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# A Simplified Computational Methodology of Finding Wiener Number and Hyper-Wiener Number of Triangular Graphs

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***Abstract: The Wiener index of a graph G is denoted by W (G) and defined as the sum of distances between all pairs of vertices in simple connected graph. Wiener index is used for modelling the shape of organic molecules and for calculating several of their physicochemical properties. In this paper we have considered the popular topological indices Wiener vector, Hyper-Wiener Vector, Wiener Matrix Sequence, Hyper Weiner Matrix sequence, Wiener number and Hyper Wiener number of graphs two connected graphs are computed and presented in a well defined form and also it is expressed in a simplified form using interpolation of sequences. Also in this paper, some topological indices of triangular graphs are computed using interpolation of sequence.***
***Keywords: Wiener matrix sequence, hyper-Wiener vector, Weighted Wiener polynomial, Wiener polynomial sequence.***

## I. INTRODUCTION

Let G= (V (G), E (G)) be a simple connected graph of order n and size m. The degree of any vertex is the number of first neighbor of v is denoted by deg(v). The distance of between any two vertices u and v of graph is defined as the length of the shortest path connecting u, v is d (u, v). Topological indices are particularly suitable ,if properties like shape or degree of branching are expected to have an influence on the property predicted. Calculation is often based on the distance matrix of molecular graph. A disadvantage of topological indices is that they are unrelated to observable physical properties. Quantitative structure –Activity and structure property relationships (QSAR/QSPR) use chemo-metric methods to study how a given biological activity or a physicochemical property varies as a function of topological descriptors describing the chemical structure of molecules. With these studies it is possible to replace costly and time taking biological tests or experiments of a given physiochemical property with models involving topological descriptors. In this paper we have considered the two connected graphs and their popular topological indices using the Wiener vector, Hyper-wiener Vector, Wiener Matrix Sequence, Hyper Wiener Matrix Sequence, Wiener polynomial Sequence and Hyper-wiener polynomial, Wiener number and Hyper Wiener number are calculated. In this paper, some topological indices of triangular graphs are computed using matrix method and also using the method of interpolation of sequence.

## II. WIENER VECTOR AND HYPER WIENER VECTOR

For a connected graph G with n vertices, denoted by 1,2,….n, let $W_k = \sum_{i<j,dij=k} dij$, k=1,2,……The vector $(W_1,W_2,…..)$is called the Wiener Vector of G, denoted by WV(G). Clearly; the sum of all components of the Wiener vector of G is just equal to the Wiener number of G.For a connected graph G with n vertices, denoted by 1,2,..n, let $W_{ij,k} = \max\{d_{ij} - k + 1, 0\}$ where $d_{ij}$ is the distance between vertices i and j . Then ${}^kW = \sum_{i<j} W_{ij,k}, k = 1,2.....$ are called the higher Wiener numbers of G. The vector $({}^1W, {}^2W,….)$ is called the hyper-Wiener vector Of G, denoted by HWV(G).

Let D be the distance matrix of a connected graph of G.

Let $W^{(1)}$=D for k=1,2,…$W^{(K+1)}$is obtained from $W^{(k)}$by leaving zeroes in place and replacing each nonzero entry x of $W^{(k)}$ by x-1. Let D be the distance matrix of a connected graph G, and let $W^1,W^2$,…..be the Wiener matrix sequence of G. The Hyper-Wiener matrix $W^{(H)}$of G is defined as $$W^{(H)} = \sum_{k=1,2,...} W^{(K)}$$

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From the definitions, we can see that (i, j) entry of $W^{(k)}$ is just equal to $d_{ij}$-k+1, so the sum of entries of upper triangle of $W^{(k)}$ is just equal to ${}^{k}W$. Moreover , the sum of entries of upper triangle of the hyper-Wiener matrix $W^{(H)}$ is just equal to the hyper-Wiener number R. However, the hyper-Wiener matrix is applicable for any connected structure.

## III. INTERPOLATION METHOD

For a function 'f', defined on the line of real numbers, difference operator $\Delta$ is defined as $\Delta f(x) = f(x+1) - f(x)$. For each natural number, by $\Delta^0 f = I(f) = f$ and $\Delta^n f = \Delta^{n-1}(\Delta f)$, powers of $\Delta$ is defined inductively. One of the interesting features of a polynomial is that, if the degree of polynomial $f(x) is\ n$, then we have $\Delta^{n+1} f = 0$. This enables us to calculate many topological indices for infinite families of graphs, which have a common form. The lemma is given by "For a real value function f, $\Delta^n f = 0$, if and only if, $f(x)=a_0(x) + a_1(x)x + ... + a_n(x)x^n$, so that, $a_0$, $a_1$, ... , $a_n$, are all 1-periodic functions. ". If for a sequence of real numbers, such as $A = \{a_n\}_{n\geq1}$, we define, $\Delta A = a_{n+1} - a_n\}_{n\geq1}$, then according to the above Lemma it can be concluded that the generator of sequence is a polynomial of degree k, if and only if $\Delta^k(A)$ is a fixed sequence while $\Delta^{k-1}A$ is not a fixed sequence.

According to the above issues, it is concluded that having some of the first values of indices, is enough to obtain a closed formula for a topological index in a family of graphs which has a common form, where the closed formula is a polynomial or a function of polynomials. To determine this closed formula which is in the form of a polynomial, it is enough to obtain differences sequences several times as far as the fixed sequence is obtained. If such a fixed sequence is not achieved, it will be concluded that the closed formula not in a form of polynomial. As an example suppose that, the sequence, $A = \{a_n\}_{n\geq1}$ in order, is listed by following sentences: $A = \{3,6,11,18,27, \dots \}$;$\Delta A = \{3,5,7,9 \dots \}$;$\Delta^2 A = \{2,2,2,2 \dots \}$.Therefore, the generator of sequence is a quadratic polynomial using the integration on $\Delta^2 A = \{2\}_{n\geq1}$, $\Delta A = \{2n^1 + C_1\}_{n\geq1}$, will be obtained. Given the fact that the first term of $\Delta A$, is equal to 3, it is obvious that $C_1$=1. Therefore, we get $\Delta A = \{2n^1 + C_1\}_{n\geq1}$. The integration is repeated one more time and we get $A = \{n^2+n+2\}_{n\geq1.}$

## IV. WIENER INDEX ON TRIANGULAR GRAPH

The sequences of simple triangular graphs are shown in Figure 1 to 4.

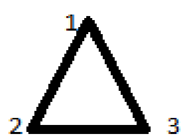


Figure:1

$$W^{(1)} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$W^{(1)}$=3

$$W^{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$W^{(2)} = 0$

$WV(G) = (3,0); HWV(G) = (3,0).$

$W = 3; R = 3.$

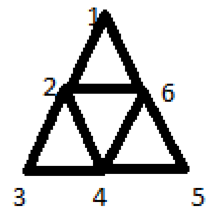


Figure:2

$$W^{(1)} = \begin{pmatrix} 0 & 1 & 2 & 2 & 2 & 1 \\ 1 & 0 & 1 & 1 & 2 & 1 \\ 2 & 1 & 0 & 1 & 2 & 2 \\ 2 & 1 & 1 & 0 & 1 & 1 \\ 2 & 2 & 2 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 & 1 & 0 \end{pmatrix}$$

$$W^{(1)} = 21$$

$$W^{(2)} = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$W^{(2)} = 6$$

$$W^{(3)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$W^{(3)} = 0$$

$$W^{(H)} = \begin{pmatrix} 0 & 1 & 3 & 3 & 3 & 1 \\ 1 & 0 & 1 & 1 & 3 & 1 \\ 3 & 1 & 0 & 1 & 3 & 3 \\ 3 & 1 & 1 & 0 & 1 & 1 \\ 3 & 3 & 3 & 1 & 0 & 1 \\ 1 & 1 & 3 & 1 & 1 & 0 \end{pmatrix}$$

$$WV(G) = (9, 12, 0);\ HWV(G) = (21, 6, 0)$$

$$W = 21, R = 27.$$

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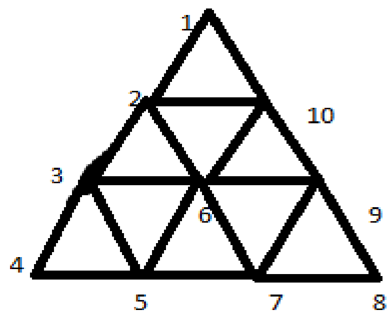


Figure:3

$$W^{(1)} = \begin{pmatrix} 0 & 1 & 2 & 3 & 3 & 2 & 3 & 3 & 2 & 1 \\ 1 & 0 & 1 & 2 & 2 & 1 & 2 & 3 & 2 & 1 \\ 2 & 1 & 0 & 1 & 1 & 1 & 2 & 3 & 2 & 2 \\ 3 & 2 & 1 & 0 & 1 & 2 & 2 & 3 & 3 & 3 \\ 3 & 2 & 1 & 1 & 0 & 1 & 1 & 2 & 2 & 2 \\ 2 & 1 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 1 \\ 3 & 2 & 2 & 2 & 1 & 1 & 0 & 1 & 1 & 2 \\ 3 & 3 & 3 & 3 & 2 & 2 & 1 & 0 & 1 & 2 \\ 2 & 2 & 2 & 3 & 2 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 2 & 3 & 2 & 1 & 2 & 2 & 1 & 0 \end{pmatrix}$$

$W^{(1)} = 81.$

$$W^{(2)} = \begin{pmatrix} 0 & 0 & 1 & 2 & 2 & 1 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 2 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 & 1 \\ 2 & 1 & 0 & 0 & 0 & 1 & 1 & 2 & 2 & 2 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 2 & 2 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$W^{(2)} = 36$

$$W^{(3)} = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$W^{(3)} = 9$

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$$W^{(4)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$W^{(4)} = 0$

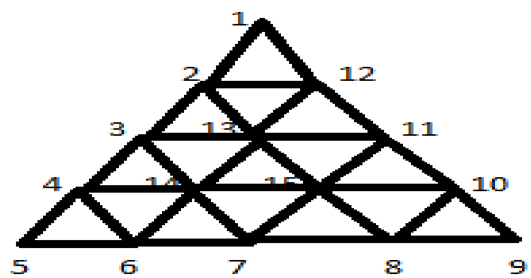


Figure: 4

$$W^{(1)} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 4 & 4 & 4 & 4 & 3 & 2 & 1 & 2 & 3 & 3 \\ 1 & 0 & 1 & 2 & 3 & 3 & 3 & 3 & 4 & 3 & 2 & 1 & 1 & 2 & 2 \\ 2 & 1 & 0 & 1 & 2 & 2 & 2 & 3 & 4 & 3 & 2 & 2 & 1 & 1 & 2 \\ 3 & 2 & 1 & 0 & 1 & 1 & 2 & 3 & 4 & 3 & 3 & 3 & 2 & 1 & 2 \\ 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 4 & 4 & 4 & 3 & 2 & 3 \\ 4 & 3 & 2 & 1 & 1 & 0 & 1 & 2 & 3 & 3 & 3 & 3 & 2 & 1 & 2 \\ 4 & 3 & 2 & 2 & 2 & 1 & 0 & 1 & 2 & 2 & 2 & 3 & 2 & 1 & 1 \\ 4 & 3 & 3 & 3 & 3 & 2 & 1 & 0 & 1 & 1 & 2 & 3 & 2 & 2 & 1 \\ 4 & 4 & 4 & 4 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 3 & 3 & 2 \\ 3 & 3 & 3 & 3 & 4 & 3 & 2 & 1 & 1 & 0 & 1 & 2 & 2 & 2 & 1 \\ 2 & 2 & 2 & 3 & 4 & 3 & 2 & 2 & 2 & 1 & 0 & 1 & 1 & 2 & 1 \\ 1 & 1 & 2 & 3 & 4 & 3 & 3 & 3 & 3 & 2 & 1 & 0 & 1 & 2 & 2 \\ 2 & 1 & 1 & 2 & 3 & 2 & 2 & 2 & 3 & 2 & 1 & 1 & 0 & 1 & 1 \\ 3 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 3 & 2 & 2 & 2 & 1 & 0 & 1 \\ 3 & 2 & 2 & 2 & 3 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 0 \end{pmatrix}$$

$W^{(1)} = 231.$

$$W^{(2)}=\begin{pmatrix}
0&0&1&2&3&3&3&3&3&2&1&0&1&2&2\\
0&0&0&1&2&2&2&2&3&2&1&0&0&1&1\\
1&0&0&0&1&1&1&2&3&2&1&1&0&0&1\\
2&1&0&0&0&0&1&2&3&2&2&2&1&0&1\\
3&2&1&0&0&0&1&2&3&3&3&3&2&1&2\\
3&2&1&0&0&0&0&1&2&2&2&2&1&0&1\\
3&2&1&1&1&0&0&0&1&1&1&2&1&0&0\\
3&2&2&2&2&1&0&0&0&0&1&2&1&1&0\\
3&3&3&3&3&2&1&0&0&0&1&2&2&2&1\\
2&2&2&2&3&2&1&0&0&0&0&1&1&1&0\\
1&1&1&2&3&2&1&1&1&0&0&0&0&1&0\\
0&0&1&2&3&2&2&2&2&1&0&0&0&1&1\\
1&0&0&1&2&1&1&1&2&1&0&0&0&0&0\\
2&1&0&0&1&0&0&1&2&1&1&1&0&0&0\\
2&1&1&1&2&1&0&0&1&0&0&1&0&0&0
\end{pmatrix}$$

$$W^{(2)}=126$$

$$W^{(3)}=\begin{pmatrix}
0&0&0&1&2&2&2&2&2&1&0&0&0&1&1\\
0&0&0&0&1&1&1&1&2&1&0&0&0&0&0\\
0&0&0&0&0&0&0&1&2&1&0&0&0&0&0\\
1&0&0&0&0&0&0&1&2&1&1&1&0&0&0\\
2&1&0&0&0&0&0&1&2&2&2&2&1&0&1\\
2&1&0&0&0&0&0&0&1&1&1&1&0&0&0\\
2&1&0&0&0&0&0&0&0&0&0&1&0&0&0\\
2&1&1&1&1&0&0&0&0&0&0&1&0&0&0\\
2&2&2&2&2&1&0&0&0&0&0&1&1&1&0\\
1&1&1&1&2&1&0&0&0&0&0&0&0&0&0\\
0&0&0&1&2&1&0&0&0&0&0&0&0&0&0\\
0&0&0&1&2&1&1&1&1&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&1&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
1&0&0&0&1&0&0&0&0&0&0&0&0&0&0
\end{pmatrix}$$

$$W^{(3)}=51$$

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$$W^{(4)} = \begin{pmatrix} 0&0&0&0&1&1&1&1&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&1&1&1&1&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&1&1&1&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$W^{(4)} = 12$$

$$W^{(5)} = \begin{pmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

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$W^{(5)} = 0$

$$W^{(H)} = \begin{pmatrix} 0 & 1 & 3 & 6 & 10 & 10 & 10 & 10 & 10 & 6 & 3 & 1 & 3 & 6 & 6 \\ 1 & 0 & 1 & 3 & 6 & 6 & 6 & 6 & 10 & 6 & 3 & 1 & 1 & 3 & 3 \\ 3 & 1 & 0 & 1 & 3 & 3 & 3 & 6 & 10 & 6 & 3 & 3 & 1 & 1 & 3 \\ 6 & 3 & 1 & 0 & 1 & 1 & 3 & 6 & 10 & 6 & 6 & 6 & 3 & 1 & 3 \\ 10 & 6 & 3 & 1 & 0 & 1 & 3 & 6 & 10 & 10 & 10 & 10 & 6 & 3 & 6 \\ 10 & 6 & 3 & 1 & 1 & 0 & 1 & 3 & 6 & 6 & 6 & 6 & 3 & 1 & 3 \\ 10 & 6 & 3 & 3 & 3 & 1 & 0 & 1 & 3 & 3 & 3 & 6 & 3 & 1 & 1 \\ 10 & 6 & 6 & 6 & 6 & 3 & 1 & 0 & 1 & 1 & 3 & 6 & 3 & 3 & 1 \\ 10 & 10 & 10 & 10 & 10 & 6 & 3 & 1 & 0 & 1 & 3 & 6 & 6 & 6 & 3 \\ 6 & 6 & 6 & 6 & 10 & 6 & 3 & 1 & 1 & 0 & 1 & 3 & 3 & 3 & 1 \\ 3 & 3 & 3 & 6 & 10 & 6 & 3 & 3 & 3 & 1 & 0 & 1 & 1 & 3 & 1 \\ 1 & 1 & 3 & 6 & 10 & 6 & 6 & 6 & 6 & 3 & 1 & 0 & 1 & 3 & 3 \\ 3 & 1 & 1 & 3 & 6 & 3 & 3 & 3 & 6 & 3 & 1 & 1 & 0 & 1 & 1 \\ 6 & 3 & 1 & 1 & 3 & 1 & 1 & 3 & 6 & 3 & 3 & 3 & 1 & 0 & 1 \\ 6 & 3 & 3 & 3 & 6 & 3 & 1 & 1 & 3 & 1 & 1 & 3 & 1 & 1 & 0 \end{pmatrix}$$

$W^{(H)} = 420$

$WV(G) = (30, 72, 81, 48, 0);$

$HWV(G) = (231, 126, 51, 12, 0)$

$W = 231;\ R = 420.$

## V. WIENER POLYNOMIAL SEQUENCE AND HYPER-WIENER POLYNOMIAL

Let D be the distance matrix of a graph G, let l be the largest entry of D , let 2d $_k$ be the number of such entries of D that are equal to k. The Wiener polynomial W(G, x) of G is then given by

$$W(G,x) = \sum_{k=1}^{l} d_k x^k$$

W(G, x) related to the Wiener index of a graph G, called the Wiener polynomial $W(G,x) = \sum_{\{u,v \subseteq V(G)} x^{d(u,v)}$ where d (u, v) denotes the distance between vertices u and v. The above two formulae give the same polynomial of G that

$W = W'(G,1), R = W'(G,1) = \frac{1}{2} W''(G,1)$ .

A hyper Hosoya polynomial is given by $HH(G,x) = \sum_{k=1}^{l} [k + 1/2] d_k x^k$ and showed that $R(G) = HH'(G,1)$. It can be found that the Wiener vector ($W_1$,$W_2$,…..) consists of the coefficients of the derivative $W'$(G,x) of the Wiener polynomial, where $W_k$ is equal to the coefficient $kd_{k^k}$ of $x^{k-1}$ in $W'(G,x)$. The hype-Wiener vector cannot be obtained from the hyper Hosoya polynomial

HH (G, x). Let G be a connected graph with n vertices. The k[th] Wiener polynomial of G, $1 \le k \le dia(G)$, is defined by $W_k(G,x) = \sum_{\{u,v\} \subseteq V(G)} x^{\max[d(u,v)-k+1,0]}$, where dia(G) is the diameter of G. The polynomial sequence $W_1(G,x), W_2(G,x), W_3(G,x), \ldots$

## VI. WIENER POLYNOMIAL SEQUENCE OF G

The Wiener polynomial of a graph was first introduced by Hosoya. Let D be the distance matrix of a graph G, let l be the largest entry of D, and let $2d_k$ be the number of such entries of D that are equal to k. The Wiener polynomial W(G,x) of G is then given by

$$W(G,x) = \sum_{k=1}^{n} d_k x^k$$

Later Sagan and Yeh and collegues defined a generating function W(G,x) related to the Wiener index of a graph G, called the Wiener polynomial:

$$W(G,x) = \sum_{\{u,v\} \subseteq V(G)} x^{d(u,v)},$$

where d(u,v) denotes the distance between vertices u and v.

$$W = W'(G,1),$$

$$R = W'(G,1) + \frac{1}{2} W''(G,1).$$

The weighted hyper-wiener number $R_W(G)$ of a graph G is defined as $R_W(G) = \sum_{k=1,2,\ldots} {}^{k}W_{y_k}$ where $y_k$ is the weight of ${}^{k}W$. In additon we will introduce a novel weighted hyper- wiener polynomial HW(G,x,y) of a graph G, so that the hyper-wiener vector and the weighted hyper-wiener number can be obtained from the polynomial. The Weighted hyper-Wiener polynomial HW(G,x,y) of a graph G defined as, $HW(G,x,y) = \sum_{k=1,2,\ldots} W_k(G,x) y_k$ where $y = (y_1, y_2, \ldots)$. From the above definition we have that $\frac{d}{dx}[HW(G,x,y)]/x = 1 = R_w(G) = \sum_{K=1,2,\ldots} {}^{k}W_{Y_K}$ where the coefficients of $y_k$ in the weighted hyper Wiener number $R_w(G)$ are equal to the k[th] Wiener number . Hence , the hyper –Wiener vector can be given by coefficients of d/dx=[HW(G,x,y]/ x=1.

In a particular graph, we have d/dx=[HW(G,x,y]/ x=1,y=(1,1,1,…..) = R(G) .

For a graph G, the Wiener vector, the Wiener polynomial sequence, the Weighted hyper-Wiener polynomial of G can be given as follows:

From Figure (1),

$$W'(G,x) = 3$$
$$WV(G) = (3,0), W(G) = 3.$$

$$W^{(1)}(G,x) = 3x$$
$$W^{(2)}(G,x) = 0$$

$$HW(G,x,y) = (3x)y_1$$
$$\frac{d}{dx}[HW(G,x,y)]_{x=1} = 3y_1,$$
$$HWV(G) = 3,$$
$$\frac{d}{dx}[HW(G,x,y)]_{x=1,y=(1,1,\ldots)} = R(G) = 3$$

For graph G in Figure (2), the Wiener Vector, the Wiener polynomial sequence, the Weighted hyper-Wiener polynomial of G can be given as follows:

$$W'(G,x) = 12x+9,$$
$$WV(G) = (9,12,0), W(G) = 21.$$
$$W^{(1)}(G,x) = 6x^2+9x,$$
$$W^{(2)}(G,x) = 6x.$$

$$HW(G,x,y) = (6x^2+9x)y_1 + (6x)y_2.$$
$$\frac{d}{dx}[HW(G,x,y)]_{x=1} = (12x+9)y_1 + 6y_2$$
$$= 21y_1 + 6y_2$$
$$HWV(G) = (21,6,0),$$
$$\frac{d}{dx}[HW(G,x,y)]_{x=1,y=(1,1\ldots)} = R(G) = 27$$

For a graph G in Figure 3, the Wiener vector, the Wiener polynomial sequence, the weighted hyper-

Wiener polynomial of G can be given as follows:

$$W'(G,x) = 27x^2+36x+18,$$
$$WV(G) = (18,36,27,0,0), W(G) = 81.$$
$$W^{(1)}(G,x) = 9x^3+18x^2+17x,$$
$$W^{(2)}(G,x) = 9x^2+18x+0$$
$$W^{(3)}(G,x) = 9x+0$$

$$HW(G,x,y) = (9x^3+18x^2+17x)y_1 + (9x^2+18x)y_2 + (9x)y_3$$
$$= (27x^2+36x+17)y_1 + (18x+18)y_2 + (9)y_3$$

$$\frac{d}{dx}[HW(G,x,y)]_{x=1} = 81y_1 + 36y_2 + 9y_3$$
$$HWV(G) = (81,36,9,0)$$
$$\frac{d}{dx}[HW(G,x,y)]_{x=1,y=(1,1,\ldots)} = R(G) = 126.$$

For graph G in Figure 4, the Wiener vector, the wiener polynomial sequence, the Weighted hyper-wiener polynomial of G can be given as follows:

$$W'(G,x) = 48x^3 + 81x^2 + 72x + 30,$$
$$WV(G) = (30,72,81,48,0), W(G) = 231.$$
$$W^{(1)}(G,x) = 12x^4 + 25x^3 + 34x^2 + 30x$$
$$W^{(2)}(G,x) = 12x^3 + 27x^2 + 36x + 30$$
$$W^{(3)}(G,x) = 12x^2 + 27x + 36$$
$$W^{(4)}(G,x) = 12x + 27$$

$$HW(G,x,y) = (12x^4 + 25x^3 + 34x^2 + 30x)y_1 + (12x^3 + 27x^2 + 36x + 30)y_2$$
$$+(12x^2 + 27x + 36)y_3 + (12x + 27)y_4$$
$$\frac{d}{dx}[HW(G,x,y)]_{x=1} = 231y_1 + 126y_2 + 51y_3 + 12y_4$$
$$HWV(G) = (231,126,51,12),$$
$$\frac{d}{dx}[HW(G,x,y)]_{x=1,y=(1,1......)} = R(G) = 420$$

## VII. TECHNIQUE OF INTERPOLATION

In similar manner the calculation the first nine values of wiener index of the triangular graphs have the following sequence as A={0,3,21,81,231,564,1134,2142,3762}
The difference sequence are obtained as follows:

$$DA = \{3,18,60,150,315,588,1008,1620,....\}$$
$$D^2A = \{15,42,90,165,273,420,612,.....\}$$
$$D^3A = \{27,48,75,108,147,192,......\}$$
$$D^4A = \{21,27,33,39,45,.....\}$$
$$D^5A = \{6\}_{k\geq 0}$$

According to these sequences, the generator of A is a fifth-degree polynomial.

Calculating the integral, we have

$$D^5A = \{6\}_{k\geq 0},$$
$$D^4A = \{6k^1 + 21\}_{k\geq 0}$$

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$$D^3A = \{3k^2 + 21k^1 + 27\}_{k\geq 0}$$

$$D^2A = \{k^3 + \frac{21}{2}k^2 + 27k^1 + 15\}_{k\geq 0}$$

$$DA = \left\{\frac{1}{4}k^4 + \frac{7}{2}k^3 + \frac{27}{2}k^2 + 15k^1 + 3\right\}_{k\geq 0}$$

$$\Rightarrow A = \left\{\frac{1}{20}k^5 + \frac{7}{8}k^4 + \frac{9}{2}k^3 + \frac{15}{2}k^2 + 3k^1\right\}_{k\geq 0}$$

$$= \left\{\frac{1}{40}(2k^5 + 15k^4 + 40k^3 + 45k^2 + 18k\right\}_{k\geq 0}$$

From the above it can be found that if the k[th] Wiener polynomial is given, the (k+1)[th] Wiener polynomial can be obtained from $W^{(k)}(G,x)$ by replacing exponent k of $x^k$ by k-1 for k=1,2,3,... for each integer $k \geq 0$,

$$W(G_{k+1}) = \frac{1}{40}[2k^5 + 15k^4 + 40k^3 + 45k^2 + 18k]$$

## VIII. CONCLUSION

The topological indices are used to study quantitative structure relationships–property or activity (QSPR/QSAR). The topological invariants such as the Wiener vector, the Hyper-wiener Vector, the Wiener Matrix Sequence, the Hyper Wiener Matrix Sequence, the Wiener polynomial Sequence and the Hyper-wiener polynomial, the Wiener number and the Hyper Wiener number of graphs are derived in this work. Whenever the closed formula of a topological index of an infinite family of graphs is in the form of a polynomial, the degree of polynomial as well as the polynomial itself can be calculated using the successive difference method. These sequences will be helpful for similarity research analysis and multiply regression analysis in the study of structure-property relationship.

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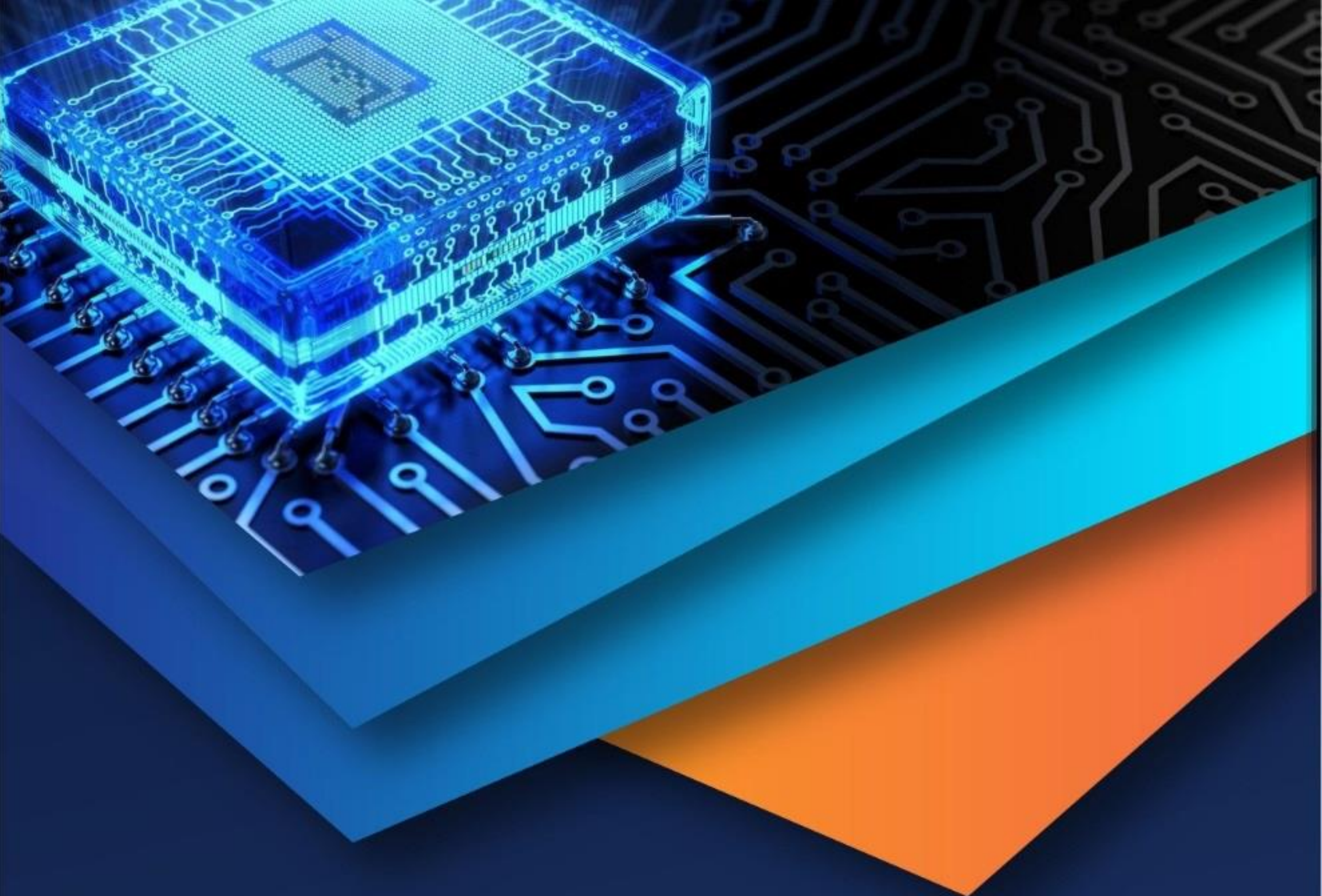


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