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## **The Long-Wavelength Limit**

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Abstract: For optical frequencies, the wavelength (or decay length) of the transverse field both inside and outside the metal is much longer than the typical length scale on which longitudinal fields and induced charge densities (and, of course, the unperturbed charge density) at the surface vary. Even at and above the plasma frequency the longitudinal fields decay (due to damping effects) much faster than transverse fields, This is easily visualized within the hydrodynamic approximation of which yields for the z component of the wave vector of the longitudinal field,  $P_{\tau}$ , and of the transverse field,  $P_{t}$ , Keywords: longitudinal field, hydrodynamic approximation, transverse field

## I. INTRODUCTION

In this section one shows that in the LWL the surface response properties can be expressed in terms of two functions  $d_{\perp}(\omega)$  and  $d_{\Pi}(\omega)$ , which are related to mean values of the nonlocal dielectric tensor and its inverse.

$$p_{\ell}^{2} = \frac{5}{3v_{F}^{2}} \left[ \omega \left( \omega + i\gamma \right) - \frac{\omega_{n}^{2}}{\varepsilon_{b}} \right] - k_{x}^{2}$$
(4.21a)

and

$$p_{t}^{2} = \frac{\omega \varepsilon_{b}}{\omega + i\gamma} \frac{1}{c^{2}} \Big[ \omega \big( \omega + i\gamma \big) - \frac{\omega_{n}^{2}}{\varepsilon_{b}} \Big] - k_{x}^{2}$$
(4.21b)

respectively. Realistic values of the damping satisfy  $(V_F/C)^2 \omega_p \Box \gamma \Box \omega_p$  and the decay length of the plasma waves is typically by a factor  $V_F/C \Box 10^{-2}$  smaller than the decay length or wavelength of transverse waves, even at the plasma frequency. For typical s-p metals the plasma frequency is of the order of magnitude  $\overset{0}{\hbar} \omega_p \Box$  ev or less. For frequencies in this range (e.g.  $\omega < 1.5\omega_p$ ) the transverse fields inside the metal vary only on a scale  $\geq 10^3 \dot{A}$ . Here and in the following we assume, not explicitly state otherwise, that the effective thickness  $\overset{0}{d}$  of the surface region in which deviations from the asymptotic transverse field occur is of the order of several or, at most, several tens of  $\dot{A}$  angstroms i.e. we consider clean surfaces or surfaces covered with thin films (width  $\leq 10 \dot{A}$ ) and assume for  $\omega > \omega_p a$  elastic damping of plasma waves. Then  $\overset{0}{d}\omega/c \ll 1$ , and in the rest of

this Sect. certain only leading order terms with respect to the small parameter  $d\omega/c$  consider the long-wavelength limit (LWL), which has extensively been discussed FEIBELMAN<sup>1-3</sup>.

In the LWL, simplifications of the following type occur. Field components, which are already continuous in the local Fresnel

treatment, very slowly in the surface region and are essentially constant over a distance d. Since the bulk response to transverse field is assumed to be local anyhow, these field components can be taken out of the integrals which specify the nonlocal constitutive equations. So that these effectively reduce to local equations even in the surface region, Arguments this type have extensively been discussed in the literature <sup>1-3</sup>.

On considers first the simpler case of 8 *polarizations*. In the LWL one may consider  $E_y(z)$  as constant in the surface region, which implies

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$$E_{y}(\xi_{2}) - E_{y}(\xi_{1}) = E_{y}^{>}(a) - E_{y}^{<}(a) = 0$$
(4.22)

so that  $E_y^0(z;a) = E_y(z)$  and  $n_y^{(n)}(a) = 0$  from (4.9b), since the response in the bulk where only transverse fields survive, is assumed local and since  $E_y(z)$  is practically constant in the surface region, where nonlocal effects can occur, the constitutive equation.

$$D_{y}(z) = \int_{-\infty}^{+\infty} dz' \varepsilon_{yy}(z,z') E_{y}(z') \approx \int_{-\infty}^{+\infty} dz' \varepsilon_{yy}(z,z') E_{y}(z')$$
(4.23)

reduces effectively to a local one. The effective local dielectric function

$$\varepsilon_{yy}(z) = \int_{-\infty}^{\infty} dz' \varepsilon_{yy}(z,z') E_y(z)$$
(4.24)

interpolates smoothly between the values  $\varepsilon_a$  for  $z < \xi_1$  and  $\varepsilon_t$  for  $z > E_2$ . As a consequence of (4.22-24), one obtain for (4.9a)

$$\delta_{y}^{(n)}(a) = \int_{-\infty}^{+\infty} dz \left(z-a\right)^{n-1} \left\{ \varepsilon_{yy}(z) - \left[\varepsilon_{a}\theta(a-z) + \varepsilon_{t}\theta(z-a)\right] \right\}$$
(4.25)

Where, for  $\xi_1 < a < \xi_2$ , contributions 6to the integral come from  $\xi_1 < z < \xi_2$ . With  $\overset{\square}{d} = \xi_2 - \xi_1$  we estimate the order of magnitude  $\delta_y^{(n)} \square d^{-n} (n = 1, 2)$  so that in the LWL only  $\delta_y^{(1)}$  contributes to (4.18-20) whereas  $p_t \delta_y^{(2)} |<<|\delta_y^{(1)}$ . Expanding the phase factors in (4.18-20) our obtain in the LWL ( $[a] \le d$ ), using  $p_t^2 - p_a^2 = (\varepsilon_t - \varepsilon_a) \omega^2 / c^2$ ,

$$r_{s} = \frac{p_{t} - p_{a}}{p_{t} + p_{a}} [1 + 2ip_{a}d_{\Pi}(\omega)]$$
(4.26)

$$t_{s} = \frac{2p_{a}}{p_{t} + p_{a}} [1 - i(p_{t} - p_{a})d_{\Pi}(\omega)]$$
(4.27)

where

$$d_{\Pi}(\omega) = \frac{\delta_{y}^{(1)}(a)}{\varepsilon_{a} - \varepsilon_{t}} + a$$
(4.28)

For a = 0, (4.26) reduces to a result of BAGCHI et. a110, who used the notation  $\wedge_y(\omega)$  for  $\delta_y^{(1)}(0)$ . The surface function  $d_{\Pi}(\omega)$ , (4.28), is independent of a, as is easily seen from the derivative of (4.25) with respect to a. It has been introduced (for  $\varepsilon_a = 1$ ) previously by FEIBELMAN and may be written in several equivalent forms. Integrating by parts one obtains from (4.25) for instance,

$$d_{\Pi}(\omega) = \frac{1}{\varepsilon_{t} - \varepsilon_{a}} \int_{-\infty}^{+\infty} dz \, z \frac{d}{dz} \varepsilon_{yy}(z)$$
(4.29)

One want to emphasize that the assumption  $E_y(z) = \text{const.}$  and the neglect of  $n_y^{(n)}$  correct in lowest order of  $\omega d/c$ . This can be seen as follows. Using  $k_x^2 n_y^{(2)} \ll (\omega)/c^2$  as input, we obtain from the generalized matching condition (4.18)

$$\delta_{y}^{(n)}(a) - E_{y}^{<}] / E_{y}^{>}(0) = \omega^{2} \delta_{y}^{(2)} / c^{2} - (\omega \tilde{d} / c)^{2}$$

Marked of (4.22). this yields the estimate  $[E_y(z) - E_y^0(z;a)] / E_y^>(0) \square (\omega \tilde{d}/c)^2$  for  $\xi_1 < z < \xi_2$  and from (4.9b),  $n_y^{(n)} \square \stackrel{-n}{d} (\omega \tilde{d}/c)^2$ . Since  $k_x \stackrel{\square}{d} | << 1$ , the leading order is remin unaffected. It can p polarization the situation is a little more



complicated, since in principle dielectric tensor  $\varepsilon_{\mu\nu}(q_x; z, z')$  is not diagonal, e.g.,  $\varepsilon_{xz} \neq 0$ , and x and z components of electric and displacement field are coupled. But in the LWL this coupling and can be neglected, as has been shown explicitly in a recent RPA calculation and as is plausible from the following argument. Since the isotropic inverse dielectric constant  $\varepsilon_t$  correctly describes the response deep inside, the coupling can occur only in the surface region  $(z, z' \Box d)$ . From rotational of the metal around the surface normal one can easily show that  $\varepsilon_{xz}$  and must be proportional to  $q_x$ . Then the no diagonal elements of the dielectric is must be much smaller, typically by a factor of the order  $q_x d <<1$ , then the elements. By similar arguments one can show that in the LWL  $\varepsilon_{xx}$  and  $\varepsilon_{yy}$ . The reason for these simplifications to hold is that  $q_x$  is much smaller typical electronic momenta,  $q_x << k_F$ . (This should not be confused with the limit  $q_x <<\omega/c$ , i.e. nearly perpendicular incidence, which is neither reduced nor implied.)

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