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Elliptically Polarized Dust Alfven Wave Excited by an Ion Beam

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***Abstract:** An ion beam propagating through dusty plasma drives electromagnetic dust Alfven waves to instability via slow cyclotron interaction. We study the combined effect of number density of dust grains and ion beam on the growth rate of the dust Alfven wave instability in dusty plasma. It is shown that the presence of charged dust grains and ion beam modify the dispersion relation of low frequency Alfven wave. The dust grain concentration and the fluctuating charge on dust grains reduce the frequency of the dust Alfven wave. The damping rate of Alfven wave is reduced in the presence of ion beam. The dielectric constant and refractive index increase with an increase in dust grain number density. The value of maximum growth rate increases with an increase in unstable parallel wave number, unstable frequency and the beam density.*

***Keywords:** Alfven, cyclotron, dispersion, growth rate, refractive index*

I. INTRODUCTION

The study of wave propagation in dusty plasma is important for many applications as well as for understanding the processes in the Earth's ionosphere, space and cometary plasmas [1-4]. The dust grains are usually negatively charged, with a large number of electrons on each dust grain, and they have a mass that is significantly larger than the positive ion mass. The presence of charged dust grains can change the existing plasma modes or introduce new eigenmodes. Waves in dusty plasmas have been studied theoretically [5-8] and experimentally [9-12] by a number of workers.

The dominant and the lowest frequency transverse mode of magnetized plasma is the Alfven wave. Several theoretical models have proposed Alfven waves as a possible accelerating mechanism for solar and stellar winds [13-16]. In laboratory and space plasmas, Alfven waves may be excited by many sources such as external antennae, energetic charged particle beams, nonuniform plasma parameters, electrostatic and electromagnetic waves, etc. But these Alfven waves are strongly modified in a dusty plasma, when a fraction of charge settles down on the dust grain surface. Even if the proportion of charge on the dust grains compared to that of free electrons is quite small, it can have a large effect on shear or polarized Alfven waves propagating at frequencies well below the ion cyclotron frequency. In this paper, we study the electromagnetic dust Alfven waves in three component dusty plasmas with electrons, ions and dust grains. In Sec. 2, we carry out the instability analysis and find the expression for growth rate using first order perturbation technique. Results and discussions are given in Sec. 3. And the conclusion part is given in Sec. 4.

II. INSTABILITY ANALYSIS

We consider a dusty plasma whose constituents are electrons, ions and negatively charged dust grains, immersed in a static magnetic field B_s in the z-direction. An ion beam is considered propagating along z-axis parallel to the magnetic field with density n_{bo}, mass m_i and equilibrium beam velocity $v_{bo}\hat{z}$. At equilibrium, the plasma system is quasineutral, i.e., $n_{bo}+n_{io}= n_{eo}+ Z_d n_{do}$, where n_{io}, n_{eo}, n_{do} are the number densities of ions, electrons and dust grains, respectively, Z_d (= Q_{do}/e) is the number of electrons residing on the surface of negatively charged dust grains; Q_{do} being the equilibrium dust grain charge and e is the magnitude of the electronic charge. Let us consider an elliptically polarized electromagnetic Alfven wave, propagating parallel to the external static magnetic field with propagation wave vector k_z. We assume the t, z variations of Alfven fields as $\mathbf{E},\mathbf{B} \propto \exp[-i(\omega t - k_z z)]$ and consider $\mathbf{E}$ = (E_x, E_y,0).

The perturbed magnetic field of the wave is $\mathbf{B} = \frac{c}{\omega}\mathbf{k}\times\mathbf{E}$.

The equation of motion for the perturbed plasma electrons is

$$\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla(\mathbf{v}) = -\frac{e}{m_e}\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}_s+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot \qquad (1)$$

On linearizing Eq. (1), we obtain the perturbed electron velocities as

$$v_{e1x} = -\frac{e\left(i\omega\, E_x + \omega_{ce}\, E_y\right)}{m_e\left(\omega^2 - \omega_{ce}^2\right)}, \tag{2}$$

$$v_{e1y} = \frac{e\left(\omega_{ce}E_x - i\omega\, E_y\right)}{m_e\left(\omega^2 - \omega_{ce}^2\right)}, \tag{3}$$

Using Eqs. (2), (3) and (4), we get the perturbed plasma electron current density as:

$$\mathbf{J_{e1}} = n_{eo}\frac{e^2}{m_e}\frac{\left(i\omega\, E_x + \omega_{ce}\, E_y\right)}{\left(\omega^2 - \omega_{ce}^2\right)}\hat{x} - n_{eo}\frac{e^2}{m_e}\frac{\left(\omega_{ce}E_x - i\omega\, E_y\right)}{\left(\omega^2 - \omega_{ce}^2\right)}\hat{y}\,. \tag{4}$$

The response of the plasma ions and dust grains can be obtained from Eqs. (2), (3) and (4), by replacing e, $m_{e,}$ ω_{ce}, n_{e0} by -e, m_i, $-\omega_{ci}$, n_{i0} for ions and by Q_{d0}, m_d, $-\omega_{cd}$ and n_{d0} for dust grains. The frequency of the dust Alfven mode is such that $\omega < \omega_{cd} << \omega_{ci} << \omega_{ce}$; ω_{cd} , ω_{ci} and ω_{ce} are the dust, ion and electron cyclotron frequencies respectively.

The response of beam ions to the perturbation is also governed by the equation of motion, which on linearization yields the perturbed beam velocities as

$$v_{b1x} = \frac{e}{i\, m_i}\frac{\varpi^2}{\omega\left(\varpi^2 - \omega_{ci}^2\right)}E_x - \frac{e}{m_i}\frac{\varpi\ \omega_{ci}}{\omega\left(\varpi^2 - \omega_{ci}^2\right)}E_y\,, \tag{5}$$

$$v_{b1y} = \frac{e}{m_i}\frac{\varpi\ \omega_{ci}}{\omega\left(\varpi^2 - \omega_{ci}^2\right)}E_x - \frac{i\,e}{m_i}\frac{\varpi^2}{\omega\left(\varpi^2 - \omega_{ci}^2\right)}E_y\,, \tag{6}$$

where $\varpi = \omega - k_z v_{b0}$.

Using the perturbed beam velocities given by Eqs. (5) and (6), we obtain the perturbed beam current density as

$$\mathbf{J_{b1}} = \left[\frac{ie^2 n_{bo}}{m_i}\frac{\varpi^2}{\omega\left(\varpi^2 - \omega_{ci}^2\right)}E_x + \frac{e^2 n_{bo}}{m_i}\frac{\varpi\ \omega_{ci}}{\omega\left(\varpi^2 - \omega_{ci}^2\right)}E_y\right]\hat{x}$$

$$-\left[\frac{e^2 n_{bo}}{m_i}\frac{\varpi\ \omega_{ci}}{\omega\left(\varpi^2 - \omega_{ci}^2\right)}E_x - \frac{i\,e^2 n_{bo}}{m_i}\frac{\varpi^2}{\omega\left(\varpi^2 - \omega_{ci}^2\right)}E_y\right]\hat{y}\,. \tag{7}$$

The wave equation which governs the mode structure of low frequency dust Alfven waves is given as

$$\nabla^2\mathbf{E} - \nabla\left(\nabla\cdot\mathbf{E}\right) + \left(\frac{\omega^2}{c^2}\right)\mathbf{E} = -\frac{4\pi i\omega}{c^2}\mathbf{J_1}\,. \tag{8}$$

Writing the x and y components of Eq. (8), we get

$$AE_x + iBE_y = 0\,, \tag{9}$$

$$AE_y - iBE_x = 0\,, \tag{10}$$

where

$$A = -k_z^2c^2 + \omega^2 - \frac{\omega_{pe}^2\omega^2}{\left(\omega^2 - \omega_{ce}^2\right)} - \frac{\omega_{pi}^2\omega^2}{\left(\omega^2 - \omega_{ci}^2\right)} - \frac{\omega_{pd}^2\omega^2}{\left(\omega^2 - \omega_{cd}^2\right)} - \frac{\omega_{pb}^2\varpi^2}{\left(\varpi^2 - \omega_{ci}^2\right)},$$

$$B=\frac{\omega\,\omega_{ce}\omega_{pe}^{2}}{\left(\omega^{2}-\omega_{ce}^{2}\right)}-\frac{\omega\,\omega_{ci}\omega_{pi}^{2}}{\left(\omega^{2}-\omega_{ci}^{2}\right)}+\frac{\omega\,\omega_{cd}\omega_{pd}^{2}}{\left(\omega^{2}-\omega_{cd}^{2}\right)}+\frac{\bar{\omega}\,\omega_{ce}\omega_{pb}^{2}}{\left(\bar{\omega}^{2}-\omega_{ci}^{2}\right)},$$

$$\omega_{pb}^{2}=\frac{4\pi n_{b0}e^{2}}{m},\omega_{pe}^{2}=\frac{4\pi n_{e0}e^{2}}{m_{e}},\omega_{pi}^{2}=\frac{4\pi n_{i0}e^{2}}{m_{i}}\text{ and }\omega_{pd}^{2}=\frac{4\pi\, n_{do}Q_{do}^{2}}{m_{d}}.$$

A non-trivial solution of Eqs. (9) and (10) demands that the determinant of coefficients of E_x and E_y must vanish, i.e., $A^2-B^2=0$.

It gives two distinct modes of wave propagation. We examine here right-hand polarized electromagnetic dust Alfven waves i.e. $A-B=0$, or

$$k_{z}^{2}-\frac{\omega^{2}}{c^{2}}\in_{R}=-\frac{\omega_{pb}^{2}\bar{\omega}}{c^{2}\left(\bar{\omega}+\omega_{ci}\right)}\ . \tag{11}$$

where $\in_{R}=1+\dfrac{\omega_{pe}^{2}}{\omega\omega_{ce}}+\dfrac{\omega_{pi}^{2}}{\omega_{ci}^{2}}-\dfrac{\omega_{pi}^{2}}{\omega\omega_{ci}}-\dfrac{\omega_{pd}^{2}\left(\omega+\omega_{cd}\right)}{\omega^{3}}$.

The refractive index of the dusty plasma can be written as

$$\mu=\left[1+\frac{\omega_{pe}^{2}}{\omega\omega_{ce}}+\frac{\omega_{pi}^{2}}{\omega_{ci}^{2}}-\frac{\omega_{pi}^{2}}{\omega\omega_{ci}}-\frac{\omega_{pd}^{2}\left(\omega+\omega_{cd}\right)}{\omega^{3}}\right]^{1/2}. \tag{12}$$

Eq. (11) can be rewritten as

$$\omega^{2}-\frac{k_{z}^{2}c^{2}}{\in_{R}}=\frac{\omega_{pb}^{2}\bar{\omega}}{\in_{R}\left(\bar{\omega}+\omega_{ci}\right)}. \tag{13}$$

Eq. (13) gives the dispersion relation of elliptically polarized dust Alfven waves in a dusty plasma. In the absence of beam Eq. (13) gives

$$\omega=k_{z}V_{A}\left[1+\frac{V_{A}^{2}}{c^{2}}+\frac{n_{do}m_{d}}{n_{io}m_{i}}\right]^{-1/2}, \tag{14}$$

where $V_{A}=c\dfrac{\omega_{ci}}{\omega_{pi}}$, is the Alfven speed.

Assuming perturbed quantities $\omega=\dfrac{k_{z}c}{\in_{R}}+\Delta$ and $\omega=k_{z}v_{bo}-\omega_{ci}+\Delta$ in Eq.(12), we get

Growth rate $\gamma=\mathrm{Im}(\Delta)$

or
$$\gamma=\left[\frac{\omega_{pb}^{2}\omega_{ci}}{2k_{z}c\sqrt{\in_{R}}}\right]^{1/2}. \tag{15}$$

III. RESULTS AND DISCUSSION

The parameters used in the present calculations are: ion plasma density n_{i0} = 10^{12} cm^{-3}, magnetic field B_s = 100 G, mass of ion m_i=39×1836m_e (potassium-plasma), mass of dust grain m_d = 10^{-15} kg, beam density n_{b0}=10^9cm^{-3} and beam velocity v_{b0} = 3.5×10^7 cm/s. The dust grain number density of negatively charged dust grains n_{d0} has been varied from 0.05×10^4 cm^{-3} to 2×10^4 cm^{-3}.

In Fig. 1, we have plotted the dispersion curves of dust Alfven waves in magnetized dusty plasma using Eq. (14), for different values of dust grain number density n_{d0}. We have also plotted the beam mode via slow cyclotron interaction for an ion beam travelling inside the dusty plasma. The velocity of the beam is chosen in such a way so that they intersect the dispersion curves of dust Alfven waves in the required frequency range. The frequencies and the corresponding wave numbers of the unstable mode obtained from the points of intersection between the beam mode and the dust Alfven modes are given as Table 1. The unstable frequencies of the Alfven waves in presence of dust grains and the parallel wave vector k_z decrease with increase in number density of negatively charged dust grains. The phase velocity and the group velocity of the wave decreases with an increase in the concentration of dust grains.

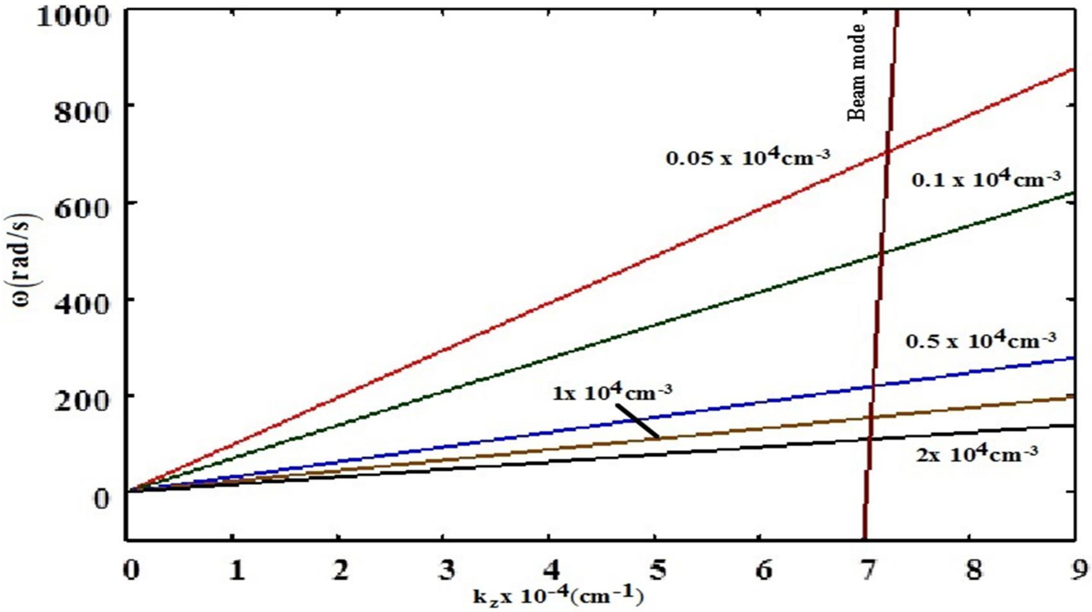

Fig. 1 Dispersion curves of dust Alfven waves over a magnetized dusty plasma for different values of n_{do} and beam mode with velocity = 3.5×10^7 cm/s.

Table 1 Unstable wave frequencies ω(rad./s) and axial wave numbers k_z(cm^{-1}) for different values of n_{do} from Fig. 1.

$n_{do}\times10^4$ (cm^{-3})	$k_z\times10^{-4}$ (cm^{-1})	ω (rad./sec)
0.05	7.214	706.21
0.1	7.163	496.98
0.5	7.076	221.91
1	7.061	157.10
2	7.044	112.88

Using Eq. (15), we have plotted in Fig. 2 the growth rate γ (rad./sec) of the dust Alfven waves as a function of dust grain concentration n_{do} for the same parameters used for plotting dispersion curves plus unstable wave frequencies and wave numbers of the dust Alfven waves (from Table 1) in addition to beam density $n_{bo} = 3\times10^9 cm^{-3}$. From Fig. 2, it can be seen that the growth rate of the unstable mode decreases with n_{do}. The growth rate γ (in rad./sec) of the unstable wave decreases by a factor ~ 1.75 when n_{do} changes from 0.1 to 1.0. When a large number of dust grains are settled down in plasma, the dust grains tend to increase the dielectric constant ε_R, thereby reducing the growth rate of dust Alfven waves.

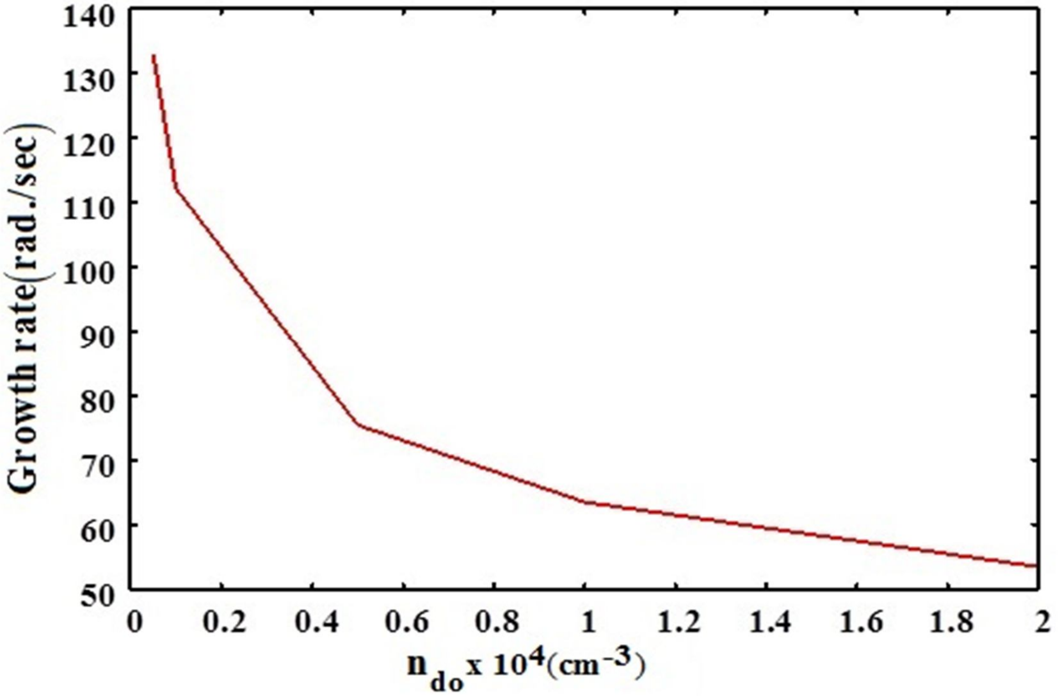

Fig. 2 Growth/Damping rate of the unstable alfven mode as a function of dust particle density n_{do}.

Fig. 3 shows the variation of growth rate as a function of parallel wave number k_z, plotted using Eq. (15), for the same parameters used for plotting dispersion curves plus unstable wave frequencies and wave numbers of the dust Alfven waves (from Table 1) in addition to beam density $n_{bo} = 3\times10^9$ cm^{-3}. The growth rate of the unstable mode increases with the beam density and is directly proportional to the beam density (cf. Eq. 15). In Fig. 4, we have plotted the refractive index of dusty plasma as a function of dust particle number density n_{do} during propagation of dust Alfven waves, using Eq. (12).

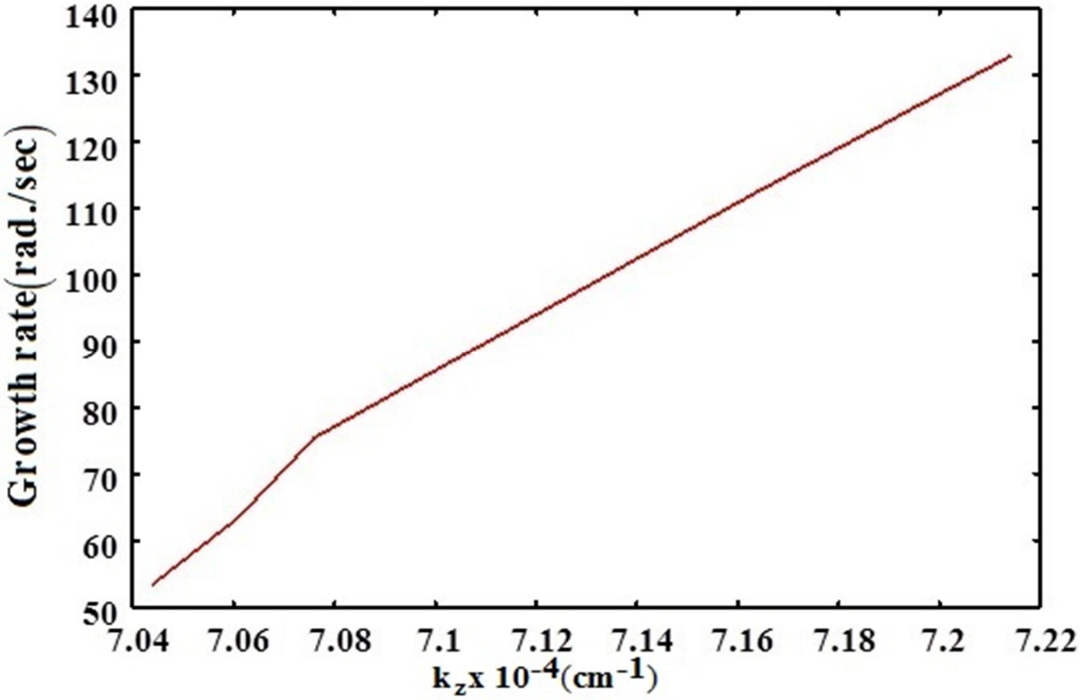

Fig. 3 Growth rate of the unstable dust alfven mode as a function of unstable wave number k_z.

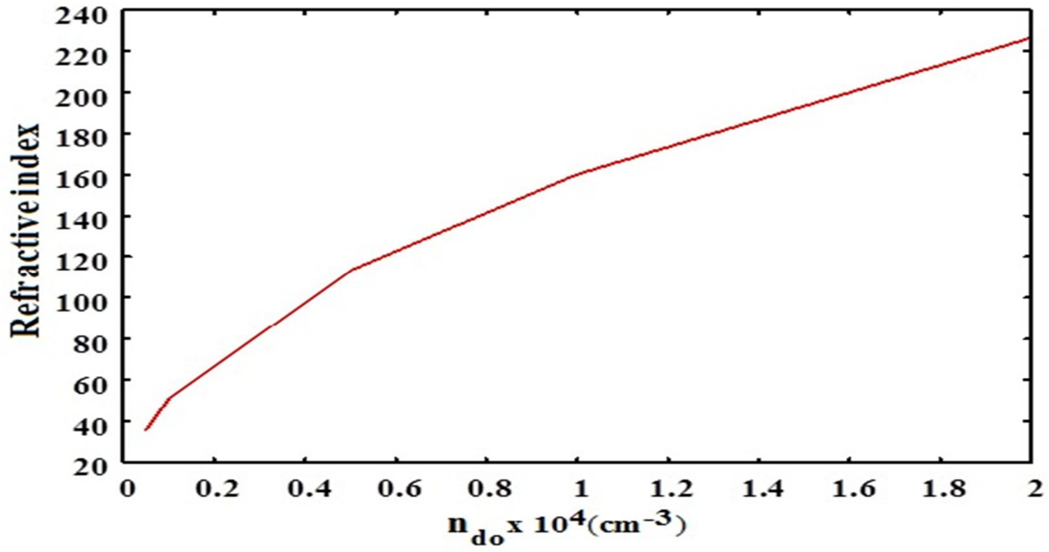

Fig. 4 Refractive index of dusty plasma as a function of dust partcle number density n_{do} during propagation of dust Alfven waves.

IV. CONCLUSION

In conclusion, we may say that an ion beam propagating through a magnetised dusty plasma drives elliptically polarized dust Alfven waves to instability via slow cyclotron interaction. The growth rate and mode frequencies were evaluated based on typical dusty plasma parameters and it is found that the frequency and the growth rate of the dust Alfven mode decreases with an increase in the number density of dust grains. An increase in the dust population reduces the growth rate of the dust Alfven waves through the effect of capturing electrons, modifying the dispersion properties of dust Alfven waves in dusty plasmas. As the number density of dust grains n_{d0} increases, the electron plasma density n_{e0} decreases with respect to ion plasma density n_{i0}. The presence of negatively charged dust grains reduces the critical electron drift and hence decreases the growth rate.

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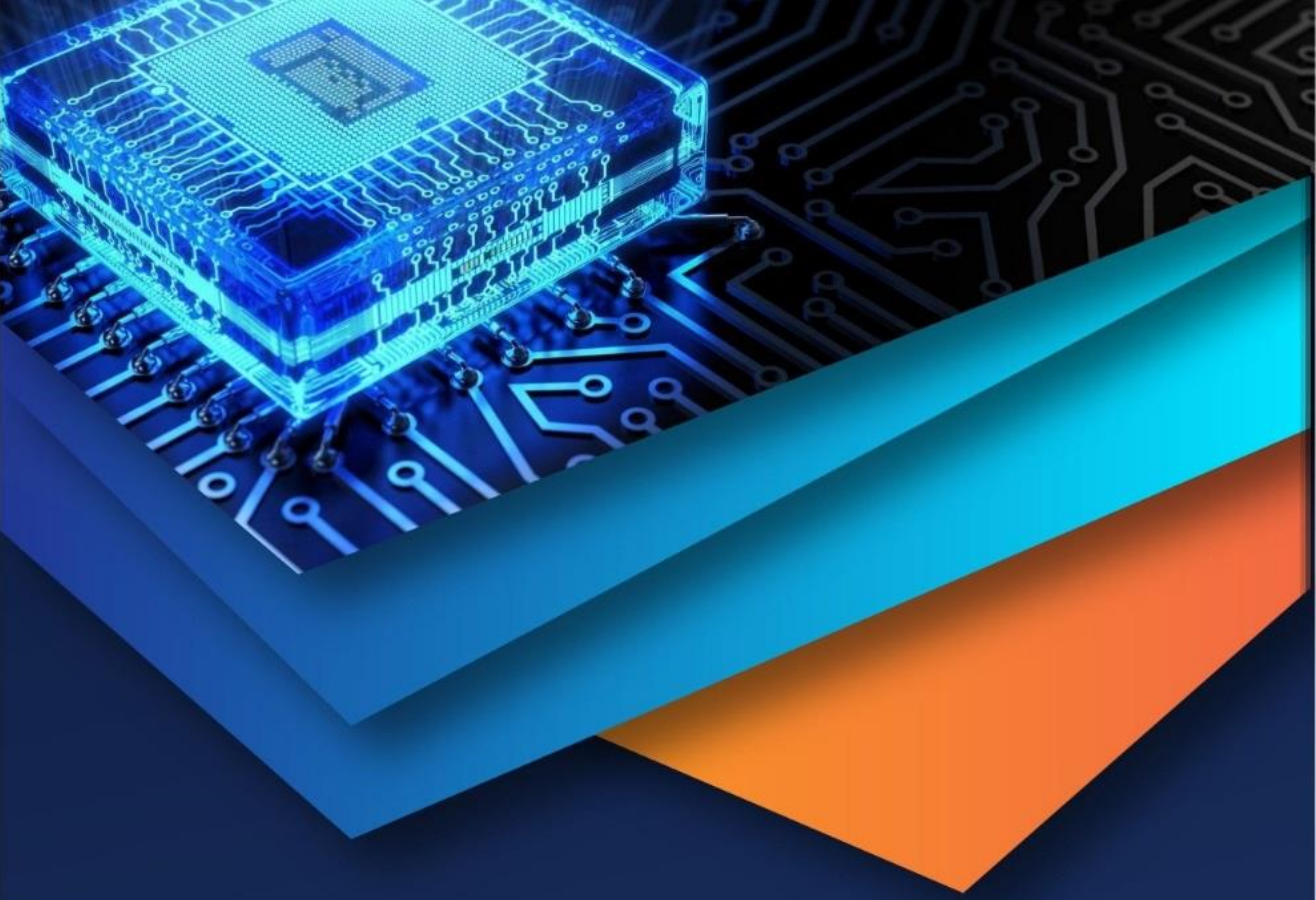

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