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Equation for Resilience in a Linearly Tapering Hollow Shaft of Circular Cross Section Subjected to Torsion

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Abstract: In this paper, new equations have been developed to obtain the resilience for linearly tapering hollow circular shaft subjected to pure torsion. The resilience equations have been developed from the strain energy equations. While deriving the equations some ratios have been introduced and their significance is also detailed. Two resilience equations have been developed, first resilience equations is obtained by considering the average volume and neglecting suitable terms and second resilience equation is obtained by considering original volume. Conclusions are made on the new equations developed.

I. INTRODUCTION

When a member is subjected to moment about it axis then it is considered to be under torsion. The torsional moment is also called as torque or twisting moment because the effect of torsional moment on the member is to twist the member. In the field of engineering majority of members are subjected torsion. Some of the examples include shafts transmitting power from a motor to machine, from engine to the rear axle of automobile, from a turbine to electric motors, ring beam of circular water tanks and beams of grid flooring system. If the cross sections of members are subjected to only torsional moments and not accompanied by bending moment and axial forces, then the member is under pure torsion. While developing the theory of pure torsion assumption made are the material is assumed to be homogeneous and isotropic, the stresses developed are assumed to be within the elastic limit, cross sections are plane before applying twisting moment are assumed to remain plane even after the application of twisting moment (no warping takes place), radial lines are assumed to remain radial even after applying torque and the twist along the shaft is assumed as uniform.

A. *Strain energy in a linearly tapering hollow shaft*

When a torque $\mathcal{T}$ is applied to a linearly tapering hollow shaft, it gets twisted by an angle θ. Thus the twisting moment does the work on the shaft and this work done is stored as strain energy in the shaft. When T is applied gradually the angle of rotation increases linearly and reaches the value of θ. From the torsion equation, for a given shaft the relationship between $\mathcal{T}$ and θ is linear as shown as shown in Figure 1.

Strain Energy = $\mathcal{U}$ = Area under the curve

$$\mathcal{U} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$\mathcal{U} = \frac{1}{2} \times \text{Angle of twist} \times \text{Torque}$$

$$\mathcal{U} = \frac{1}{2} \times \theta \times \mathcal{T}$$

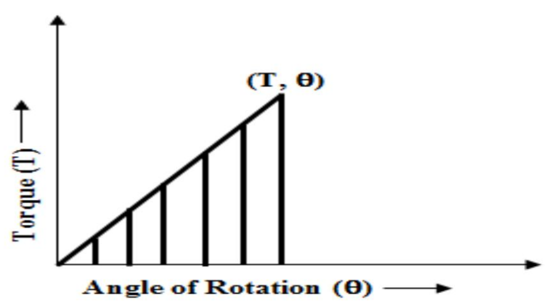

Figure 1: Relationship between torque and angle of twist (linear relationship)

From the equation of torsion we have the angle of twist as

$$\theta = \frac{\mathcal{T}\mathcal{L}}{\mathcal{G}\mathcal{J}}$$

where, $\mathcal{T}$ = Torque
$\mathcal{L}$ = Length
$\mathcal{G}$ = Modulus of rigidity or shear modulus
$\mathcal{J}$ = Polar moment of inertia

Therefore, strain energy is given by,

$$\mathcal{U} = \frac{1}{2}\left(\frac{\mathcal{T}\mathcal{L}}{\mathcal{G}\mathcal{J}}\right)\mathcal{T} = \frac{1}{2}\left(\frac{\mathcal{T}^2\mathcal{L}}{\mathcal{G}\mathcal{J}}\right)$$

Consider a linearly tapering hollow shaft having circular cross section, tapering from diameter $\phi_{1_{i/o}}$ to $\phi_{2_{i/o}}$ to as shown in Figure 2.

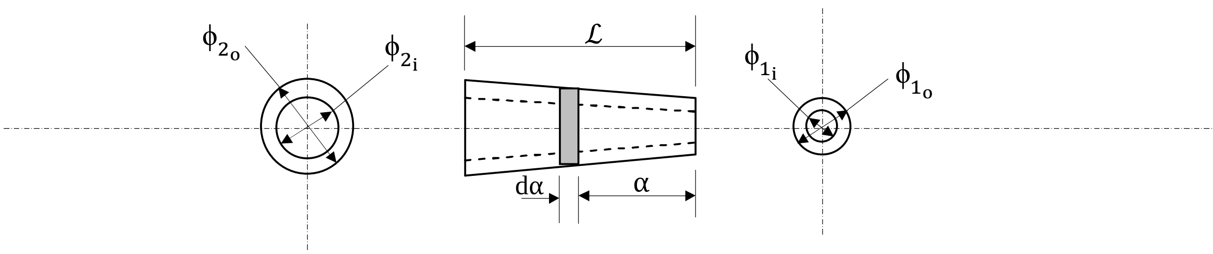

Figure 2: Linearly tapering hollow shaft of circular cross section

where, ϕ_{1_i}= inner diameter of shaft at end 1
ϕ_{2_i}= inner diameter of shaft at end 2
ϕ_{1_o}= outer diameter of shaft at end 1
ϕ_{2_o}= outer diameter of shaft at end 2

Rate of change of inner diameter, $(\phi_i)_{\mathcal{R}} = \frac{\phi_{2_i} - \phi_{1_i}}{\mathcal{L}}$

Rate of change of outer diameter, $(\phi_o)_{\mathcal{R}} = \frac{\phi_{2_o} - \phi_{1_o}}{\mathcal{L}}$

Inner diameter at a distance α from the end of a bar of diameter $\phi_{1_{i/o}}$, $\phi_i = \phi_{1_i} + \left(\frac{\phi_{2_i} - \phi_{1_i}}{\mathcal{L}}\right)\alpha$

Outer diameter at a distance α from the end of a bar of diameter $\phi_{1_{i/o}}$, $\phi_o = \phi_{1_o} + \left(\frac{\phi_{2_o} - \phi_{1_o}}{\mathcal{L}}\right)\alpha$

Polar moment of inertia for hollow circular cross section of outer diameter ϕ_o and inner diameter ϕ_i,

$$\mathcal{J} = \frac{\pi}{32}(\phi_o^4 - \phi_i^4)$$

$$\mathcal{J} = \frac{\pi}{32}\left\{\left[\phi_{1_o} + \left(\frac{\phi_{2_o} - \phi_{1_o}}{\mathcal{L}}\right)\alpha\right]^4 - \left[\phi_{1_i} + \left(\frac{\phi_{2_i} - \phi_{1_i}}{\mathcal{L}}\right)\alpha\right]^4\right\}$$

We know torque, $\mathcal{T} = \frac{\tau\mathcal{J}}{\xi}$

where, $\mathcal{T}$ = Torque
ξ = Radius or distance of element from center of shaft
τ = Shear stress in the element at radius r
$\mathcal{J}$ = Polar moment of inertia

Here, consider the element at the outer surface of the hollow shaft, then $\xi = {}^{\phi_o}/_2$.

$$\mathcal{T} = \frac{(\tau)\frac{\pi}{32}\left\{\left[\phi_{1_o} + \left(\frac{\phi_{2_o} - \phi_{1_o}}{\mathcal{L}}\right)\alpha\right]^4 - \left[\phi_{1_i} + \left(\frac{\phi_{2_i} - \phi_{1_i}}{\mathcal{L}}\right)\alpha\right]^4\right\}}{\left({}^{\phi_o}/_2\right)}$$

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$$\mathcal{T} = \frac{(\tau)\frac{\pi}{16}\left\{\left[\phi_{1_o} + \left(\frac{\phi_{2_o}-\phi_{1_o}}{\mathcal{L}}\right)\alpha\right]^4 - \left[\phi_{1_i} + \left(\frac{\phi_{2_i}-\phi_{1_i}}{\mathcal{L}}\right)\alpha\right]^4\right\}}{\left[\phi_{1_o} + \left(\frac{\phi_{2_o}-\phi_{1_o}}{\mathcal{L}}\right)\alpha\right]}$$

Therefore, strain energy equation can be written as

$$\mathcal{U} = \frac{1}{2}\left(\frac{\mathcal{L}}{\mathcal{G}\mathcal{J}}\right)\left\{\frac{(\tau)\times\frac{\pi}{16}\times\left\{\left[\phi_{1_o} + \left(\frac{\phi_{2_o}-\phi_{1_o}}{\mathcal{L}}\right)\alpha\right]^4 - \left[\phi_{1_i} + \left(\frac{\phi_{2_i}-\phi_{1_i}}{\mathcal{L}}\right)\alpha\right]^4\right\}}{\left[\phi_{1_o} + \left(\frac{\phi_{2_o}-\phi_{1_o}}{\mathcal{L}}\right)\alpha\right]}\right\}^2$$

$$\mathcal{U} = \frac{1}{2}\left(\frac{\mathcal{L}}{\mathcal{G}\mathcal{J}}\right)\left\{\frac{(\tau)\times\frac{\pi}{16}\times\left\{\left[\phi_{1_o} + \left(\frac{\phi_{2_o}-\phi_{1_o}}{\mathcal{L}}\right)\alpha\right]^4 - \left[\phi_{1_i} + \left(\frac{\phi_{2_i}-\phi_{1_i}}{\mathcal{L}}\right)\alpha\right]^4\right\}}{\left[\phi_{1_o} + \left(\frac{\phi_{2_o}-\phi_{1_o}}{\mathcal{L}}\right)\alpha\right]}\right\}^2$$

$$\mathcal{U} = \frac{1}{2}\left(\frac{\tau^2\times\pi^2\times\mathcal{L}}{16^2\mathcal{G}}\right)\left\{\frac{\left\{\frac{\left\{\left[\phi_{1_o}+\left(\frac{\phi_{2_o}-\phi_{1_o}}{\mathcal{L}}\right)\alpha\right]^4-\left[\phi_{1_i}+\left(\frac{\phi_{2_i}-\phi_{1_i}}{\mathcal{L}}\right)\alpha\right]^4\right\}}{\left[\phi_{1_o}+\left(\frac{\phi_{2_o}-\phi_{1_o}}{\mathcal{L}}\right)\alpha\right]}\right\}^2}{\frac{\pi}{32}\left\{\left[\phi_{1_o} + \left(\frac{\phi_{2_o}-\phi_{1_o}}{\mathcal{L}}\right)\alpha\right]^4 - \left[\phi_{1_i} + \left(\frac{\phi_{2_i}-\phi_{1_i}}{\mathcal{L}}\right)\alpha\right]^4\right\}}\right\}$$

$$\mathcal{U} = \left(\frac{\tau^2\times\pi\times\mathcal{L}}{16\mathcal{G}}\right)\left\{\frac{\left\{\frac{\left\{\left[\phi_{1_o}+\left(\frac{\phi_{2_o}-\phi_{1_o}}{\mathcal{L}}\right)\alpha\right]^4-\left[\phi_{1_i}+\left(\frac{\phi_{2_i}-\phi_{1_i}}{\mathcal{L}}\right)\alpha\right]^4\right\}}{\left[\phi_{1_o}+\left(\frac{\phi_{2_o}-\phi_{1_o}}{\mathcal{L}}\right)\alpha\right]}\right\}^2}{\left\{\left[\phi_{1_o} + \left(\frac{\phi_{2_o}-\phi_{1_o}}{\mathcal{L}}\right)\alpha\right]^4 - \left[\phi_{1_i} + \left(\frac{\phi_{2_i}-\phi_{1_i}}{\mathcal{L}}\right)\alpha\right]^4\right\}}\right\}$$

$$\mathcal{U} = \left(\frac{\tau^2\times\pi\times\mathcal{L}}{16\mathcal{G}}\right)\left\{\frac{\left\{\left[\phi_{1_o} + \left(\frac{\phi_{2_o}-\phi_{1_o}}{\mathcal{L}}\right)\alpha\right]^4 - \left[\phi_{1_i} + \left(\frac{\phi_{2_i}-\phi_{1_i}}{\mathcal{L}}\right)\alpha\right]^4\right\}}{\left[\phi_{1_o} + \left(\frac{\phi_{2_o}-\phi_{1_o}}{\mathcal{L}}\right)\alpha\right]^2}\right\}$$

On solving we get,

$$\boldsymbol{Strain\ Energy, \mathcal{U}} = \left(\frac{\boldsymbol{\tau^2\times\pi\times\mathcal{L}}}{\boldsymbol{16\mathcal{G}}}\right)\left\{\frac{\left\{\left[\boldsymbol{\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)}\right]^4 - \left[\boldsymbol{\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)}\right]^4\right\}}{\left[\boldsymbol{\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)}\right]^2}\right\}$$

In the strain energy equation mentioned above, the ratio $\left(\frac{\alpha}{\mathcal{L}}\right)$ has a significant meaning. If $\alpha = 0$, i.e. starting point of bar then, the ratio $\left(\frac{\alpha}{\mathcal{L}}\right)$ becomes zero. If $\alpha = L$, i.e. end point of the bar then, the ratio $\left(\frac{\alpha}{\mathcal{L}}\right)$ becomes one. If the midpoint is considered then the ratio $\left(\frac{\alpha}{\mathcal{L}}\right)$ becomes 0.5.

For the ratio $\left(\frac{\alpha}{\mathcal{L}}\right) = 0$, the strain energy equation can be written as below.

$$\mathcal{U} = \left(\frac{\tau^2\times\pi\times\mathcal{L}}{16\mathcal{G}}\right)\left\{\frac{\left\{\left(\phi_{1_o}\right)^4 - \left(\phi_{1_i}\right)^4\right\}}{\left(\phi_{1_o}\right)^2}\right\}$$

For the ratio $\left(\frac{\alpha}{\mathcal{L}}\right) = 1$, the strain energy equation can be written as below.

$$\mathcal{U} = \left(\frac{\tau^2\times\pi\times\mathcal{L}}{16\mathcal{G}}\right)\left\{\frac{\left\{\left(\phi_{2_o}\right)^4 - \left(\phi_{2_i}\right)^4\right\}}{\left(\phi_{2_o}\right)^2}\right\}$$

For the ratio $\left(\frac{\alpha}{\mathcal{L}}\right) = 0.5$, the strain energy equation can be written as below.

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$$\mathcal{U} = \left(\frac{\tau^2 \times \pi \times \mathcal{L}}{64\mathcal{G}}\right)\left\{\frac{\left\{\left(\phi_{1_o} + \phi_{2_o}\right)^4 - \left(\phi_{1_i} + \phi_{2_i}\right)^4\right\}}{\left(\phi_{1_o} + \phi_{2_o}\right)^2}\right\}$$

By neglecting the term $\left(\frac{\alpha}{\mathcal{L}}\right)$ in strain energy equation, we get

$$\mathcal{U} = \left(\frac{\tau^2 \times \pi \times \mathcal{L}}{16\mathcal{G}}\right)\left[\frac{\left(\phi_{1_o} + \phi_{2_o}\right)^4 - \left(\phi_{1_i} + \phi_{2_i}\right)^4}{\left(\phi_{1_o} + \phi_{2_o}\right)^2}\right]$$

$$\mathcal{U} = \left(\frac{\tau^2 \times \pi \times \mathcal{L}}{16\mathcal{G}}\right)\left\{\frac{\left[\left(\phi_{1_o} + \phi_{2_o}\right)^2\right]^2 - \left[\left(\phi_{1_i} + \phi_{2_i}\right)^2\right]^2}{\left(\phi_{1_o} + \phi_{2_o}\right)^2}\right\}$$

Multiply and divide by $\left(\frac{\pi}{16}\right)^2$, we get

$$\mathcal{U} = \left(\frac{\tau^2 \mathcal{L}}{\mathcal{G}}\right)\left\{\frac{\left[\frac{\pi}{4} \times \left(\frac{\phi_{1_o} + \phi_{2_o}}{2}\right)^2\right]^2 - \left[\frac{\pi}{4} \times \left(\frac{\phi_{1_i} + \phi_{2_i}}{2}\right)^2\right]^2}{\frac{\pi}{4} \times \left(\frac{\phi_{1_o} + \phi_{2_o}}{2}\right)^2}\right\}$$

$$\mathcal{U} = \left(\frac{\tau^2 \mathcal{L}}{\mathcal{G}}\right)\left\{\frac{\left[\frac{\pi}{4} \times \left(\phi_{o_{avg}}\right)^2\right]^2 - \left[\frac{\pi}{4} \times \left(\phi_{i_{avg}}\right)^2\right]^2}{\frac{\pi}{4} \times \left(\phi_{o_{avg}}\right)^2}\right\}$$

where,

Average outer diameter, $\phi_{o_{avg}} = \frac{\phi_{1_o} + \phi_{2_o}}{2}$

Average inner diameter, $\phi_{i_{avg}} = \frac{\phi_{1_i} + \phi_{2_i}}{2}$

$$\mathcal{U} = \left(\frac{\tau^2 \mathcal{L}}{\mathcal{G}}\right)\left[\frac{\left(\Gamma_{o_{avg}}\right)^2 - \left(\Gamma_{i_{avg}}\right)^2}{\Gamma_{o_{avg}}}\right]$$

where,

Average outer area, $\Gamma_{o_{avg}} = \frac{\pi}{4} \times \left(\phi_{o_{avg}}\right)^2 = \frac{\pi}{4} \times \left(\frac{\phi_{1_o} + \phi_{2_o}}{2}\right)^2$

Average inner area, $\Gamma_{i_{avg}} = \frac{\pi}{4} \times \left(\phi_{i_{avg}}\right)^2 = \frac{\pi}{4} \times \left(\frac{\phi_{1_i} + \phi_{2_i}}{2}\right)^2$

$$\mathcal{U} = \left(\frac{\tau^2 \mathcal{L}}{\mathcal{G}}\right)\left[\frac{\left(\Gamma_{o_{avg}} + \Gamma_{i_{avg}}\right)\left(\Gamma_{o_{avg}} - \Gamma_{i_{avg}}\right)}{\Gamma_{o_{avg}}}\right]$$

$$\mathcal{U} = \left(\frac{\tau^2 \mathcal{L}}{\mathcal{G}}\right)\left[\left(\Gamma_{o_{avg}} - \Gamma_{i_{avg}}\right)\left(\frac{\Gamma_{o_{avg}} + \Gamma_{i_{avg}}}{\Gamma_{o_{avg}}}\right)\right]$$

$$\boldsymbol{Strain\ Energy, \mathcal{U}} = \left(\frac{\boldsymbol{\tau^2}}{\boldsymbol{\mathcal{G}}}\right)\boldsymbol{\Psi_{avg}(1 + \varsigma)}$$

where,

Average volume, $\Psi_{avg} = \mathcal{L}\left(\Gamma_{o_{avg}} - \Gamma_{i_{avg}}\right)$

Ratio of average inner and outer areas, $\varsigma = \frac{\Gamma_{i_{avg}}}{\Gamma_{o_{avg}}}$

In the strain energy equation mentioned above the ratio of areas $\varsigma = \frac{\Gamma_{i_{avg}}}{\Gamma_{o_{avg}}}$ has significance details. The value of ς can never be equal to one i.e. $\varsigma \neq 1$ and can never be greater than one i.e. $\varsigma \ngtr 1$. The value of ς should always be less than 1 i.e. $\varsigma < 1$. If the value of ς

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is equal to zero i.e. $\varsigma = 0$, then it indicates inner area is zero i.e. $\Gamma_{i_{avg}} = 0$, this implies the shaft is a linearly tapering solid shaft. Thus for the solid shaft the strain energy equation reduces to the following form.

$$\boldsymbol{Strain\ Energy}, \boldsymbol{\mathcal{U}} = \left(\frac{\boldsymbol{\tau}^2}{\boldsymbol{\mathcal{G}}}\right)\boldsymbol{\Psi}_{\mathbf{avg}}$$

B. Resilience In The Linearly Tapering Hollow Shaft

The strain energy per unit volume is called as resilience. The maximum strain energy per unit volume, which can be stored by a body without undergoing permanent deformation is known as proof resilience. Proof resilience is strain energy per unit volume in the body corresponding to stress at elastic limit.

1) Considering average volume: Consider the strain energy equation of linearly tapering shaft developed above as

$$\mathcal{U} = \left(\frac{\tau^2}{\mathcal{G}}\right)\Psi_{avg}(1+\varsigma)$$

This equation of strain energy is obtained by considering the linearly tapering shaft of average diameter. By considering this equation, equation of resilience can be developed as below.

Resilience, Λ = Strain energy per unit volume

$$\Lambda = \frac{\left(\frac{\tau^2}{\mathcal{G}}\right)\Psi_{avg}(1+\varsigma)}{\text{Volume}}$$

Since the linearly tapering hollow shaft is considered as the straight hollow shaft of average inner diameter $\left(\frac{\phi_{1_i}+\phi_{2_i}}{2}\right)$ and average outer diameter $\left(\frac{\phi_{1_o}+\phi_{2_o}}{2}\right)$, the volume will be average volume. Hence the resilience equation is given as below for the linearly tapering shaft considered as straight hollowe shaft of average diameters.

$$\Lambda = \frac{\left(\frac{\tau^2}{\mathcal{G}}\right)\Psi_{avg}(1+\varsigma)}{\Psi_{avg}}$$

$$\textbf{Resilience}, \quad \boldsymbol{\Lambda} = \left(\frac{\boldsymbol{\tau}^2}{\boldsymbol{\mathcal{G}}}\right)(\mathbf{1}+\boldsymbol{\varsigma})$$

In the resilience equation mentioned above the ratio of areas $\varsigma = \frac{\Gamma_{i_{avg}}}{\Gamma_{o_{avg}}}$ has significance details. The value of ς can never be equal to one $(\varsigma \neq 1)$ i.e. outer area can never be equal to inner area which does not indicate a hollow shaft. The value of ς can never be greater than one $(\varsigma \ngtr 1)$ i.e. inner area can never be greater than outer area. The value of ς should always be less than $(\varsigma < 1)$ i.e. outer area is always greater than inner area. If the value of ς is equal to zero i.e. $\varsigma = 0$, then it indicates inner area is zero i.e. $\Gamma_{i_{avg}} = 0$, this implies the shaft is a linearly tapering solid shaft. Thus for the solid shaft the resilience equation reduces to the following form.

$$\textbf{Resilience}, \quad \boldsymbol{\Lambda} = \left(\frac{\boldsymbol{\tau}^2}{\boldsymbol{\mathcal{G}}}\right)$$

Consider the strain energy equation of linearly tapering shaft developed above as

$$\mathcal{U} = \left(\frac{\tau^2\times\pi\times\mathcal{L}}{16\mathcal{G}}\right)\left\{\frac{\left\{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4-\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4\right\}}{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^2}\right\}$$

This equation of strain energy is obtained by considering the original volume of the linearly tapering hollow shaft. By considering this equation, equation of resilience can be developed as below.

Resilience, Λ = Strain energy per unit volume

$$\Lambda = \frac{\left(\frac{\tau^2\times\pi\times\mathcal{L}}{16\mathcal{G}}\right)\left\{\frac{\left\{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4-\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4\right\}}{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^2}\right\}}{\text{Volume}}$$

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Since the original volume of the linearly tapering hollow shaft is to be considered, the volume will be obtained by multiplying the length to the integrated area between two inner and outer diameters. Hence the resilience equation is given as below for the linearly tapering hollow shaft considering the original volume Ψ.

$$\Lambda = \frac{\left(\frac{\tau^2\times\pi\times\mathcal{L}}{16\mathcal{G}}\right)\left\{\frac{\left\{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4-\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4\right\}}{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^2}\right\}}{\frac{\mathcal{L}\pi}{4}\int_{\phi_1}^{\phi_2}(\phi_o^2-\phi_i^2)d\phi}$$

$$\Lambda = \left(\frac{\tau^2}{4\mathcal{G}}\right)\frac{\left\{\frac{\left\{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4-\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4\right\}}{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^2}\right\}}{\int_{\phi_1}^{\phi_2}(\phi_o^2-\phi_i^2)d\phi}$$

$$\Lambda = \left(\frac{\tau^2}{4\mathcal{G}}\right)\frac{\left\{\frac{\left\{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4-\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4\right\}}{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^2}\right\}}{\left[\frac{\phi_o^3}{3}-\frac{\phi_i^3}{3}\right]_{\phi_1}^{\phi_2}}$$

$$\Lambda = \left(\frac{\tau^2}{4\mathcal{G}}\right)\frac{\left\{\frac{\left\{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4-\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4\right\}}{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^2}\right\}}{\left[\frac{\phi_{2_o}^3}{3}-\frac{\phi_{2_i}^3}{3}\right]-\left[\frac{\phi_{1_o}^3}{3}-\frac{\phi_{1_i}^3}{3}\right]}$$

$$\Lambda = \frac{\left(\frac{3\tau^2}{4\mathcal{G}}\right)}{(\phi_{2_o}^3-\phi_{2_i}^3-\phi_{1_o}^3+\phi_{1_i}^3)}\int_0^{\mathcal{L}}\left\{\frac{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4-\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4}{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^2}\right\}d\alpha$$

$$\Lambda = \frac{\left(\frac{3\tau^2}{4\mathcal{G}}\right)}{(\phi_{2_o}^3-\phi_{2_i}^3-\phi_{1_o}^3+\phi_{1_i}^3)}\int_0^{\mathcal{L}}\left\{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^2-\frac{\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4}{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^2}\right\}d\alpha$$

$$\Lambda = \frac{\left(\frac{3\tau^2}{4\mathcal{G}}\right)}{(\phi_{2_o}^3-\phi_{2_i}^3-\phi_{1_o}^3+\phi_{1_i}^3)}\left\{\left[\frac{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^3}{3\left(\frac{\phi_{2_o}}{\mathcal{L}}-\frac{\phi_{1_o}}{\mathcal{L}}\right)}\right]_0^{\mathcal{L}}-\int_0^{\mathcal{L}}\frac{\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4}{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^2}d\alpha\right\}$$

$$\Lambda = \frac{\left(\frac{3\tau^2}{4\mathcal{G}}\right)}{(\phi_{2_o}^3-\phi_{2_i}^3-\phi_{1_o}^3+\phi_{1_i}^3)}\left\{\left[\frac{\phi_{2_o}^3-\phi_{1_o}^3}{3\left(\frac{\phi_{2_o}}{\mathcal{L}}-\frac{\phi_{1_o}}{\mathcal{L}}\right)}\right]-\mathrm{K}\right\}$$

$$\Lambda = \frac{\left(\frac{3\tau^2}{4\mathcal{G}}\right)}{(\phi_{2_o}^3-\phi_{2_i}^3-\phi_{1_o}^3+\phi_{1_i}^3)}\left\{\left(\frac{\mathcal{L}}{3}\right)\left(\frac{\phi_{2_o}^3-\phi_{1_o}^3}{\phi_{2_o}-\phi_{1_o}}\right)-\mathrm{K}\right\}$$

$$\Lambda = \frac{\left(\frac{3\tau^2}{4\mathcal{G}}\right)}{[(\phi_{2_o}^3-\phi_{2_i}^3)-(\phi_{1_o}^3-\phi_{1_i}^3)]}\left\{\left(\frac{\mathcal{L}}{3}\right)\left(\frac{\phi_{2_o}^3-\phi_{1_o}^3}{\phi_{2_o}-\phi_{1_o}}\right)-\mathrm{K}\right\}$$

where,

$$\mathrm{K} = \int_0^{\mathcal{L}}\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^{-2}d\alpha$$

$$K = \left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4 \int_0^{\mathcal{L}} \left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^{-2} d\alpha - \int_0^{\mathcal{L}} \left\{4\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^3 \left(\frac{\phi_{2_i}}{\mathcal{L}} - \frac{\phi_{1_i}}{\mathcal{L}}\right)\left\{\int_0^{\mathcal{L}}\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^{-2} d\alpha\right\}\right\} d\alpha$$

$$K = \left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4 \left[\frac{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^{-1}}{\frac{\phi_{1_o}}{\mathcal{L}}-\frac{\phi_{2_o}}{\mathcal{L}}}\right]_0^{\mathcal{L}} - \int_0^{\mathcal{L}} \left\{4\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^3 \left(\frac{\phi_{2_i}}{\mathcal{L}} - \frac{\phi_{1_i}}{\mathcal{L}}\right)\left[\frac{\left[\phi_{1_o}\left(1-\frac{\alpha}{\mathcal{L}}\right)+\phi_{2_o}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^{-1}}{\frac{\phi_{1_o}}{\mathcal{L}}-\frac{\phi_{2_o}}{\mathcal{L}}}\right]_0^{\mathcal{L}}\right\} d\alpha$$

$$K = \left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4 \left[(\mathcal{L})\frac{\phi_{2_o}^{-1}-\phi_{1_o}^{-1}}{(\phi_{1_o}-\phi_{2_o})}\right] \int_0^{\mathcal{L}} \left\{4\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^3 \left(\frac{\phi_{2_i}}{\mathcal{L}} - \frac{\phi_{1_i}}{\mathcal{L}}\right)\left[(\mathcal{L})\frac{\phi_{2_o}^{-1}-\phi_{1_o}^{-1}}{(\phi_{1_o}-\phi_{2_o})}\right]\right\} d\alpha$$

$$K = \left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4 \left(\frac{\mathcal{L}}{\phi_{1_o}\phi_{2_o}}\right) - \int_0^{\mathcal{L}} \left\{4\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^3 \left(\frac{\phi_{2_i}}{\mathcal{L}} - \frac{\phi_{1_i}}{\mathcal{L}}\right)\left(\frac{\mathcal{L}}{\phi_{1_o}\phi_{2_o}}\right)\right\} d\alpha$$

$$K = \left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4 \left(\frac{\mathcal{L}}{\phi_{1_o}\phi_{2_o}}\right) - \left(\frac{\mathcal{L}}{\phi_{1_o}\phi_{2_o}}\right)\int_0^{\mathcal{L}} 4\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^3 \left(\frac{\phi_{2_i}}{\mathcal{L}} - \frac{\phi_{1_i}}{\mathcal{L}}\right) d\alpha$$

$$K = \left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4 \left(\frac{\mathcal{L}}{\phi_{1_o}\phi_{2_o}}\right) - \left(\frac{\mathcal{L}}{\phi_{1_o}\phi_{2_o}}\right)\left\{\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4\right\}_0^{\mathcal{L}}$$

$$K = \left(\frac{\mathcal{L}}{\phi_{1_o}\phi_{2_o}}\right)\left\{\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4 - (\phi_{2_i}^4 - \phi_{1_i}^4)\right\}$$

Substituting the above equation for K in resilience equation Λ, we get

$$\Lambda = \frac{\left(\frac{3\tau^2}{4\mathcal{G}}\right)}{\left[(\phi_{2_o}^3 - \phi_{2_i}^3) - (\phi_{1_o}^3 - \phi_{1_i}^3)\right]}\left\{\left(\frac{\mathcal{L}}{3}\right)\left(\frac{\phi_{2_o}^3 - \phi_{1_o}^3}{\phi_{2_o} - \phi_{1_o}}\right) - \left(\frac{\mathcal{L}}{\phi_{1_o}\phi_{2_o}}\right)\left\{\left[\phi_{1_i}\left(1-\frac{\alpha}{\mathcal{L}}\right) + \phi_{2_i}\left(\frac{\alpha}{\mathcal{L}}\right)\right]^4 - (\phi_{2_i}^4 - \phi_{1_i}^4)\right\}\right\}$$

Since the resilience is for the entire shaft $\alpha = L$. Hence the resilience equation for hollow tapering shaft of circular cross section further reduces to the following final form.

$$\Lambda = \frac{\left(\frac{3\tau^2}{4\mathcal{G}}\right)}{\left[(\phi_{2_o}^3 - \phi_{2_i}^3) - (\phi_{1_o}^3 - \phi_{1_i}^3)\right]}\left\{\left(\frac{\mathcal{L}}{3}\right)\left(\frac{\phi_{2_o}^3 - \phi_{1_o}^3}{\phi_{2_o} - \phi_{1_o}}\right) - \left(\frac{\mathcal{L}}{\phi_{1_o}\phi_{2_o}}\right)\left[\phi_{2_i}^4 - (\phi_{2_i}^4 - \phi_{1_i}^4)\right]\right\}$$

$$\textbf{Resilience}, \boldsymbol{\Lambda} = \frac{\left(\frac{\tau^2\mathcal{L}}{4\mathcal{G}}\right)}{\left[(\phi_{2_o}^3 - \phi_{2_i}^3) - (\phi_{1_o}^3 - \phi_{1_i}^3)\right]}\left[\left(\frac{\phi_{2_o}^3 - \phi_{1_o}^3}{\phi_{2_o} - \phi_{1_o}}\right) - \left(\frac{3}{\phi_{1_o}\phi_{2_o}}\right)\phi_{1_i}^4\right]$$

The resilience equation developed for hollow tapering shaft of circular cross section has significance importance. For solid shaft, inner diameters ϕ_{1_i} and ϕ_{2_i} are equal to zero. Hence, the resilience equation for the solid tapering shaft reduces to the hollowing form.

$$\Lambda = \frac{\left(\frac{\tau^2\mathcal{L}}{4\mathcal{G}}\right)}{(\phi_{2_o}^3 - \phi_{1_o}^3)}\left[\left(\frac{\phi_{2_o}^3 - \phi_{1_o}^3}{\phi_{2_o} - \phi_{1_o}}\right)\right]$$

$$\textbf{Resilience}, \boldsymbol{\Lambda} = \frac{\tau^2}{\mathcal{G}}\left[\frac{\mathcal{L}}{4(\phi_{2_o} - \phi_{1_o})}\right]$$

II. CONCLUSIONS

The two strain energy equations and the corresponding resilience equations have been developed for the linearly tapering hollow shaft of circular cross section. The resilience equation $\left(\frac{\tau^2}{\mathcal{G}}\right)(1 + \varsigma)$ has been developed from the strain energy equation by neglecting term $\left(\frac{\alpha}{\mathcal{L}}\right)$ and considering average volume. This equation reveals that resilience is independent of the length of the shaft and only dependent on the average cross sectional areas, applied torque and shear modulus and varies only with the variation in cross section. Another resilience equation developed from the strain energy equation $\frac{\left(\frac{\tau^2 \mathcal{L}}{4\mathcal{G}}\right)}{\left[\left(\phi_{2_o}^3 - \phi_{2_i}^3\right) - \left(\phi_{1_o}^3 - \phi_{1_i}^3\right)\right]}\left[\left(\frac{\phi_{2_o}^3 - \phi_{1_o}^3}{\phi_{2_o} - \phi_{1_o}}\right) - \left(\frac{3}{\phi_{1_o}\,\phi_{2_o}}\right)\phi_{1_i}^4\right]$ has been developed from the strain energy equation by considering original volume without any assumptions and without neglecting any terms. This equation reveals that the resilience is dependent not only on the applied torque and shear modulus but also on the dimensions of the shaft namely length and diameters. Further the resilience equations developed for hollow shaft have been validated by reducing those equations to the solid shafts and comparing with the equations mentioned in the reference [6]. The reduced equations are $\left(\frac{\tau^2}{\mathcal{G}}\right)$ and $\frac{\tau^2}{\mathcal{G}}\left[\frac{\mathcal{L}}{4(\phi_{2_o} - \phi_{1_o})}\right]$ and these equations exactly match with resilience equations developed for the solid shafts in the reference paper [6]. Hence the conclusion can be made that, the resilience equations developed for the hollow tapering shaft of circular cross section are correct and can be applied directly for the practical application. The major outcome of the resilience equation developed for the linearly tapering hollow shaft of circular cross section is that, the resilience value is inversely proportional to the tapering angle if length, shear modulus and applied torque are kept constant. The application of these equations for tapering hollow shaft of circular cross section can be found in the shafts used for variable speed transmission systems.

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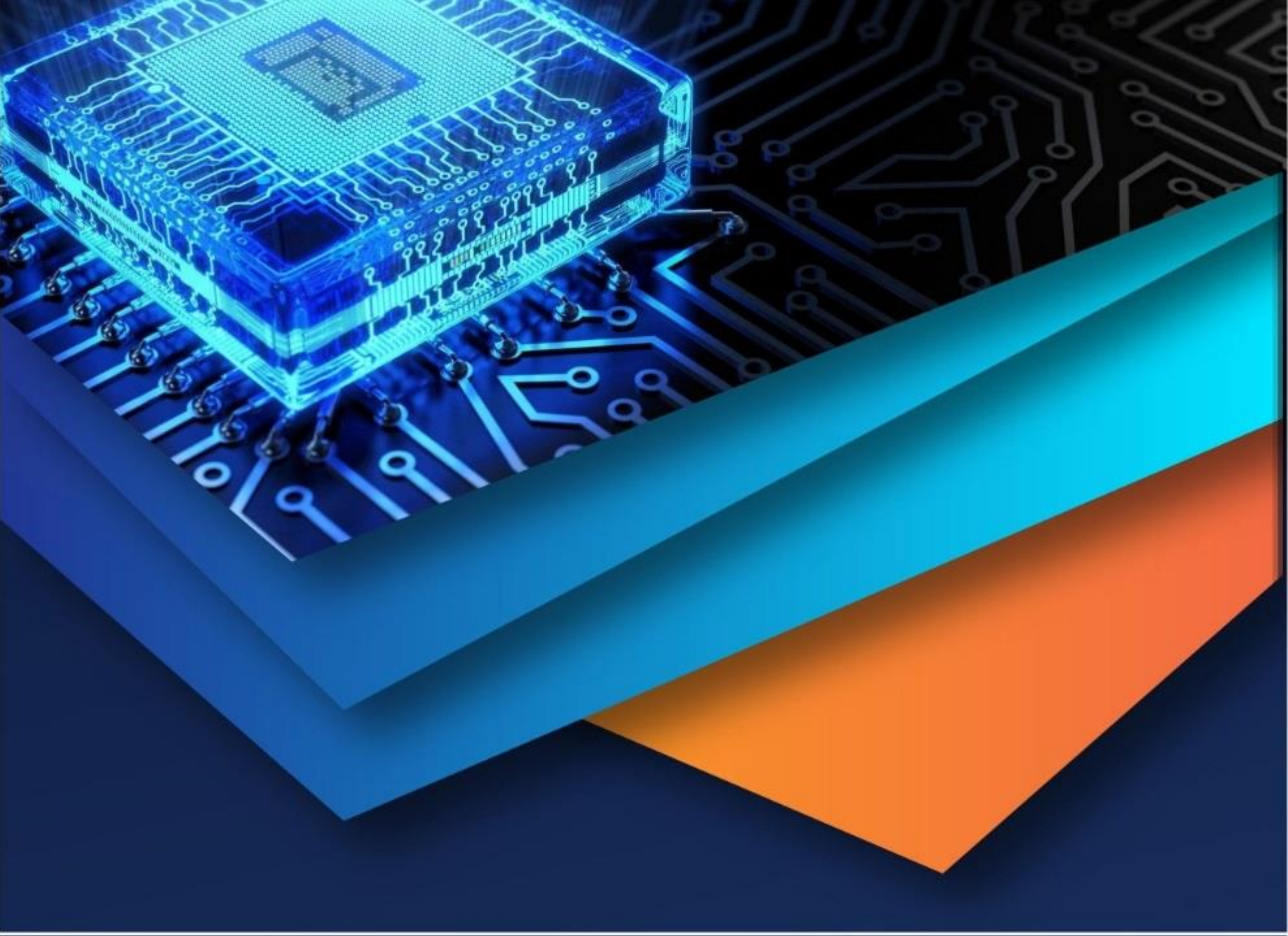

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