Design & Development of an Offset Unfurlable Parabolic Reflector for a Satellite Antenna

Haresh K. Devjani¹, Pratik D. Patel², Akshay Mahitkar³, Y. D. Patel⁴, A. R. Srinivas⁵

Mechanical Engineering Department, A. D. Patel Institute of Technology, Karamsad. Space Applications Centre-ISRO, Ahmedabad.

Abstract: Satellite based communication is on rise day-by-day. The utilities of satellite communication are penetrating in many walks of human life. The rise in application of satellite has also increased the need of large data transfer at faster rate. This, in addition, has generated the demand of large size deployable reflector to have wide coverage, larger bandwidth and high gain antenna. The primary requirement of deployable/unfurlable reflectors stems from the limitations of the launch vehicle fairing size. The technological development of reflectors is moving forward in two directions, larger size and high accuracy. The progression of research activities in space exploration proposes various concepts to increase the size and accuracy of the reflectors. Typical large deployable structure includes inflatable reflectors, solid deployable reflectors and mesh deployable reflectors. Out of these, the mesh deployable reflector has been a consistent area of interest due to its inherent advantages of lightweight and low stowed area. Mesh deployable reflectors consists of pliable gold plated molybdenum wire knitted in the form of mesh to form the reflecting surface. However, the deployed surface accuracy (~ 5 mm) and frequency range (usually 2-4 GHz) of mesh deployable reflectors is restricted by the manufacturing limitations of fine wire mesh. The present work proposes a novel approach for the development of reflector surface for an offset parabolic unfurlable reflector and elucidates the method of obtaining the offset reflector surface from a fabric. The work also includes the method for testing the reflector surface and improvement in the surface accuracy of the reflector.

Keywords: Mesh deployable, Unfurlable, CFRP, Reflectors, Space Antennas, Photogrammetry.

I. INTRODUCTION

The increase in satellite-based global communication has led to the need of large diameter, cheaper, lighter, more durable and more precise reflectors to for insertion into the Earth orbit. The size of non-deployable reflectors is constrained (4.7 meters) by the shroud size of the launch vehicles. Therefore, the deployable reflectors are gaining fame due to their inherent advantage of low mass, reduced stowed volume at maximum achievable deployed diameter and surface accuracy. Amongst the solid and mesh deployable reflectors, the latter is being widely used after its first successful development in 2003, Astromesh Reflector [1]. The mesh deployable reflector surface consists of a fine gold plate molybdenum wire knitted together with tricot knot to form a mesh grid. Due to the manufacturing limitations and realization infeasibility of thin wire to form closed loop cells, the applicability of this technology is limited to 2-4 GHz in S-band [2]. The present work entails the development of reflector surface for an unfurlable reflector using a fabric. The same idea can be extended to develop the reflector surface from CFRP fabric. The use of CFRP as a solid or fixed reflector surface has been demonstrated earlier in CFRP skin antenna reflector [3] and Ultra-thin deployable reflector [4]. Hence, the novel idea to use CFRP fabric as the reflector surface for 6.0m unfurlable reflector has been developed and the method to obtain a reflector surface from a fabric has been demonstrated.

II. CALCULATIONS FOR PARABOLIC REFLECTOR DEVELOPMENT

The idea to develop a prime focus parabolic reflector shape from a flat surface has been established by in [5]. The same technique has been extended to develop the surface for offset parabolic reflector.

A. Central focus parabolic reflector

The work involves joining an ‘n’ petal flower shaped flattened surface along the radial direction to obtain a paraboloid. The work is primarily based on the premise that the radius of actual paraboloid is less than that of flattened paraboloid. Hence, the difference between their circumferences can be used to determine the length of arc to be deducted from a plain circle to obtain the flower shape as shown in Figure 1. The total length of arc is equally divided into number of petals so that uniform shape can be obtained from a flat surface with minimum wrinkle formation. The basic calculation involved in the formation of central focus parabolic reflector surface is discussed below.
Arc length of a curve,
\[ l = \int_{x_1}^{x_2} \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \]  
(1)

The equation of the parabola with focal length ‘a’ and symmetric about x-axis can be presented in the following form:
\[ x^2 = 4a \cdot y \]  
(2)

Hence, from equation (2), using, \( \frac{dy}{dx} = \frac{x}{2a} \) and substituting in equation (1), we can obtain the arc length as:
\[ l = \int_{x_1}^{x_2} \sqrt{1 + \left( \frac{x}{2a} \right)^2} \, dx \]  
(3)

![Fig. 1 Flattened Parabola (n=12)](image)

By using the limits \( x_1 = 0 \) and \( x_2 = x \), the radius ‘\( R \)’ of flattened parabola corresponding to the parabolic radius ‘\( x \)’ can be obtained. Therefore, two circles with different radii ‘\( x \)’ and ‘\( R \)’ will provide the required circumference difference. Total arc length to be deducted from the circle:

\[ C_1 = 2 \cdot \pi \cdot R - 2 \cdot \pi \cdot x \]
\[ C_2 = 2 \cdot \pi \cdot (R - x) \]  
or
\[ C_2 = \frac{C_1}{2} = \frac{2 \cdot \pi \cdot x}{2} \]  
(4)

This compensation is equally divided into ‘\( n \)’ number of petals to form a uniform surface with minimum surface errors and wrinkles:
\[ C_2 = \frac{C_1}{2n} = \frac{2 \cdot \pi \cdot x}{2 \cdot n} \]  
(5)

B. Offset Parabolic surface

The offset-feed parabola is the subset of prime-focal parabola which is obtained by extruding a circle at some pre-defined offset distance from the center of the parabola. The main advantage of the offset-feed parabola is that the feed structure does not obstruct the field-of-view (FOV) of the reflector resulting in increased gain and directivity of the waves. For an offset feed reflector surface as shown in Figure 2, first, the total compensations are calculated for parent central focus parabola with the same procedure. Then the compensation required for the subset (offset) parabola is calculated using the angle subtended by the boundary of offset parabola with the arcs intersecting it at different radii.

![Fig. 2 Modelling of Offset-Feed Reflector](image)

The included angle out of 360 degrees indicates the portion out of total compensation to be made for offset-feed parabola:

\[ \text{compensation for offset reflector} = \frac{\text{included angle}}{360} \cdot \text{total compensation} \]  
(5)
III. ACTUAL DEVELOPMENT OF THE REFLECTOR

After optimizing for a good number of combinations, the 6.0 m offset reflector surface is divided into 122 parts, considering cutting accuracy, handling ease, capability of stitching, minimization of wrinkles and maintaining a required level of accuracy in post-stitched surface. A nylon fabric was used to simulate the actual shape of the reflector after stitching. Efforts were made to reduce the formation of wrinkles by providing sufficient compensations during the stitching. A cutting plan for 6.0 meter offset reflector and the reflector surface made up of a white nylon fabric is as shown in the Figure 3. The surface of reflector was used in multiple deployment cycles of reflector.

A. Strategy for Shape Adjustment of the Reflector

The method discussed in previous section provides an approximate surface of the reflector which can be treated as the initial surface of the reflector. The control of local deviations in the reflector surface is done with the help of strings. The strings are used to tie the surface of reflector to a similar inverted surface. By adjusting the length of the strings, the deviations in the surface can be controlled locally as well as globally as shown in the Figure 4.

IV. MEASURING THE REFLECTOR SURFACE

The measurement of surface accuracy was carried out using photogrammetry to determine the RMS error in the reflector surface. The testing includes setting up the target points on the reflector surface and determining the location of those target points with respect to a coordinate system using digital image associativity. A total of 122 target points were placed on the reflector surface at specified locations after the final adjustment of the surface. The plot of the target points on the reflector surface in the CAD model is as shown in the Figure 5. The final output data from the testing was a cloud of points along with their coordinates. These coordinates were then used to generate the surface of the reflector.
A. Orientation of reflector surface

The cloud of points was then fitted in a closed form surface by different tools available. The ideal reflector surface is an offset reflector; hence it was necessary to orient the cloud of data points before carrying out the surface fitting. One method to transform the coordinates of the reflector is using rotation angles $\beta$ in the $x$-$z$ plane and $\gamma$ in the $y$-$z$ plane [6]. The transformed coordinates $x'$, $y'$ and $z'$ with origin translated to $(a, b, c)$ are given as:

$$
\begin{pmatrix}
  x' \\
  y' \\
  z'
\end{pmatrix} = \begin{pmatrix}
  1 & 0 & 0 \\
  0 & \cos \beta & -\sin \beta \\
  0 & \sin \beta & \cos \beta
\end{pmatrix} \begin{pmatrix}
  x - a \\
  y - b \\
  z - c
\end{pmatrix}
$$

(7)

This transformation can be represented as:

$$
\mathbf{B} = \mathbf{T} \cdot \mathbf{A}
$$

(8)

where set $\mathbf{B}$ represents the transformed set of points and set $\mathbf{A}$ represents the original set of points.

The original set of points $\mathbf{A}$ were obtained from measurement results. The transformation matrix $\mathbf{T}$ was obtained using the transformation of three points from set $\mathbf{A}$ to set $\mathbf{B}$ as shown in Figure 6. Any plane can be defined using three points and hence the transformation of the plane passing through three points was carried out to determine the orientation of the reflector surface. Three points were identified from the testing results and the corresponding point coordinates were extracted from ideal reflector surface in the CAD model. The required transformation matrix is given by:

$$
\mathbf{T} = \left( \mathbf{B} \cdot \mathbf{A} \right)^{-1}
$$

(9)
B. Surface fitting of Point Cloud

Using the same transformation matrix $T$, all the points in set $A$ were transformed into the required orientation as shown in Figure 7. For best fitting of the data points, least squared approximations were used. The equation of a general quadratic in a Cartesian space can be given as [7]:

$$f(x, y, z) = (x^2 + y^2) \frac{a}{4} z^2 + x + y + z = 0$$

(10)

Fig. 7 Final MATLAB plot of oriented target points

The above equation can be represented in matrix form as [8]:

$$x_0^T A x_0 + g^T x_0 = d$$

(11)

where,

$$A = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \frac{1}{4f^2}
\end{bmatrix}, \quad g = [0, 0, -1]^T, \quad d = 0 \quad \text{and} \quad x_0 = [x, y, z]^T.$$

Using the above matrix form, the least square approximations for surface fitting can be carried out with the following objective function:

$$\text{Minimize Error,} \quad E = \frac{1}{2} \sum_{i=1}^{n} (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2$$

(12)

where $(x, y, z)$ represents the points obtained from the testing and $(x, y, z)$ represents the points on the surface of the ideal reflector. The error $E$ represents the root mean square error in surface fitting.

V. RESULTS

Least-squared approximations of the target points to best fit the offset parabolic reflector surface was carried out using MATLAB. The root mean square error in best fitting the reflector surface after first iteration of photogrammetric testing was 18.22 mm. The result from the first iteration of photogrammetric testing was used to obtain a residual plot. Residual plot determines the deviations of different target points from the ideal reflector surface as shown in the Figure 8. The best fitting of the cloud of points was carried out based on equation of parabola as given by equation (10) for a focal length of 4380 mm. Using the residual plot of 1st iteration of photogrammetry, the shape adjustments in the reflector surface was carried out with the help of string and tie-cables as depicted in Figure 4. After adjusting the reflector surface for minimum deviation, the 2nd iteration of the photogrammetric testing was carried out.
The same procedure was used to best fit the reflector surface for the data obtained from 2nd iteration of photogrammetry. The root mean square in best fitting the parabolic surface from the cloud of points obtained in the second iteration of photogrammetry was 6.17 mm. The overall improvement in the surface accuracy of the reflector surface was 66.13% in single iteration of testing.

VI. SUMMARY AND CONCLUSION
The final RMS error in the reflector surface after 2nd iteration of photogrammetry was found to be 6.17 mm, which can be reduced to below 5 mm by adjusting 10 identified points from the testing results, making the Unfurlable reflector suitable for the S-band range (2-4 GHz). The final best-fit surface of the reflector is as shown in Figure 9. Further, it can be concluded that with each iteration of testing, the RMS errors can be reduced using the residual plots to obtain the desired level of accuracy as per the operating frequency of the reflector. The suggested future directions for the work may include the development of reflector surface using CFRP fabric. With the applicability of CFRP as a reflector material and the method to realize a reflector surface from a fabric as suggested in the present work, the surface for unfurlable reflector can be developed using CFRP fabric.

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