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Cayley-Hamilton Theorem for Fuzzy Matrix

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***Abstract*— The classical Cayley-Hamilton theorem says that every square matrix satisfies its own characteristic equation. The Cayley-Hamilton theorem has been extended to rectangular matrices, block matrices, pairs of commuting matrices, standard and singular two-dimensional linear (2-D) systems. The Cayley-Hamilton theorem and its generalizations have been used in control systems, electrical circuits, systems with delays, singular systems, 2-D linear systems etc., In this paper the new approach of Cayley-Hamilton theorem was done using the fuzzy matrices. For this, the Characteristic equation of fuzzy matrix, Fuzzy Eigen values and Eigen vectors have been derived.**
***Keywords*— Characteristic Equation, Eigen values, Eigen Vectors, Cayley-Hamilton theorem and Fuzzy Matrix.**

I. INTRODUCTION

The classical Cayley-Hamilton theorem (Gantmacher 1974, Kaczork 1988, Lancaster 1969) says that every square matrix satisfies its own characteristic equation. The Cayley-Hamilton theorem has been extended to rectangular matrices (Kaczorek 1995, Kaczorek 1988), block matrices (Kaczorek 1995), pairs of commuting matrices (Chang and Chan 1992, Lewis1982, lewis 1986, Kaczorek 1995) and standard and singular two-dimensional linear (2-D) systems (Kaczorek 1992, 1993, Kaczorek 1995, Smark and Barnett 1989, Theodoru 1989). The Cayley-Hamilton theorem and its generalizations have been used in control systems, electrical circuits, systems with delays, singular systems, 2-D linear systems etc., (Kaczorek 1992, 1993, Buslowicz 1981, Buslowicz 1982, Kaczorek 1994, Lewis 1982, Mertizions and Christodoulous 1986). The Cayley-Hamilton theorem has been extended to n-dimenstional (n-D) real polynomial matrices (Kaczorek 2005). An extension of the Cayley-Hamilton theorem for discrete-time linear systems with delay has been given in (Buslowicz and Kaczorek 2004). In this paper the new approach of Cayley-Hamilton theorem has been given using fuzzy matrices. For this, Characteristic equation of fuzzy matrix, Fuzzy Eigen values and Eigen vectors have been derived.

II. PRELIMINARIES

A. Cayley-Hamilton theorem for Square and Rectangular Matrices

Let $C^{n\times m}$ be the set of complex $(n\times m)$ matrices.

Theorem 1: (Cayley-Hamilton theorem). Let $A\in C^{n\times m}$ and

$p(s)=\det\left[\mathrm{I}_n s-A\right]=\sum_{i=0}^{n}a_i s^i \quad (a_n=1)$ be the characteristic polynomial of A, where I_n is the $(n\times n)$ identity matrix. Then

$p(A)=\sum_{i=0}^{n}a_i A^i=0_n$, where 0_n is the $(n\times n)$ matrix.

The classical Cayley-Hamilton theorem has extended to rectangular matrices as follows (Kaczorek 1988).

Theorem 2: (Cayley-Hamilton theorem for rectangular matrices). Let $A=\left[A_1 A_2\right]\in C^{n\times m}$, $A_1\in C^{m\times m}$, $A_2\in C^{m\times(n-m)}$, $n>m$ and

$p_{A_1}=\det\left[\mathrm{I}_{\mathrm{m}}\, s-A_1\right]=\sum_{i=0}^{m}a_i s^i \quad (a_m=1)$ be the characteristic polynomial of A_1.

Then $\sum_{i=0}^{m}a_{m-i}\left[A_i^{n-i}\,A_1^{n-i-1}A_2\right]=0_{mn}s^i$, where 0_{mn} is the $(n\times m)$ matrix.

Theorem 3: Let $A=\begin{bmatrix}A_1\\ A_2\end{bmatrix}\in C^{m\times n}$, $m>n$ and let the characteristic polynomial of A_1 have the form

$p_{A_1}=\det\left[\mathrm{I}_{\mathrm{m}}\, s-A_1\right]=\sum_{i=0}^{m}a_i s^i \quad (a_m=1)$. Then $\sum_{i=0}^{n}a_{n-i}\begin{bmatrix}A_1^{m-i}\\ A_2A_1^{m-i-1}\end{bmatrix}=0_{mn}$.

B. Cayley-Hamilton Theorem for Block Matrix

The classical Cayley-Hamilton theorem has also been extended for block matrices (Kaczorek 1998).

Theorem 4: (Cayley-Hamilton Theorem for block matrices).

Let $A = \begin{bmatrix} A_{11} & \dots & A_{1m} \\ \vdots & \ddots & \vdots \\ A_{m1} & \dots & A_{mm} \end{bmatrix} \in C^{mn \times mn}$, where $A_{ij} \in C^{n \times n}$ are commutative, i.e., $A_{ij}A_{kl} = A_{kl}A_{ij}$ for all $i, j, k, l = 1, 2, \dots, m$

Let $P(S) = \det\left[\mathrm{I_m} \otimes S - A\right] = S^m + D_1 S^{m-1} + \dots + D_{m-1}S + D_m$ be the matrix characteristic polynomial of A, where $S \in C^{n \times n}$ is the matrix (block) eigenvalue of A, $\otimes$ denotes the Kronecker product of matrices (Kaczorek 1988). Then $P(A) = \sum_{i=0}^{m} \left(\mathrm{I_m} \otimes D_{m-i}\right) A_i = 0 \quad \left(D_0 = I_n\right)$.

The matrix $P(S) = \det\left[\mathrm{I_m} \otimes S - A\right] = S^m + D_1 S^{m-1} + \dots + D_{m-1}S + D_m$ is obtained by developing the determinant of the matrix $\left[I_n \otimes S - A\right]$ considering its commuting blocks as scalar entries (Kaczorek 1988).

Theorem 5: (Cayley-Hamilton theorem for rectangular block matrices).

Let $A = \left[A_1 \;\; A_2\right] \in C^{mn \times (mn+p)}$ and let the matrix characteristic polynomial of A have the form $\det\left[\mathrm{I}_n z^{h+1} - A_0 z^h - A_1 z^{h-1} - \dots - A^h\right]$. Then $\sum_{i=0}^{m} \left(\mathrm{I_m} \otimes D_{m-i}\right)\left[A^{i+1} \quad A^i A_2\right] = 0 \;\; D_0 = I_n$ The dual theorem has the form.

Theorem 6: Let $\overline{A} = \begin{bmatrix} A \\ A_2 \end{bmatrix} \in C^{(mn+p) \otimes mn}$, $A \in C^{mn \times mn}, A_2 \in C^{p \times mn}$ and let the matrix characteristic polynomial of A have the form $\det\left[\mathrm{I}_n z^{h+1} - A_0 z^h - A_1 z^{h-1} - \dots - A^h\right]$. Then

$$\sum_{i=0}^{m} \begin{bmatrix} A \\ A_2 \end{bmatrix} \left(\mathrm{I_m} \otimes D_{m-i}\right) A_i = 0 \quad \left(D_0 = I_n\right)$$

C. Cayley-Hamilton theorem for Systems with Delays

1) Discrete time-systems

Consider the discrete-time linear system with h delays described by the equation $x_{i+1} = A_0 x_i + A_1 x_{i-1} + \dots + A_h x_{i-h} + Bu_i$, where $x_i \in C^n$, $u_i \in C^m$ are the state and input vectors, $A_k \in C^{n \times n}$, $k = 1, 2, \dots h$ and $B \in C^{n \times m}$. The characteristic polynomial of $x_{i+1} = A_0 x_i + A_1 x_{i-1} + \dots + A_h x_{i-h} + Bu_i$ has the form.

$$p(z) = \det \begin{bmatrix} I_n z - A_0 & -A_1 & \dots & -A_{h-1} & -A_h \\ -I_n & I_n z & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & -I_n & I_n z \end{bmatrix} = \det\left[I_n z^{h+1} - A_0 z^h - A_1 z^{h-1} - \dots - A_h\right]$$

$= z^N + a_{N-1} z^{N-1} + \dots + a_1 z + a_0$, $N = n(h+1)$. Let $\Phi_{i+1} = A_0 \Phi_i + A_1 \Phi_{i-1} + \dots + A_h \Phi_{i-h}$ and $\Phi_0 = I_n$ and $\Phi_i = I_0$ for $i < 0$ knowing the matrices A_k, $k = 0, 1, \dots, h$. Using the above equation, we may compute the matrices Φ_i for $i = 1, 2, \dots$.

Theorem 7: The matrices Φ_i for $i = 1, 2, \dots$ defined by $\Phi_{i+1} = A_0 \Phi_i + A_1 \Phi_{i-1} + \dots + A_h \Phi_{i-h}$ and $\Phi_0 = I_n$ and $\Phi_i = I_0$ for $i < 0$ satisfying the equation $\sum_{i=0}^{N} a_{i+k} \Phi_{i+k} = 0$ for $k = 0, 1, \dots$ $\left(a_N = 1\right)$, where a_i, $i = 0, 1, \dots, N-1$ are the coefficients of the characteristic polynomial $\det\left[I_n z^{h+1} - A_0 z^h - A_1 z^{h-1} - \dots - A_h\right]$

Proof: From definition of the inverse of the inverse matrix we have

$\left[I_n z^{h+1} - A_0 z^h - A_1 z^{h-1} - \ldots - A_h\right]_{ad} = \left[I_n z^{h+1} - A_0 z^h - A_1 z^{h-1} - \ldots - A_h\right]^{-1} \times \det\left[I_n z^{h+1} - A_0 z^h - A_1 z^{h-1} - \ldots - A_h\right]$

from $\Phi_{i+1} = A_0\Phi_i + A_1\Phi_{i-1} + \ldots + A_h\Phi_{i-h}$ and $\Phi_0 = I_n$ and $\Phi_i = I_0$ for $i < 0$ it follows that

$\left[I_n z^{h+1} - A_0 z^h - A_1 z^{h-1} - \ldots - A_h\right]^{-1} = \left[I_n z^{h+1} + A_0 z^h - A_1 z^{h-1} - \ldots - A_h\right]_{ad}$ from the above equation , we obtain

$\left[I_n z^{h+1} - A_0 z^h - A_1 z^{h-1} - \ldots - A_h\right]_{ad} = \left[I_n z^{-1} + \Phi_1 z^{-2} + \Phi_2 z^{-3} + \ldots\right]\left(z^N + a_{N-1} z^{N-1} + \ldots + a_1 z + a_0\right)$ Comparing the coefficient of the same power of $z^{-(k+1)}$ in the $\left[I_n z^{-1} + \Phi_1 z^{-2} + \Phi_2 z^{-3} + \ldots\right]$ we obtain the equation $\sum_{i=0}^{N} a_{i+k}\Phi_{i+k} = 0$ for $k = 0,1,\ldots$ $(a_N = 1)$.

Example 1: Let $h = I$ and $A_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$. The Characteristic polynomial

$\det\left[I_n z^{h+1} - A_0 z^h - A_1 z^{h-1} - \ldots - A_h\right]$ in this case, has the form $p(z) = \det\left[I_2 Z^2 - A_0 z - A_1\right]$

$$= \begin{vmatrix} z^2 - z & -1 \\ -1 & z^2 - z - 2 \end{vmatrix} = z^4 - 2z^3 - z^2 + 2z - 1.$$

Using $\Phi_{i+1} = A_0\Phi_i + A_1\Phi_{i-1} + \ldots + A_h\Phi_{i-h}$ and $A_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ we obtain

$$\Phi_1 = A_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Phi_2 = A_0\Phi_1 + A_1 = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$$

$$\Phi_3 = A_0\Phi_2 + A_1\Phi_1 = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$$

$$\Phi_4 = A_0\Phi_3 + A_1\Phi_2 = \begin{bmatrix} 2 & 5 \\ 5 & 12 \end{bmatrix}$$

Hence by $\sum_{i=0}^{N} a_{i+k}\Phi_{i+k} = 0$ for k=0 we obtain

$$\Phi_4 - 2\Phi_3 - \Phi_2 + 2\Phi_1 - I_2 = \begin{bmatrix} 2 & 5 \\ 5 & 12 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Therefore, the matrices $\Phi_1, \Phi_2, \Phi_3, \Phi_4$ satisfy the equation $= z^4 - 2z^3 - z^2 + 2z - 1$.

D. Cayley-Hamilton theorem for Singular Systems

Consider the singular systems $E\dot{x} = Ax + Bu$, where $x \in R^n$, $u \in R^m$, $A \in R^{n\times n}$, $B \in R^{n\times m}$.

It is assumed that *det E=0*, $\det[Es - A] \neq 0$ for some $s \in C$ and $EA = AE$. If the assumption $\det[Es - A] \neq 0$ holds then it is easy to show (Kaczorek 1988) that the matrices

$\bar{E} = [Es - A]^{-1} E$, $\bar{A} = [Es - A]^{-1} A$ Satisfy the conditions $\overline{EA} = \overline{AE}$

Theorem 8:

Let $\det[Es - A] = a_r s^r + a_{r-1} s^{r-1} + \ldots + a_1 s + a_0$ $(r = rank\ E < n)$ Then $\sum_{i=0}^{r} a_i A^i E^{n-i} = 0$

Proof :

Let $Adj[Es - A] = B_{n-1} s^{n-1} + \ldots + B_1 s + B_0$ be the adjoint matrix of $[Es - A]$.

From definition of the inverse matrix and $\det[Es - A] = a_r s^r + a_{r-1} s^{r-1} + \ldots + a_1 s + a_0$, $Adj[Es - A] = B_{n-1} s^{n-1} + \ldots + B_1 s + B_0$ we have $[Es - A]\left[B_{n-1} s^{n-1} + \ldots + B_1 s + B_0\right] = I_n\left(a_r s^r + a_{r-1} s^{r-1} + \ldots + a_1 s + a_0\right)$

The comparison of the coefficients of the same powers of s in the equality

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$\left[Es-A\right]\left[B_{n-1}s^{n-1}+\ldots+B_1s+B_0\right]=I_n\left(a_rs^r+a_{r-1}s^{r-1}+\ldots+a_1s+a_0\right)$ yields

$$\begin{bmatrix} E & 0 & 0 & \ldots & 0 & 0 & 0 \\ -A & E & 0 & \ldots & 0 & 0 & 0 \\ 0 & -A & E & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & -A & E & 0 \\ 0 & 0 & 0 & \ldots & 0 & -A & E \\ 0 & 0 & 0 & \ldots & 0 & 0 & -A \end{bmatrix} \begin{bmatrix} B_{n-1} \\ B_{n-2} \\ B_{n-3} \\ \vdots \\ B_2 \\ B_1 \\ B_0 \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ a_rI_n \\ \vdots \\ a_1I_n \\ a_0I_n \end{bmatrix}$$

Pre multiplying above equation by the row matrix

$$\begin{bmatrix} A^n & A^{n-1}E & A^{n-2}E^2 & \ldots & AE^{n-1} & E^n \end{bmatrix}$$

And using equation $EA=AE$ we obtain the equation $\sum_{i=0}^{r}a_iA^iE^{n-i}=0$.

III. PROPOSED DEFINITIONS AND THEOREM

In this section we give the proposed Characteristic Equations of Fuzzy matrix, Polynomial equations of fuzzy matrix, working rule to find characteristic equation of fuzzy matrix, Fuzzy Eigen Values and Eigen vectors, Properties of Fuzzy Eigen values and Eigen vectors as follows:

A. Characteristic Equation of Fuzzy Matrix

Consider the linear transformation $Y = A_F X$

In general, this transformation transforms a column vector $X=\begin{bmatrix} x_1 \\ x_2 \\ . \\ x_n \end{bmatrix}$ into the another column vector $Y=\begin{bmatrix} y_1 \\ y_2 \\ . \\ y_n \end{bmatrix}$

By means of the square fuzzy matrix A_F where

$$A_F=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ . & . & \cdots & . \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

If a vector X is transformed into a scalar multiple of the same vector. i.e., X is transformed into λX, then $Y=\lambda X = A_F X$

i.e., where I is the unit matrix of order 'n'.

$$A_F X - \lambda I X = O$$

$$(A_F-\lambda I)X = O \qquad \ldots\ldots\ldots(1)$$

$$\left[\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ . & . & \cdots & \cdots \\ . & . & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2n} \\ . & . & . & . \\ . & . & . & . \\ a_{n1} & a_{n2} & \cdots & a_{nn}-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ 0 \end{bmatrix}$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)

$$\text{i.e.,}\quad \begin{aligned} &(a_{11}-\lambda)x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = 0 \\ &a_{21}x_1 + (a_{22}-\lambda)x_2 + \ldots + a_{2n}x_n = 0 \\ &\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;. \\ &\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;. \\ &a_{n1}x_1 + a_{n2}x_2 + \ldots + (a_{nn}-\lambda)x_n = 0 \end{aligned} \qquad \ldots\ldots\ldots(2)$$

This system of equations will have a non-trivial solution, if $|A_F - \lambda I| = 0$

$$\text{i.e.,}\quad \begin{vmatrix} a_{11}-\lambda & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22}-\lambda & \ldots & a_{2n} \\ . & . & \ldots & . \\ . & . & \ldots & . \\ a_{n1} & a_{n2} & \ldots & a_{nn}-\lambda \end{vmatrix} = 0 \qquad \ldots\ldots\ldots\ldots(3)$$

The equation $|A_F - \lambda I| = 0$ or equation (3) is said to be the characteristic equation of the transformation or the characteristic equation of the matrix *A*. Solving $|A_F - \lambda I| = 0$, we get *n* roots for λ, these roots are called the characteristic roots (or) Eigen values of the matrix A_F. Corresponding to each value of λ, the equation $A_F X = \lambda X$ has a non-zero solution vector *X*. Let X_r, be the non-zero vector satisfying $A_F X = \lambda X$. When $\lambda = \lambda_r$, X_r is said to be the latent vector or Eigen vector of a matrix A_F corresponding to λ_r.

2) Characteristic polynomial of Fuzzy Matrix

The determinant $|A_F - \lambda I|$ when expanded will give a polynomial, which we call as the characteristic polynomial of fuzzy matrix A_F.

3) Working rule to find characteristic equation

Let A_F be any fuzzy square matrix of order *n*. The characteristic equation of A_F is $|A_F - \lambda I| = 0$.

B. Cayley – Hamilton theorem for fuzzy matrix

Statement: Every fuzzy square matrix satisfies its own characteristic equation.

C. Uses of Cayley – Hamilton theorem.

(i) The positive integral powers of A_F and

(ii) The inverse of a non-singular fuzzy square matrix A_F.

Example 1: Find the characteristic equation of fuzzy matrix $A_F = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.2 \end{bmatrix}$

Solution: Given: $A_F = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.2 \end{bmatrix}$

The characteristic equation of A_F is $|A_F - \lambda I| = 0$.

The characteristic equation of fuzzy matrix A_F is $\lambda^2 - S_1\lambda + S_2 = 0$

i.e., $S_1 = 0.3$, $S_2 = 0.02$

The characteristic equation of fuzzy matrix A_F is $\lambda^2 - 0.3\lambda + 0.02 = 0$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)

Example 2: Find the characteristic equation of fuzzy matrix $A_F = \begin{bmatrix} 0.2 & -0.3 & 1 \\ 0.3 & 0.1 & 0.3 \\ -0.5 & 0.2 & 0.4 \end{bmatrix}$

Solution: Given: $A_F = \begin{bmatrix} 0.2 & -0.3 & 1 \\ 0.3 & 0.1 & 0.3 \\ -0.5 & 0.2 & 0.4 \end{bmatrix}$

The characteristic equation of A_F is $|A_F - \lambda I| = 0$.

The characteristic equation of fuzzy matrix A_F is $\lambda^3 - S_1\lambda^2 + S_2\lambda + S_3 = 0$

i.e., $S_1 = 0.7$, $S_2 = 0.67$ and $S_3 = 0.187$

The characteristic equation of fuzzy matrix A_F is $\lambda^3 - 0.7\lambda^2 + 0.67\lambda + 0.187 = 0$

Example 3: Prove that Cayley Hamilton theorem for $A_F = \begin{bmatrix} 0.1 & -0.2 \\ 0.2 & 0.1 \end{bmatrix}$

Solution: Every fuzzy square matrix satisfies its own characteristic equation.

Given: $A_F = \begin{bmatrix} 0.1 & -0.2 \\ 0.2 & 0.1 \end{bmatrix}$

The characteristic equation of A_F is $|A_F - \lambda I| = 0$.

The characteristic equation of fuzzy matrix A_F is $\lambda^2 - S_1\lambda + S_2 = 0$

i.e., $S_1 = 0.2$ and $S_2 = 0.05$

The characteristic equation of the fuzzy matrix is $\lambda^2 - 0.2\lambda + 0.05 = 0$

To prove: Cayley Hamilton theorem $A^2 - 0.2A + 0.05I = 0$

$$A^2 = \begin{bmatrix} 0.1 & -0.2 \\ 0.2 & 0.1 \end{bmatrix}\begin{bmatrix} 0.1 & -0.2 \\ 0.2 & 0.1 \end{bmatrix} = \begin{bmatrix} -0.03 & -0.04 \\ 0.04 & -0.03 \end{bmatrix}$$

$$0.2A = \begin{bmatrix} 0.02 & -0.04 \\ 0.04 & 0.02 \end{bmatrix}$$

$$A^2 - 0.2A + 0.05I = \begin{bmatrix} -0.03 & -0.04 \\ 0.04 & -0.03 \end{bmatrix} - \begin{bmatrix} 0.02 & -0.04 \\ 0.04 & 0.02 \end{bmatrix} + \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$$

$$A^2 - 0.2A + 0.05I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Cayley Hamilton theorem is proved.

Example 4: Prove that Cayley Hamilton theorem for $A_F = \begin{bmatrix} 0.2 & -0.1 & 0.2 \\ -0.1 & 0.2 & -0.1 \\ 0.1 & -0.1 & 0.2 \end{bmatrix}$

Solution: Every fuzzy square matrix satisfies its own characteristic equation.

Given: $A_F = \begin{bmatrix} 0.2 & -0.1 & 0.2 \\ -0.1 & 0.2 & -0.1 \\ 0.1 & -0.1 & 0.2 \end{bmatrix}$

The characteristic equation of A_F is $|A_F - \lambda I| = 0$.

The characteristic equation of fuzzy matrix A_F is $\lambda^3 - S_1\lambda^2 + S_2\lambda + S_3 = 0$

i.e., $S_1 = 0.6$, $S_2 = 0.08$ and $S_3 = 0.003$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)

The characteristic equation of the fuzzy matrix is $\lambda^3 - 0.6\lambda^2 + 0.08\lambda - 0.003 = 0$

To prove: Cayley Hamilton theorem $A^3 - 0.6A^2 + 0.08A - 0.003I = 0$

$$A^2 = \begin{bmatrix} 0.2 & -0.1 & 0.2 \\ -0.1 & 0.2 & -0.1 \\ 0.1 & -0.1 & 0.2 \end{bmatrix}\begin{bmatrix} 0.2 & -0.1 & 0.2 \\ -0.1 & 0.2 & -0.1 \\ 0.1 & -0.1 & 0.2 \end{bmatrix} = \begin{bmatrix} 0.07 & -0.06 & 0.09 \\ -0.05 & 0.06 & -0.06 \\ 0.05 & -0.05 & 0.07 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0.07 & -0.06 & 0.09 \\ -0.05 & 0.06 & -0.06 \\ 0.05 & -0.05 & 0.07 \end{bmatrix}\begin{bmatrix} 0.2 & -0.1 & 0.2 \\ -0.1 & 0.2 & -0.1 \\ 0.1 & -0.1 & 0.2 \end{bmatrix} = \begin{bmatrix} 0.029 & -0.028 & 0.038 \\ -0.022 & 0.023 & -0.028 \\ 0.022 & -0.022 & 0.029 \end{bmatrix}$$

$$A^3 - 0.6A^2 + 0.08A - 0.003I = \begin{bmatrix} 0.029 & -0.028 & 0.038 \\ -0.022 & 0.023 & -0.028 \\ 0.022 & -0.022 & 0.029 \end{bmatrix} - \begin{bmatrix} 0.042 & -0.036 & 0.054 \\ -0.030 & 0.036 & -0.036 \\ 0.030 & -0.030 & 0.042 \end{bmatrix}$$

$$+ \begin{bmatrix} 0.016 & -0.008 & 0.016 \\ -0.008 & 0.016 & -0.008 \\ 0.008 & -0.008 & 0.016 \end{bmatrix} - \begin{bmatrix} 0.003 & 0 & 0 \\ 0 & 0.003 & 0 \\ 0 & 0 & 0.003 \end{bmatrix}$$

$$A^3 - 0.6A^2 + 0.08A - 0.003I = \begin{bmatrix} 0.045 & -0.036 & 0.054 \\ -0.030 & 0.039 & -0.036 \\ 0.022 & -0.030 & 0.045 \end{bmatrix} - \begin{bmatrix} 0.045 & -0.036 & 0.054 \\ -0.030 & 0.036 & -0.036 \\ 0.030 & -0.030 & 0.042 \end{bmatrix}$$

$$A^3 - 0.6A^2 + 0.08A - 0.003I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence Cayley Hamilton theorem is proved.

IV. CONCLUSIONS

In this paper a new approach of Cayley-Hamilton theorem for fuzzy matrix was discussed. For that we have given the characteristic equation of fuzzy matrix, Eigen values of the fuzzy matrix, and hence Cayley-Hamilton theorem was presented. Examples also were given. The application of the Cayley-Hamilton theorem are electronic circuits, measurement of string vibrating, finding out the level of heat conduction flow, algebraic and differential equations, nuclear physics, mechanics, aero dynamics and astronomy.

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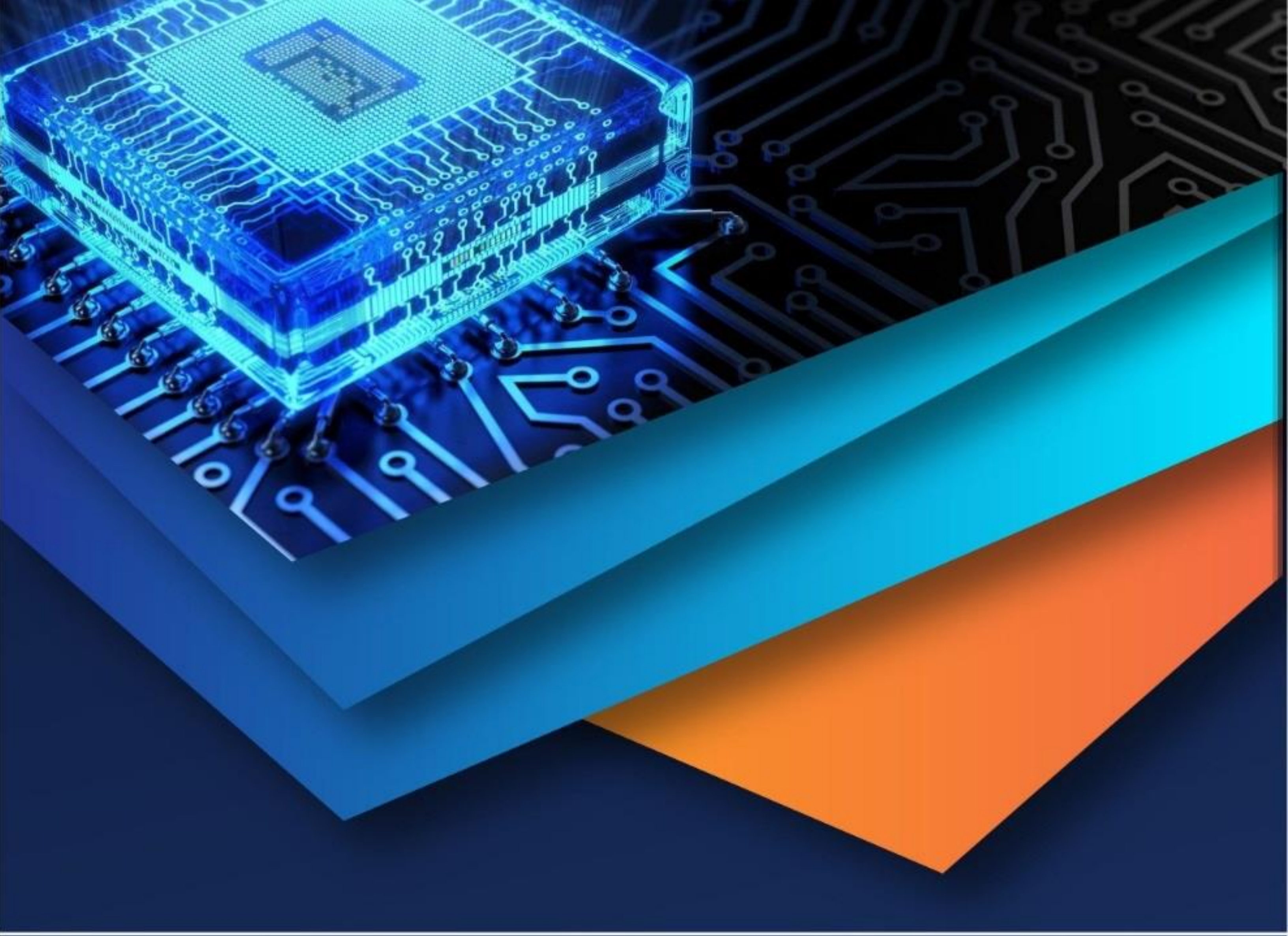

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