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Chaos in the Lorenz System

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Abstract: This paper helps us to know how the Chaos theory is applied in the Lorenz system. We extend our study by knowing the simple definitions related to the chaos theory along with the Liapunov exponent of a deterministic nonlinear system and we can see the oscillation of a points which does not repeat. Also we could identify how aperiodic patterns are recognized.
Keywords: Dynamical systems ,Liapunov exponent, Chaos, Lorenz system.

I. INTRODUCTION

The name "chaos theory" leads to believe that mathematicians have discovered some new and definitive knowledge about utterly random and incomprehensible phenomena. Chaos theory is the qualitative study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems.A dynamical system may be defined to be a simplified model for the time-varying behaviour of an actual system,and aperiodic behaviour is simply the behaviour that occurs when no variable describing the state of the system undergoes a regular repitition of the values.Aperiodic behaviour never repeats and it continues to manifest the effects of any small perturbation(a disturbance of motion,arrangement),thus any prediction of a future state in a given system that is aperiodic is impossible. Chaos theory concerns deterministic systems whose behaviors in principle can be predicted.Chaotic systems are predictable for a while and then 'appear' to become random.[1]The amount of time that the behavior of a chaotic system can be effectively predicted depends on three things:how much uncertainity can be tolerated in the forecast,how accurately its current state can be measured and a time scale depending on the dynamics of the system,called the Liapunov time. Such Liapunov times are chaotic electrical circuits,about millisecond;weather systems,a few days;the solar system,50 million years.In chaotic systems,the uncertainity in a forecast increases exponentially with elapsed time.

A. Preliminaries

Basic definitions[2] are given below**.**

1) *Linear Dynamical System:* An autonomous dynamical system $\dot{x}$=f of dimension N is called linear if the function f is linear in x,that is if f(x)=Ax,where A is an NxN constant matrix:

$$\dot{x}=Ax \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Clearly the origin $x^*=(0,0,\ldots\ldots,0)=0$ is a fixed point.

2) *Nonlinear Dynamical System:* Consider the 2-dim autonomous system $\dot{x}$=f where now f is a nonlinear function of x.Suppose that $x^*$is a fixed point:f(x^*)=0.

By expanding f(x) in Taylor series near x=x* and neglecting terms which are quadratic or of higher powers in (x-x*),we have

$$f(x)=f(x^*)+Df(x^*)(x-x^*)+\ldots\ldots\ldots=Df(x^*)(x-x^*)+\ldots.. \qquad (2)$$

where Df(x*) is the Jacobian matrix evaluated at x=x*:

$$Df(x^*)=\begin{vmatrix}\partial f1/\partial x2 & \partial f1/\partial x2\\ \partial f2/\partial x1 & \partial f2/\partial x2\end{vmatrix} \qquad (3)$$

The linear system

$$\dot{\mathbf{x}}=Df(x^*)(x-x^*), \qquad (4)$$

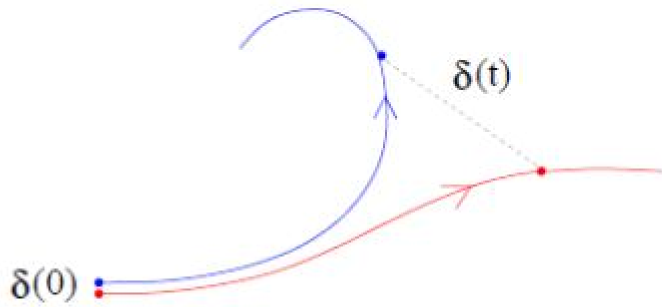
is called the linearised system of $\dot{\mathbf{x}}$=**f** near the fixed point **x*.**

B. Limit Cycles

Very important feature of nonlinear systems(which is absent in linear systems)is the possibility of limit cycles.

1) *Definition:* A cycle,or periodic orbit,of $\dot{x}$=f(x), is any closed trajectory which is not a fixed point
2) *Definition:* An isolated periodic orbit is called a limit cycle.Isolated means that the neighbouring trajectories are not closed:they either spiral toward the limit cycle or away from it. If all neighbouring trajectories approach the limit cycle,then the limit cycle is said stable,or attracting.Otherwise it called unstable,or repelling.

C. Liapunov Exponent



Let x(t) be a point on the attractor at time t,and x(t)+d(t) be another point such that initially δ(0)=|d(0)|<<1. Numerical experiments show that,as time proceeds,the seperation δ(t) between the two trajectories increases as

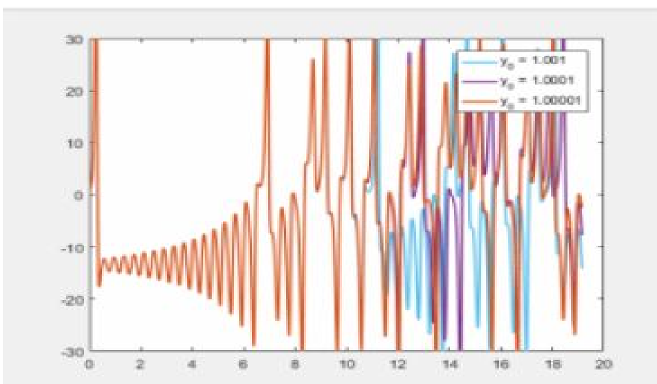
$$\delta(t) \sim \delta(0)e^{\lambda t},$$

where λ >0 is called the Liapunov exponent.

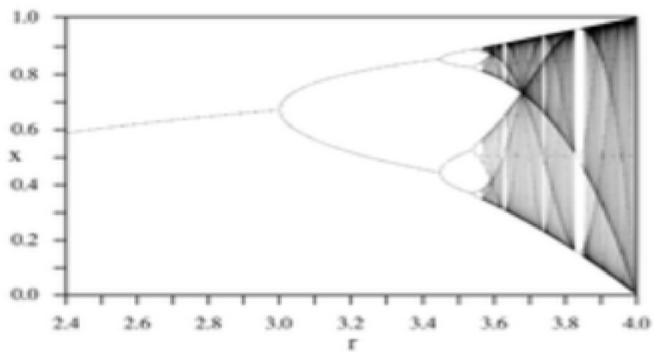
D. Chaos

Chaos is the aperiodic long-term behaviour in a deterministic system that exhibits sensitive dependence on the initial condition.

1) *Aperiodic long-term* **:** Behaviour means that trajectories do not settle down to fixed points,periodic orbitsor quasi-periodic orbits as t→∞.Deterministic means that the irregular behaviour of the system arises from its being nonlinear,not from added external noise. Chaos theory states that the apparent randomness of complex systems within it,and on programming at the initial point is known as initial conditions.The butterfly effect of chaos theory explains a smakk change of a deterministic nonlinear system. Many natural systems with chaotic behaviors like weather and climate.The applications of chaos theory includes[3][4]algorithmic trading,cryptography,[5]psychology[6] and robotics
2) *Sensitivity To Initial Conditions:* Each point in a chaotic system is arbitrarily closely approximated by other points with significantly different future paths or trajectories.Sensitivity to initial condition is popularly known as butterfly effect.

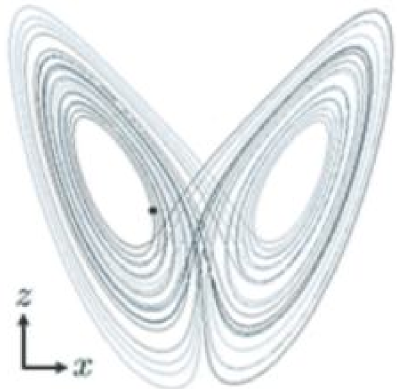


2.1b Minimum complexity of a chaotic system



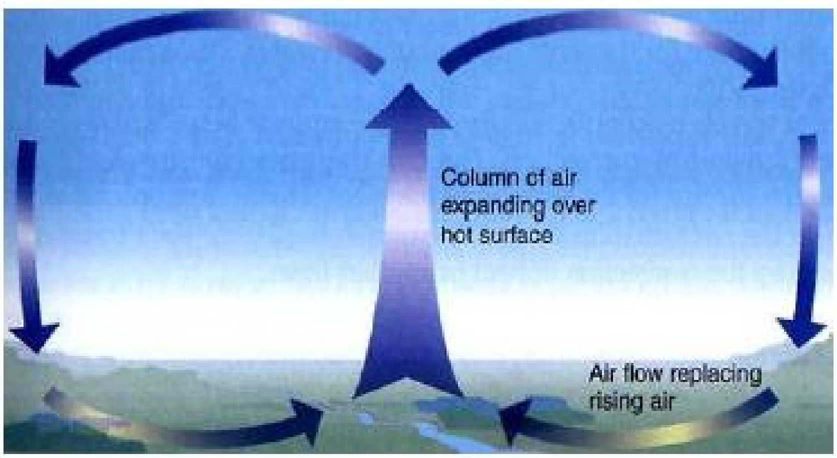
The logistic map in discrete chaotic system exhibit strange attractors without considering its dimension.

E. Lorenz System



The lorenz system is a system of ordinary differential equations first studied by Edward Lorenz.It is notable for having chaotic solutions for certain parameter values and initial conditions.In particular,the Lorenz attractor is a set of chaotic solutions of the Lorenz system which,when plotted,resemble a butterfly or figure eight.

Below the ground,air is heated which is transparent to the solar radiation.But a fluid layer which is below cooled and above heated will be unstable.The breaking effect of viscosity can be overcome,thus cool air sinks and hot air rises in the form of convection rolls.



Convection cell.

The Lorenz equation is as formulated as $\dot{\mathrm{x}}=(\dot{x},\dot{y},\dot{z})=\mathrm{f}=(f_1,f_2,f_3)$ given by:

$$\dot{x}=f_1=\sigma(y-x),$$
$$\dot{y}=f_2=-xz+rz-y,$$
$$\dot{z}=f_3=xy-bz,$$

Where $r>0$ corresponds to ΔT,$\sigma>0$ which is the ratio between energy losses due to viscosity and thermal conduction.

E. Properties of Lorenz system

Lorenz system $\dot{\mathbf{x}}=\mathbf{f}$ is dissipative,in which the volume under the flow contradicts in phase space:

$$\frac{dV}{dt}=\int_{\mathrm{v}}\nabla.\mathrm{f}dV<0.$$

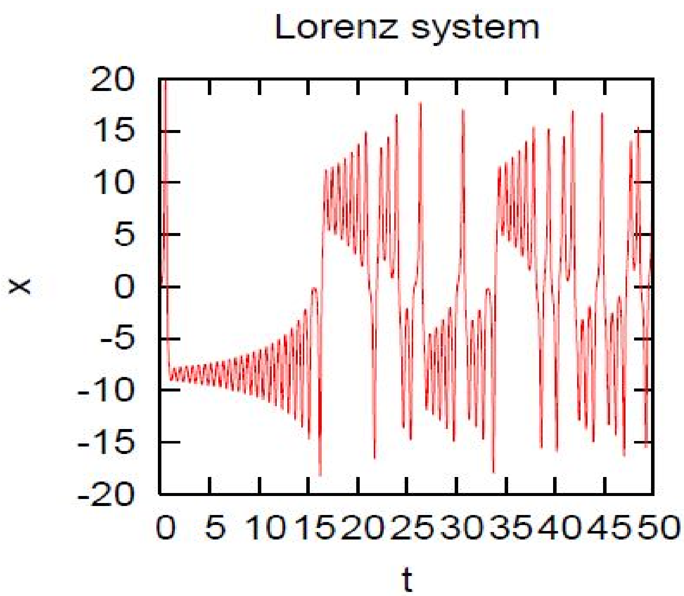
Quasi-periodic solutions are not satisfied in the Lorenz system.

Lorenz system has repelling closed orbits or no repelling fixed points.

F. Chaos In The Lorenz System

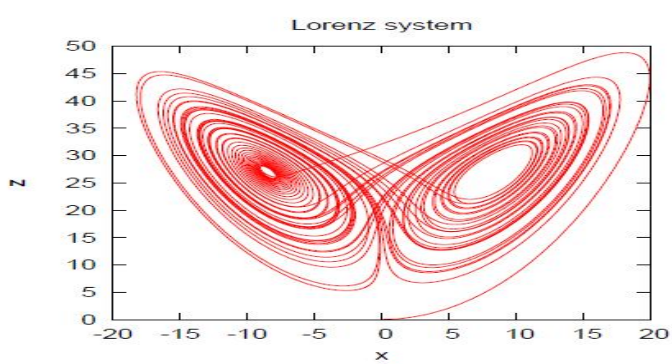
We solve the Lorenz equation numerically and the trajectories keep being repelled by C^+ and C^- like in a pinball machine.At the same time ,trajectories are confined in a set whose volume shrinks to zero for $t\to\infty$.

The below figure shows that the trajectory is an irregular oscillation which never repeats exactly.The motion is aperiodic.



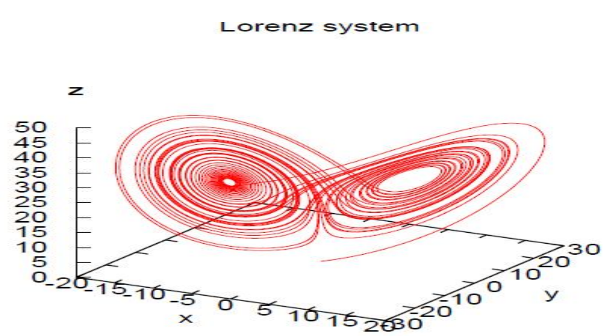
The above figure is the solution $x=x(t)$ of the Lorenz equation for $\sigma=10,r=28,b=8/3$ and initial condition $x(0)=y(0)=z(0)=0.1$.

The trajectory starts near the origin,then swings to C^+,then swings to the left spiraling around C^-many times,then shoots suddenly to C^+ again,and so on.The number of circuits around C^+ and C^-varies unpredictably from one cycle to the next.Physically,these swings correspond to reversing the rotation of the convection rolls.



The above figure shows that the solution of the Lorenz equation plotted in the x,z plane.Thus we see a strange attractor.

Hence,at the trajectory in 3-dim phase space,the attractor will be like a pair of thin wings.



It shows that the solution (x,y,z) of the Lorenz equation plotted in 3-dim.

II. CONCLUSION

Thus by applying Chaos theory in the Lorenz system helps us to know that the infinite times of oscillation of a points contracts and converges to the inner element which helps us to study of patterns.

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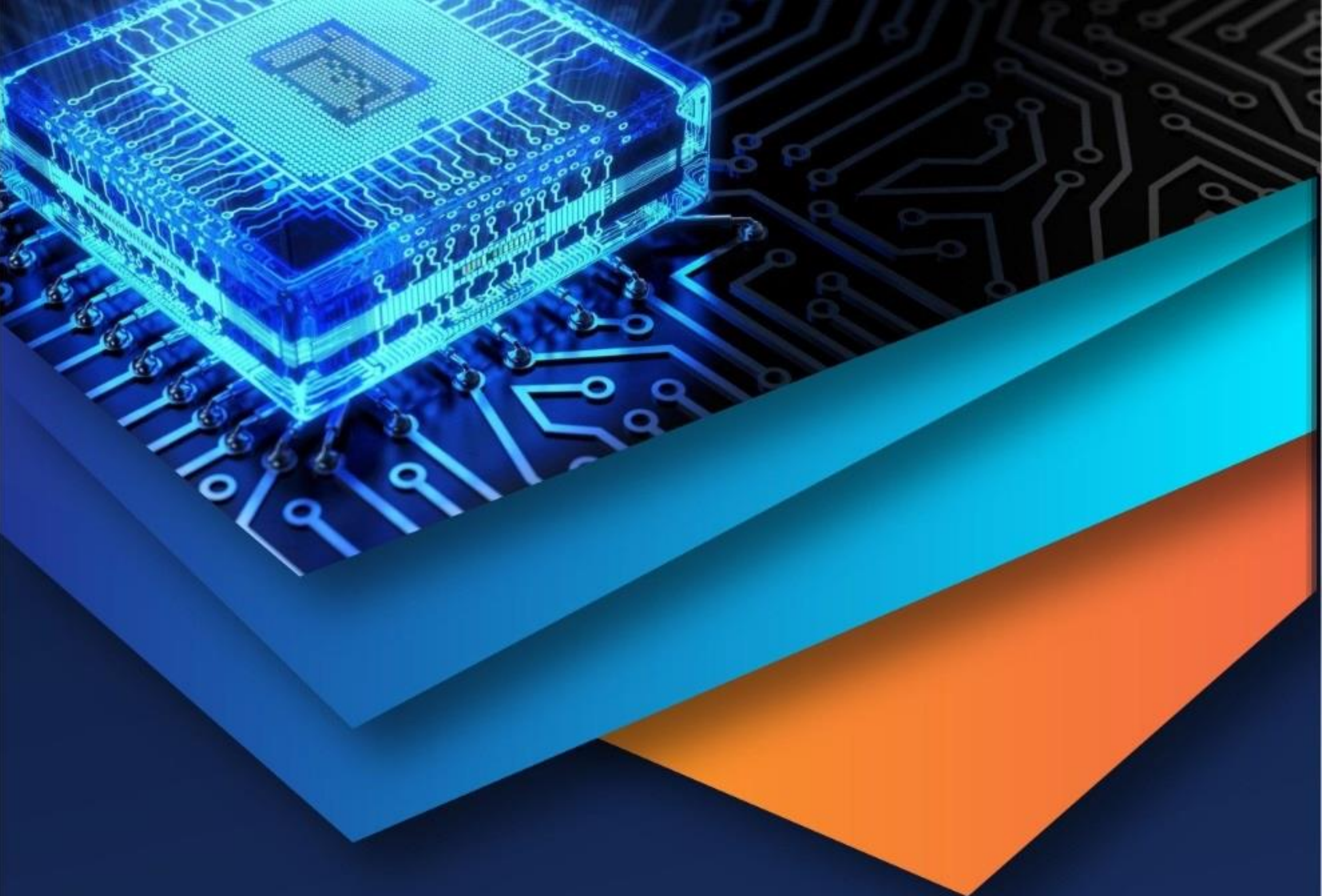

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