Strength Reduction Factors Determined by Considering Ductility and Time Period

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Abstract: Strength reduction factors (Rµ) by various investigators over the past years are studied and simplified expressions are presented. The main parameters affecting the magnitude of strength reductions are considered and models are studied. The evaluation of the results indicate that the strength reduction factors are primarily affected by ductility demand, period of the structure and soil conditions and other parameters like hysteretic behaviour, damping, etc. make expressions comparably more complicated.

Keywords: Strength reduction factor, ductility demand, time period, hysteretic behaviour, damping

I. INTRODUCTION

Recently, engineers have been showing greater interest in performance based seismic demand, in which performance of the structure is given more importance. In current codes, elastic design spectra are taken in consideration including inelastic response in somewhat implicit and indirect manner. In performance based design, inelastic response spectra are a key for seismic design to resist seismic motions. Inelastic response spectra are obtained from elastic response spectra by modifying such elastic response spectra to take account of inelastic behaviour and damping. It can be estimated by dividing elastic response spectra by strength reduction factor (Rµ),

\[ S_{\text{(inelastic)}} = \frac{S_{\text{elastic}}}{R\mu} \]

Estimation of required strengths, ductility demands or inelastic displacements is usually done using strength reductions factors (Rµ). for an elastoplastic single-degree-of freedom oscillator subjected to a given ground motion, Rµ is the ratio between the strength required for elastic behaviour and the strength for which ductility demand equals µ. If F(T, µ) is the spectrum of required strengths, then,

\[ R\mu(T) = \frac{F(T, \mu)}{F(T, 1.0)} \]

Strength reduction factors depend on the perceived seismic damage performance of the structure, characterized by ductile or brittle deformations. Strength reduction factor, also known as Response reduction factor in some codes, have been derived by several researchers considering time period of the system and ductility factor as important parameters. Different factors have been considered by different researchers for deriving strength reduction factors, like hysteretic behaviour, damping, ground motion, soil conditions and soil structure interaction.

Many research investigations conducted have determined R factors considering natural period of the system and ductility factor as great significance in proposing expressions to estimate strength reduction factor (Rµ). One such expression considering natural period and ductility factor was proposed by Vidic et. al. Simplified expressions based on the response of bilinear and stiffness degrading (Q-model) systems for 20 ground motions recorded from California, Montenegro, Chile, and Mexico City, were proposed. The simplified expressions consist of two linear segments. First segment corresponds to the short period region where R increases linearly with increasing period from R = 1 to a value that is equal or nearly equal to ductility factor. In second segment, the strength reduction factor maintains a constant value.

The proposed R-factor relations for bilinear systems and 5% mass-proportional damping are given by the following bilinear curve:

For \( T \leq T_o \)

\[ R\mu = \left( \mu - 1 \right) \left( \frac{T}{T_o} \right) + 1.0 \]

For \( T > T_o \)

\[ R\mu = \mu \]

where \( T_o \) and \( T_1 \) is given by,
After Vidic et al., many researchers tried deriving simpler expressions for strength reduction factor. Miranda\(^3\), recommended strength reductions due to non-linear behaviour influenced by the maximum tolerable displacement ductility demand, the period of the structure and the soil conditions at the site. Based on the results of a comprehensive statistical study on strength reduction factors of SDOF systems undergoing different levels of inelastic deformation when subjected to ground motions, simplified expressions were proposed for design of structures built on rock or firm sites:

\[
R_\mu = \mu + (1 - \mu) \exp \left( -\frac{4\pi T}{\mu} \right)
\]

Where \( \mu \) is the displacement ductility ratio and \( T \) is the period of vibration. This equation is said to be simpler than the equations previously proposed.

Similarly, Ordaz et al.\(^4\) presented a new rule for SDOF elasto-plastic oscillators, in which reduction factors depend only on displacement elastic spectra. Inelastic displacement for given ductility and period, \( D(T, \mu) \), is computed with,

\[
D(T, \mu) = D(T) \frac{\mu}{R_\mu(T)}
\]

Where \( D(T) \) is the elastic relative displacement spectrum.

Based on 445 ground motions from the Guerrero and Mexico City Accelerographic Arrays with elastoplastic systems having \( \beta = 5\% \) and \( \mu = 1.5, 2, 4, \) and 8, \( R_\mu \) proposed was,

\[
R_\mu(T) = 1 + \frac{D(T)\beta(\mu)}{D_{\text{max}}} (\mu - 1)
\]

Where,

\[
\beta(\mu) = 0.388 (\mu - 1)^{0.173}
\]

Tiwari et al.\(^6\) proposed a preliminary scaling model for estimating the strength reduction factors (SRF) of horizontal ground motion in terms of earthquake magnitude, strong motion duration and predominant period of the ground motion, geological site conditions, and ductility ratio. A parallel scaling model has been proposed for estimating the damage based SRF instead of ductility based SRFs. Damage and ductility supply ratio were considered as parameters in this model.

Scaling of Ductility-Based SRFs The scaling equation for \( R_\mu(T) \) is given as,

\[
\log_{10} R_\mu(T) = b_1(T) \log_{10} T_s + b_2(T) T_s + b_3(T) M + b_4(T) M^2 + b_5(T)
\]

Here, \( b(T) \)s are the period-dependent coefficients obtained for a set of geologic site conditions and ductility ratio.

Scaling of Damage-Based SRFs:
The scaling equation for \( R_0(T) \) is given as,

\[
\log_{10} R_0(T) = b_1(T) \log_{10} T_s + b_2(T)g + b_3(T)M + b_4(T)M^2 + b_5(T)
\]

Here, \( b(T) \) are the period-dependent coefficients. These coefficients also depend on ductility ratio, damage, and site category, as the role of these parameters has been considered only through variations in these coefficients.

Apart from ductility ratio, time period, site conditions, \( R \)-factor can be derived using pulse waveforms. Cuesta et al.\(^{[8]}\), in his study, derived \( R \)-factor from the response to a simple pulse, which is dependent on pulse shape, ductility (\( \mu \)) and load deformation model and the period of the system (\( T \)) relative to the characteristic period of the pulse (\( T_g \)), and were compared with those obtained using other contemporary \( R_\mu - T \) relationships for elasto-plastic, bilinear, and stiffness-degrading SDOF systems.

R-factors of 24 different pulse waveforms and their applicability to estimate the inelastic response of SDOF systems subjected to ground motions were investigated. The expression given by Vidic et al., was restated in terms of characteristic period identified by Cuesta and Aschheim and the original coefficients were modified.

\[
R = \begin{cases} 
  c_1(\mu - 1) \frac{T}{T_g} + 1 & \frac{T}{T_g} \leq 1 \\
  c_1(\mu - 1) + 1 & \frac{T}{T_g} > 1
\end{cases}
\]

where \( T_g \) is given by,

\[
T_g = 2\pi \frac{(S_v)_{max}}{(S_d)_{max}}
\]

And \( c_1 = 1.3 \) for an accurate estimate of the \( R \) factors for systems with limited stiffness degradation or \( c_1 = 1.0 \) for systems with substantial stiffness degradation.

II. CONCLUSIONS

A. The following Conclusion can be Drawn from This Study on Evaluation of Strength Reduction Factors

1) Strength reduction factors are used to reduce design forces in earthquake resistant design. Based on these investigations it can be concluded that strength reduction factor is primarily a function of the maximum tolerable displacement ductility demand, the period of the structure and the soil conditions at the site. There are other factors like type of hysteretic behavior, damping and ground motion, effecting strength reduction factor but to a much lesser degree.

2) Among all the proposed strength reduction factor (\( R_\mu \)) given by the authors, Vidic et al., provided much simplified expressions of \( R_\mu \) considering natural period of the system and prescribed ductility factor as governing factors and obtaining a bilinear curve for the \( R \)-spectrum, making it user friendly.

3) After Vidic et al., other factors like redundancy, strain hardening, element overstrength, ground motions, etc were incorporated in the expressions for strength reduction factors (\( R_\mu \)) making it complicated and robust.

REFERENCES