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Orthogonal Fuzzy Matrix using Hexagonal Fuzzy Numbers

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Abstract: In this paper, a method for solving orthogonal matrix in which the hexagonal fuzzy numbers are involved in the elements of the fuzzy matrix is proposed. Moreover, some results on orthogonal fuzzy matrices are also discussed. Numerical examples are also provided for verifying the results.
Keywords: Fuzzy matrix, hexagonal fuzzy number, hexagonal fuzzy matrix, orthogonal hexagonal fuzzy matrix

I. INTRODUCTION

Matrices play an important role in various areas in science and engineering to represent any binary relation. However, we cannot successfully use classical matrices because of various types of uncertainties present in real world situations such as problems in economics, engineering, environment science, social science, medical science etc. These types of problems are solved by using fuzzy matrices. Fuzzy matrix is a very important topic of fuzzy algebra. Fuzzy matrices plays an important role in scientific development. Fuzzy matrices were introduced by Thomson[3] in 1977 and discussed about the convergence of the powers of a fuzzy matrix. A fuzzy matrix is a matrix which has its elements from [0,1].All fuzzy matrices are matrices but every matrix is not a fuzzy matrix. This paper is organized as follows: In section 2, hexagonal fuzzy numbers , ranking function and arithmetic operations of hexagonal fuzzy numbers are given as in [1]. Section 3 deals with hexagonal fuzzy matrices and its arithmetic operations as in [2]. In section 4, orthogonal hexagonal fuzzy matrix and some theorems on the orthogonal hexagonal fuzzy matrices are discussed. In section 5, Numerical example for orthogonal hexagonal fuzzy matrix is provided. The last section draws some concluding remarks.

II. PRELIMINARIES

In this section, hexagonal fuzzy numbers and their arithmetic operations as in [1] and also hexagonal fuzzy matrices and its arithmetic operations are discussed as in [2].

A. Hexagonal Fuzzy Number

Let us consider a hexagonal fuzzy number $\tilde{A}_h = (a_1, a_2, a_3, a_4, a_5, a_6)$, where $a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6$ which are real numbers satisfying $a_2 - a_1 \le a_3 - a_2$ and $a_5 - a_4 \le a_6 - a_5$ and its membership function is given by,

$$\mu_{\tilde{A}h}(x) = \begin{cases} 0 & if\ x < a_1 \\ \frac{1}{2}\left(\frac{x - a_1}{a_2 - a_1}\right) & if\ a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_2}{a_3 - a_2}\right) & if\ a_2 \le x \le a_3 \\ 1 & if\ a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x - a_4}{a_5 - a_4}\right) & if\ a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6 - x}{a_6 - a_5}\right) & if\ a_5 \le x \le a_6 \\ 0 & if\ x \ge a_6 \end{cases}$$

B. Ranking function

Let $\mathscr{F}$(R) be the set of all hexagonal fuzzy numbers.

For $\tilde{A}_h = (a_1, a_2, a_3, a_4, a_5, a_6) \in \mathscr{F}(R)$, we define the ranking function, $\check{R}: \mathscr{F}(R) \to R$ by

$$\check{R}(\tilde{A}_h) = \left(\frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{6}\right)$$

as in [1]

C. Arithmetic Operations On Hexagonal Fuzzy Numbers

For $\tilde{A}_h=(a_1,a_2,a_3,a_4,a_5,a_6)$ and $\tilde{B}_h=(b_1,b_2,b_3,b_4,b_5,b_6)$ in $\mathscr{F}(R)$, we define

1) *Addition :* $\tilde{A}_h + \tilde{B}_h = (a_1+b_1 , a_2+b_2 , a_3+b_3 , a_4+b_4 , a_5+b_5 , a_6+b_6)$
2) *Subtraction:* $\tilde{A}_h - \tilde{B}_h=(a_1-b_6 , a_2-b_5 , a_3-b_4 , a_4-b_3 , a_5-b_2 , a_6-b_1)$
3) *Multiplication:* $\tilde{A}_h * \tilde{B}_h=(a_1\check{R}(\tilde{B}_h), a_2\check{R}(\tilde{B}_h) , a_3\check{R}(\tilde{B}_h) , a_4\check{R}(\tilde{B}_h) , a_5\check{R}(\tilde{B}_h) , a_6\check{R}(\tilde{B}_h))$
4) *Division*: $\tilde{A}_h / \tilde{B}_h = (a_1/ \check{R}(\tilde{B}_h), a_2/ \check{R}(\tilde{B}_h), a_3/ \check{R}(\tilde{B}_h), a_4/ \check{R}(\tilde{B}_h), a_5/ \check{R}(\tilde{B}_h), a_6/ \check{R}(\tilde{B}_h))$

Where $\check{R}(\tilde{B}_h) = \dfrac{b_1 + b_2 + b_3 + b_4 + b_5 + b_6}{6}$

III. HEXAGONAL FUZZY MATRIX

A fuzzy matrix $\tilde{A}=(\tilde{a}_{hij})_{mxn}$ of order mxn is called a Hexagonal Fuzzy Matrix , if the elements of the matrix are Hexagonal Fuzzy Numbers i.e., of the form $(a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}, a_{ij5}, a_{ij6})$

Example

$$\tilde{A} = \begin{bmatrix} (0,1,2,2,3,4) & (2,3,4,4,5,6) \\ (4,5,6,6,7,8) & (6,7,8,8,9,10) \end{bmatrix}$$

is a Hexagonal Fuzzy Matrix.

A. Arithmetic Operations on Hexagonal Fuzzy Matrices

Let $\tilde{A} = (\tilde{a}_{hij})_{mxn}$ and $\tilde{B} = (\tilde{b}_{hij})_{mxn}$ be two hexagonal fuzzy matrices on same order, then we have the following

1) *Addition :* $\tilde{A} + \tilde{B} = (\tilde{a}_{hij} + \tilde{b}_{hij})$
2) *Subtraction:* $\tilde{A} - \tilde{B} = (\tilde{a}_{hij} - \tilde{b}_{hij})$
3) *Multiplication:* $\tilde{A}*\tilde{B} = (\tilde{c}_{hij})_{mxk}$ where, $(\tilde{c}_{hij})_{mxk} = \sum_{p=1}^{n} \tilde{a}_{hip} \ \tilde{b}_{hpj}$,

   i=1,2,3,…….,m ; j=1,2,3,……..,k
4) *Scalar Multiplication:* $k\tilde{A}=k(\tilde{a}_{hij})$, where k is a scalar.

IV. ORTHOGONAL HEXAGONAL FUZZY MATRIX

A Square fuzzy matrix, $\tilde{A} = (\tilde{a}_{hij})$ is said to be Orthogonal hexagonal fuzzy Matrix if,

$\tilde{A}\,\tilde{A}^T = \tilde{A}^T\,\tilde{A} = \tilde{I}$ where, $\tilde{A}^T = (\tilde{a}_{hji})$ represents the Transpose of $\tilde{A}$, $\tilde{I}$ is the identity fuzzy Matrix

A. Theorem 4.1

If A and B are orthogonal hexagonal fuzzy matrices of the same order. Then AB and BA are also orthogonal hexagonal fuzzy matrix.

1) *Proof*

If $\tilde{A} = (\tilde{a}_{hij})$ and $\tilde{B} = (\tilde{b}_{hij})$ be orthogonal hexagonal fuzzy matrix.

Then, $\tilde{A}\,\tilde{A}^T = \tilde{A}^T\,\tilde{A} = \tilde{I}$ and $\tilde{B}\,\tilde{A}^T = \tilde{A}^T\,\tilde{A} = \tilde{I}$

$(\tilde{A}\tilde{B})(\tilde{A}\tilde{B})^T = \tilde{A}\tilde{B}\tilde{B}^T\tilde{A}^T = \tilde{A}(\tilde{B}\tilde{B}^T)\tilde{A}^T = \tilde{A}\tilde{I}\tilde{A}^T = \tilde{A}\tilde{A}^T = \tilde{I}$

Similarly, we can show that $(\tilde{A}\tilde{B})^T(\tilde{A}\tilde{B}) = \tilde{I}$

Also, $(\tilde{B}\tilde{A})(\tilde{B}\tilde{A})^T = \tilde{B}\tilde{A}\tilde{A}^T\tilde{B}^T = \tilde{B}(\tilde{A}\tilde{A}^T)\tilde{B}^T = \tilde{B}\tilde{I}\tilde{B}^T = \tilde{B}\tilde{B}^T = \tilde{I}$

Similarly, we can show that $(\tilde{B}\tilde{A})^T(\tilde{B}\tilde{A}) = \tilde{I}$

$\therefore$ $\tilde{B}\tilde{A}$ is Orthogonal Hexagonal Fuzzy Matrix.

B. Theorem 4.2

If $\tilde{A}$ is orthogonal hexagonal fuzzy matrix, then $\tilde{A}^T$ is also orthogonal hexagonal fuzzy matrix.

Let $\tilde{A} = (\tilde{a}_{hij})$ be an orthogonal hexagonal fuzzy Matrix. Then, $\tilde{A}\tilde{A}^T$

Taking Transpose on both sides,

$(\tilde{A}\tilde{A}^T)^T = \tilde{I}^T$

$(\tilde{A}^T)^T(\tilde{A}^T) = \tilde{I}$

$\tilde{A}\tilde{A}^T = \tilde{I}$

$\therefore$ $\tilde{A}^T$ is an Orthogonal Hexagonal Fuzzy Matrix.

V. NUMERICAL EXAMPLE

$$\tilde{A} = \frac{1}{5}\begin{bmatrix} (4,4,4,4,4,4) & (3,3,3,3,3,3) \\ (-3,-3,-3,-3,-3,-3) & (4,4,4,4,4,4) \end{bmatrix}$$

is an Orthogonal Hexagonal Fuzzy Matrix.

A. Proof

$$\tilde{A} = \frac{1}{5}\begin{bmatrix} (4,4,4,4,4,4) & (3,3,3,3,3,3) \\ (-3,-3,-3,-3,-3,-3) & (4,4,4,4,4,4) \end{bmatrix}$$

$$\tilde{A}^T = \frac{1}{5}\begin{bmatrix} (4,4,4,4,4,4) & (-3,-3,-3,-3,-3,-3) \\ (3,3,3,3,3,3) & (4,4,4,4,4,4) \end{bmatrix}$$

$$\tilde{A}\tilde{A}^T = \frac{1}{5}\begin{bmatrix} (4,4,4,4,4,4) & (3,3,3,3,3,3) \\ (-3,-3,-3,-3,-3,-3) & (4,4,4,4,4,4) \end{bmatrix}\frac{1}{5}\begin{bmatrix} (4,4,4,4,4,4) & (-3,-3,-3,-3,-3,-3) \\ (3,3,3,3,3,3) & (4,4,4,4,4,4) \end{bmatrix}$$

$$\tilde{A}\tilde{A}^T = \frac{1}{25}\begin{bmatrix} (25,25,25,25,25,25) & (0,0,0,0,0,0) \\ (0,0,0,0,0,0) & (25,25,25,25,25,25) \end{bmatrix}$$

$$\tilde{A}\tilde{A}^T = \frac{25}{25}\begin{bmatrix} (1,1,1,1,1,1) & (0,0,0,0,0,0) \\ (0,0,0,0,0,0) & (1,1,1,1,1,1) \end{bmatrix}$$

$$\tilde{A}\tilde{A}^T = \begin{bmatrix} (1,1,1,1,1,1) & (0,0,0,0,0,0) \\ (0,0,0,0,0,0) & (1,1,1,1,1,1) \end{bmatrix}$$

$\tilde{A}\tilde{A}^T = \tilde{I}$

Similarly, $\tilde{A}^T\tilde{A} = \tilde{I}$

Therefore, the given matrix Ã is an orthogonal fuzzy matrix.

VI. CONCLUSION

In this article, orthogonal hexagonal fuzzy number matrices are defined. Moreover, some theorems of orthogonal hexagonal fuzzy matrices are discussed. Using these concepts, numerical example is also provided.

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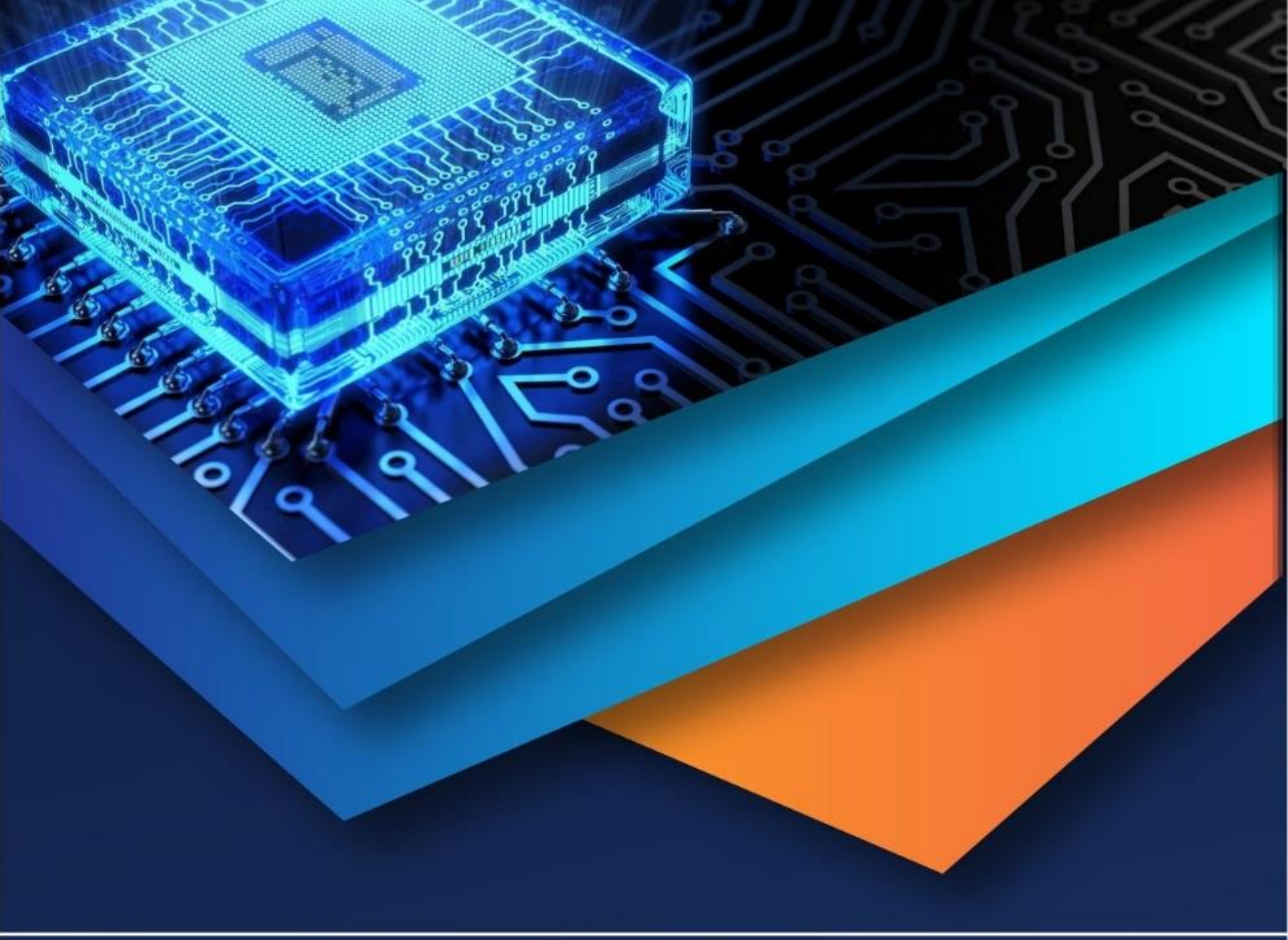

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