. *Geodetic Polynomial*

Let $\mathcal{G}$ (G,i) be the family of geodetic sets of the graph G with cardinality I and $\mathcal{G}_e(G,i) = |\mathcal{G}\ (G,i)|$. Then the geodetic polynomial $\mathcal{G}$ (G,x) of G is defined as

$$\mathcal{G}\ (\ G, x) = \sum_{i=\mathcal{G}(G)}^{|V(G)|} \mathcal{G}_e\ (G,i)x^i$$

Where $\mathcal{G}$ (G) is the geodetic number of G.

1) *Example 1.2.1:*Consider The Following Graph**.**



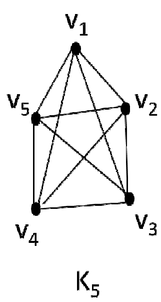
d(v_1,v_5) = 3 ; $v_1 - v_2 - v_4 - v_5$, $v_1 - v_3 - v_4 - v_5$, I (u,v) = $\{v_1,v_2,v_3,v_4,v_5\}$

$|\mathcal{G}_e(G,2)| = 1$,$|\mathcal{G}_e(G,3)| = 2$,$|\mathcal{G}_e\ (G,4)| = 3$,$|\mathcal{G}_e\ (G,5)| = 1$

$\mathcal{G}$ (G,x) $= \sum_{i=\mathcal{G}(G)}^{|V(G)|} \mathcal{G}_e\ (G,i)x^i = \sum_2^5\ \mathcal{G}_e\ (G,i)x^i$

$\mathcal{G}$ (G,x) $= x^2 + 2x^3 + 3x^4 + x^5$

2) *Geodetic Polynomial of Complete Graph*
3) *Result1.3.1.[***5]** $\mathcal{G}(K_n,x) = x^{n,}$ where K_n is the complete graph.
4) *Example1.3.2.*Consider the graph K_5



$\mathcal{G}$ (K_5, x) = $\sum \mathcal{G}_e\ (K_5,i)x^i$ = $1.x^5 = x^5$

C. *Geodetic Polynomial of complete bipartite Graph*

1) *Result1.3.1[5]:* $\boldsymbol{\mathcal{G}\left(K_{m,n},x\right) = x^n(1+x)^m + x^m(1+x)^n - x^{m+n}}$ **,m $\geq$ 2 ,n $\leq$ 4**
2) *Example1.3.2.*Consider the graph $K_{2,3}$

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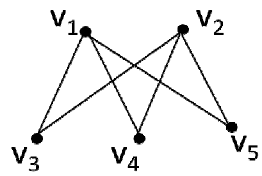
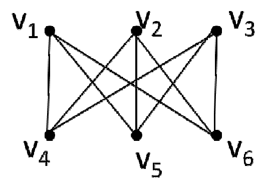


$\mathcal{G}$ ($K_{2,3}$, x) $= 1.x^2 + 4.x^3 + 5.x^4 + x^5$

3) Result1.3.2[5]
The geodetic polynomial of $K_{n,n}$is
a) $\mathcal{G}(K_{n,n},x) = x^n(2(1+x)n - x^n)$, $2 \leq n \leq 4$.
b) $\mathcal{G}(K_{n,n},x) = ((1+x)^n - 1)^2 - 2nx(1+x)^n + 2x^n(1+nx) + 2nx + n^2x^2$, $n > 4$.

*4) Example1.3.3*consider the graph $K_{3,3}$

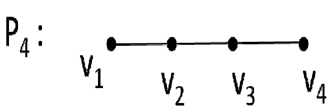


$\mathcal{G}$ ($K_{3,3}$, x) $= 2x^3 + 6x^4 + 6x^5 + x^6$

5) Result 1.3.4[5]
$\mathcal{G}$ ($K_{1,n}$, x) $= x^n + x^{n+1}$, $n > 1$

From the Figure ,it is easy to observe that $\mathcal{G}(G_1,x) = (x^2 + x^3)$, $\mathcal{G}(G_2,x) = x^3$
$\mathcal{G}(G,x) = (x^2 + x^3).x^3$
$\mathcal{G}(G,x) = x^5 + x^6$.

D. Geodetic Polynomial of Path
1) Result1.4.1[5]: Let $P_n$be a path with n vertices .The geodetic polynomial of P_n is
$\mathcal{G}(P_n,x) = x^2(1+x)^{n-2}$.
2) Example 1.4.2



$\mathcal{G}$(P_4, x) $= x^2 + 2x^3 + x^4$.

II. GEODETIC POLYNOMIAL OF RADIAL GRAPHS
In this section we find geodetic polynomial of some radial graphs

A. Definition 2.1
Two vertices of a graph are said to be radial to each other if the distance between them is equal to the radius of the graph.

B. Definition 2.2
The radial graph of a graph G, denoted by R(G) has the vertex set as in G and two vertices are adjacent in R(G) if and only if they are radial in G. consider the following graph

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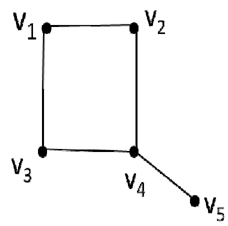
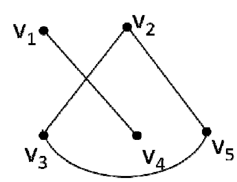


e(u) :e(v_1) = 3,e(v_2) = 2,e(v_3) = 2,e(v_4) = 2,e(v_5) = 3

The following verticex pairs are radial to each other.

(v_1, v_4) , (v_2, v_3), (v_2, v_5), (v_3, v_5)

Radial graph :



C. Result2.3[4]
Let $P_n$be any path on n ≥ 5 vertices then

$$R(P_n) = \begin{cases} \left(\frac{n}{2}\right)K_2 & \text{if n is even} \\ P_3 \cup \left(\frac{n-3}{2}\right)K_2 & \text{if n is odd} \end{cases}$$

*D. Result 2.5[4]*R($K_{m,n}$) = $K_m \cup K_n$
E. Result2.6[4]

Let C_n be any cycle on n ≥ 4 vertices then

$$R(C_n) = \begin{cases} \left(\frac{n}{2}\right)K_2 & \text{if n is even} \\ \cong C_n & \textit{if n is odd} \end{cases}$$

*F. Result 2.8 [4]*Every path P_n, n ≠ 4 is a radial graph ,(i.e) R ($\overline{P}_n$) = P_n
G. Theorem2.9

$\mathcal{G}$(R (P_n),x) = x^nif n is even, n ≥ 5

$\mathcal{G}$(R (P_n),x) = $x^{n-1} + x^n$if n is odd, n ≥ 5

Proof:

Since R(P_n) =$\left(\frac{n}{2}\right) K_2$, n is even and n ≥ 5 and $\mathcal{G}$(K_2) = x^2

Therefore the geodetic polynomial of R(P_n) is x^n

$\mathcal{G}(R(P_n),x)$ is x^n ,if n is even and n ≥ 5.

Since $R(P_n) = P_3 \cup \left(\frac{\square-3}{2}\right)\square_2$,If n is odd , n ≥ 5 and the result 2.3[3]

the geodetic polynomial of $R(\square_{\square})$ is

$\square(R(P_n),x) = \square(P_3).\{\ \square(K_2,x).\ \square(K_2,x)\ \ldots\ldots.\frac{\square-3}{2}\text{times}\ \}$

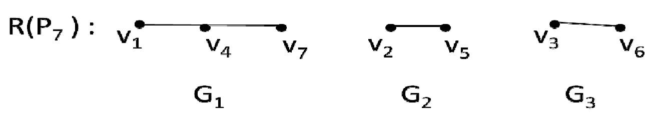
$= (\ \square^2 + \square^3).\square^2.\square^2.\square^2\ldots\ldots.\frac{\square-3}{2}$ times

$\square(R(P_n),x) = (\ \square^2 + \square^3\ ).\square^{2\left(\frac{\square-3}{2}\right)}$

$= (\ \square^2 + \square^3\ ).\square^{\square-3}$

$\square(R(P_n),x) = \square^{\square-1} + \square^{\square}$

H. Example 2.10



$\square(G_1,x) = \square^2 + \square^3$, $\square(G_2,x) = \square^2$, $\square(G_3,x) = \square^2$

$\square(R(P_n),x) = \left(\square^2 + \square^3\right).\square^2.\square^2$

$\square(R(P_n),x) = \square^6 + \square^7$

I. Theorem 2.11

$\square(\ R(\square_{\square,\square}),\ x\ ) = \square^{\square+\square}$

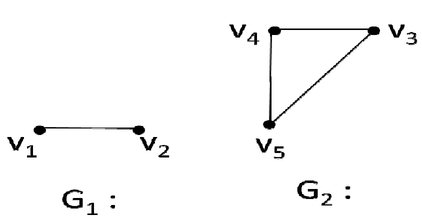
Proof:

Since $R(K_{m,n}) = K_m \cup K_n$ by the result 2.5[3] the geodetic polynomial of $R(K_{m,n})$

$\square(\ R(\square_{\square,\square}),\ x\ ) = \square((\ \square_{\square} \cup \square_{\square}),\ x\ )$

$= \square(\ \square_{\square},\ x\ ).\ \square(\square_{\square},\ x\ ) = \square^{\square}.\ \square^{\square} = \square^{\square+\square}$.

J. Example2.12 Consider the graph $R(K_{2,3})$



$\square(\ G_1,\ x\ ) = \square^2, \square(\ G_2,\ x\ ) = \square^3$

$\square(\ R(\square_{2,3}),\ x\ ) = \square^2.\ \square^3 = \square^5$

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III. DETOUR GEODETIC POLYNOMIAL OF GRAPHS

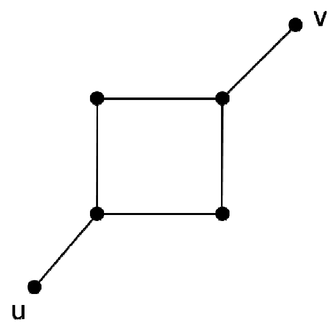
In this section we find detour geodetic polynomial of some graphs

A. Definition 3.1

Let G be a connected graph. The length of a longest u-v path between two vertices u and v in G is called the detour distance D(u, v) between u and v. A u-v path of length D(u, v) is a u-v detour.

B. Example 3.2

For a graph G given in the following figure, D(u, v) = 4.



C. Definition 3.3

A vertex w is said to lie on a u-v path P if w is a vertex of P. The closed interval $I_D[u,v]$ consists of u, v and all vertices lying in some u-v detour of G. For any $S \subseteq V$, $I_D[S] = \cup_{□,□\in□} I_D[u,v]$.

D. Definition 3.4

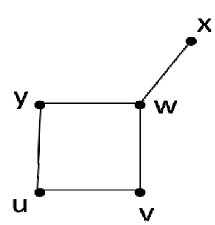
A set of S of vertices is called a detour geodetic set of G if $I_D[S] = V$. The minimum cardinality of a detour geodetic set is the detour number and is denoted by $d_g(G)$. A detour set of minimum cardinality $d_g(G)$ is called a minimum detour set of G or detour geodetic basis of G. Equivalently, a set S of vertices is called a detour geodetic set of G if every vertex of G lies on some detour joining a pair of vertices of G.

E. Definition 3.5

The upper detour geodetic number $dg^+(G)$ of G is the maximum cardinality of a minimum detour geodetic set of G.

F. Example 3.6

For the graph G given below



S = {x, u}, S_1 = {x, v}, S_2 = {x, y} are three minimum detour geodetic set so that$d_g(G) = 2$.

G. Definition 3.7

Let D □(G, i) be the family of detour geodetic sets of the graph G with cardinality i and let D $□_e$ (G, i) = |D □(G, i)|. Then the detour geodetic polynomal D □(G, x) of G is defined as

$$D □(G,x) = \sum_{□=d_g(G)}^{d_g+(G)} D □_e(G, i)x^i$$

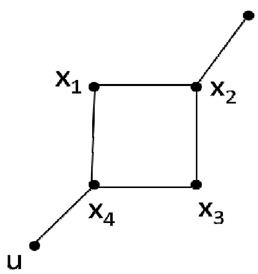
Where $d_g(G)$ is the Detour number of G.

H. *Example 3.8*

For the graph G, given in the above Figure S = {x, u}, S1 = {x, v}, S2 = {x, y}, dg(G) = dg+(G)the detour geodetic polynomial of the graph G is Defined byD□(□, x) = $\sum_{□=2}$ □□e (G, i) . $□^{□}$= D□(□, x) = $3x^2$.

I. *Example 3.9*

For the Graph G, Given in the Figure



D(u,v) = 4

IV. DETOUR GEODETIC POLYNOMIAL OF RADIAL GRAPH

In this section we find detour geodetic polynomial of some radial graphs

A. *Theorem 4.1*

Detour Geodetic Polynomial of $R(P_n)$ is $□^{□}$

(i.e) D□($R(P_n)$, x) = $□^{□}$ if n is even

Proof D□($R(P_n)$, x) = D□($\left(\frac{□}{2}\right)$ $□_2$, x)

= D□($□_2$, x) . D□($□_2$, x) D□($□_2$, x) ……..$\left(\frac{□}{2}\right)$times

= $□^2.□^2 . □^2$……$\left(\frac{□}{2}\right)$ times = $(□^2)^{\frac{□}{2}}$D□($R(P_n)$, x) = $□^{□}$.

B. *Theorem 4.2*

Detour geodetic polynomial of $R(P_n)$ is D□($R(P_n)$,x) = x^{n-1} if n is odd

1) *Proof* :D□($R(P_n)$,x) = D□(P_3),x) . {D□(((n-3)/2) K_2, x)}

= D□(P_3),x) . {D□(((n-3)/2) K_2, x)}….. $\frac{(n-3)}{2}$times.

= x^2 .x^2 . x^2 ………….. $\frac{(n-3)}{2}$ times.= $x^{2(n/2)}$.($x^{2(n-3)/2)}$)= x^{2+n-3}=x^{n-1}

C. *Theorem 4.3*

Detour geodetic Polynomial of $R(\bar{P}_n)$ isD□($R(\bar{P}_n)$, x) = x^2

1) Proof :Since D□($R(\bar{P}_n)$, x) = D□($\bar{P}_n$,x),

And P_n is a tree with 2 end vertices, $d_g(G)$ = d_g+(G) = 2 and

There is only one detour set with cardinality 2

D□($\bar{P}_n$), x) = x^2

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Hence $D \ (R(\overline{P}_n), x) = x^2$.

D. Theorem 4.4

The detour geodetic polynomial of $R(K_{m,n})$ is $D \ (R(K_{m,n}), x) = (mx^{m-1} + x^m) \,.\, (nx^{n-1} + x^n)$

1) Proof : Since $R(K_{m,n}) = K_m \cup K_n$

and $D \ (R(K_{m,n}), x) = D \ (K_m, x) \,.\, D \ (K_n, x)$

Now $D \ (K_m, x) = mx^{m-1} + x^m$, $D \ (K_n, x) = nx^{n-1} + x^n$

Hence $D \ (R(K_{m,n}), x) = (mx^{m-1} + x^m) \,.\, (nx^{n-1} + x^n)$.

V. CONCLUSION

Here Geodetic polynomial of path, complete graph, complete bipartite graph, radial graphs and detour geodetic polynomial of complete, complete bipartite and radial graphs have been studied. Further we can find the detour geodetic polynomial of other important graphs**.**

REFERENCES

[1] Buckley. F and Hararay, F. "Distance in Graphs", Addison Wesley, Reading (1990)
[2] Chartrand, G, and Zhang. P. "Distance in graphs" .In Handbook of graph
[3] Theory(ed.byJ.Gross and J.Yellen),CRC Press,Bocaraton,FL(2004) 873-888
[4] Chartrand, G, Harary. F and Zhang. P. (2000) "Geodetic Sets in Graphs", Discuss. Math.Graph Theory, 20 : 129-13
[5] Kathiresan. K.M and G. Marimuthu, "A study on radial graphs", ARS COMBINATORIA 96(2010) PP 353-36
[6] Vijayan. A and T. Binuselin, "An Introduction to Geodetic Polynomial of a Graph",Bulletin Pure and Applied Sciences, Volume 31E (Math and Stat.) Issue (No.1) 2012 : P 25-32.