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# Contra (i,j) (gsp)\* - continuous Function in Bitopological Space

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**Abstract:** In this paper we have introduced a new function of contra (i,j)(gsp)\*-continuous in bitopological spaces which is properly placed in between the class of closed sets and gsp-closed sets.

**Key Words:** contra (i,j) g-continuous, contra (i,j) gs-continuous,  $\alpha$ continuous, contra (i,j) gsp-continuous.

## I. INTRODUCTION

A triple  $(X, \tau_i, \tau_j)$  where  $X$  is a non-empty set and  $\tau_i$  and  $\tau_j$  are topologies on  $X$  is called a bitopological space." Kelly [20] introduced study of such spaces. In 1985, Fukutake [19] introduced the concepts of g-closed sets [10] in bitopological spaces. In the year 1994, Maki et al [12] defined  $\alpha g$ -closed sets in topological space. S.P. Arya and N. Tour [3] defined  $g_s$ -closed sets in 1990. Dontchev [8], Gnanambal [9] and Palaniappan and Rao [17] introduced gsp-closed sets. J. Dontchev [8] introduced gsp-closed sets in 1995. Levine [10] Devi et al is. In this paper the new function contra (i,j)(gsp)\*-continuous function is introduced. The concepts contra (i,j) g-continuous, contra (i,j) gs-continuous, contra (i,j)  $\alpha g$ -continuous, contra (i,j) gsp-continuous are defined few of their properties are studied.

## II. PRELIMINARIES

### A. Definition 2.1

"A subset  $A$  of topological space  $(X, \tau_i, \tau_j)$  is called

- 1) a pre-open set [14] if  $A \subseteq \text{int}(cl(A))$  and a pre-closed set if  $cl(\text{int}(A)) \subseteq A$
- 2) a semi-open set [11] if  $A \subseteq cl(\text{int}(A))$  and a semi-closed set if
- 3) a semi-pre open set [1] if  $A \subseteq cl(\text{int}(cl(A)))$  and a semi-pre closed set [1] if
- 4) an  $\alpha$ -open set [15] if  $A \subseteq \text{int}(cl(\text{int}(A)))$  and an  $\alpha$ -closed set [15] if
- 5) *regular-open*<sub>set</sub> [14] if  $\text{int}(cl(A)) = A$  and an *regular-closed*<sub>set</sub> [14] if  $A = \text{int}(cl(A))$ "

### B. Definition 2.2

"A subset  $A$  of topological space  $(X, \tau_i, \tau_j)$  is called

- 1) a generalized closed set (briefly (i,j) g-closed) [10] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in
- 2) generalized semi-closed set (briefly (i,j) gs-closed) [3] if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $\tau_i$ .
- 3) an  $\alpha$ -generalized closed set (briefly (i,j)  $\alpha g$ -closed) [12] if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $\tau_i$ .
- 4) a generalized semi pre-closed set (briefly (i,j) gsp-closed) [8] if  $spcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $\tau_i$ .

### C. Definition 2.3

"A function  $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$  is called

- 1) Contra (i,j) g-continuous [4] if  $f^{-1}(V)$  is a g-closed set of  $(X, \tau_i, \tau_j)$  for every closed set  $V$  of  $(Y, \sigma_i, \sigma_j)$ .
- 2) Contra (i,j)  $\alpha g$ -continuous [9] if  $f^{-1}(V)$  is a  $\alpha g$ -closed set of  $(X, \tau_i, \tau_j)$  for every closed set  $V$  of  $(Y, \sigma_i, \sigma_j)$ .
- 3) Contra (i,j) gs-continuous [7] if  $f^{-1}(V)$  is a gs-closed set of  $(X, \tau_i, \tau_j)$  for every closed set  $V$  of  $(Y, \sigma_i, \sigma_j)$ .
- 4) contra (i,j) gsp-continuous [8] if  $f^{-1}(V)$  is a gsp-closed set of  $(X, \tau_i, \tau_j)$  for every closed set  $V$  of  $(Y, \sigma_i, \sigma_j)$ .

### D. Definition 3

A function  $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$  is called contra (i,j) (gsp)\*-continuous if  $f^{-1}(v)$  is (i,j) (gsp)\*-closed in  $(X, \tau_i, \tau_j)$  for each open set  $v$  of  $(Y, \sigma_i, \sigma_j)$ .

*E. Theorem 3.1*

Every contra continuous function is contra (i,j) (gsp)\*-continuous

Let  $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$  be a contra continuous map

Let  $v$  be any open set in  $(Y, \sigma_i, \sigma_j)$ .

Then the inverse image of  $f^{-1}(v)$  is (i,j) (gsp)\*- closed in  $(X, \tau_i, \tau_j)$ .

Since every closed set is (i,j) (gsp)\*- closed.

$$f^{-1}(v) \text{ is (i,j) (gsp)*- closed in } (X, \tau_i, \tau_j).$$

Therefore  $f$  is contra (i,j) (gsp)\*-continuous.

Converse is not true.

*F. Theorem 3.2*

Every contra (i,j) (gsp)\*-continuous map is contra (i,j) g-continuous. But the converse is not true.

*1) Proof:* Let  $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$  be a contra (i,j) (gsp)\* continuous map.

Let  $v$  be any open set in  $(Y, \sigma_i, \sigma_j)$ .

Then the inverse image of  $f^{-1}(v)$  is (i,j) (gsp)\*- closed in  $(X, \tau_i, \tau_j)$ .

Since every (i,j) (gsp)\*- closed set is (i,j) g-closed.

$$f^{-1}(v) \text{ is (i,j) g-closed in } (X, \tau_i, \tau_j).$$

Therefore  $f$  is contra (i,j) g-continuous.

*G. Example 3.3*

Let  $X = \{a, b, c\} = Y$ ,  $\tau_i = \{\varphi, X, \{c\}, \{a, c\}\}$ ,  $\tau_j = \{\varphi, X, \{b\}, \{a, b\}\}$ ,  $\sigma_i = \varphi, Y, \{a, c\}$ ,

$$\sigma_j = \varphi, Y, \{b, c\}.$$

Let  $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$  be the identity map.

Let us prove that  $f$  is contra (i,j) g- continuous. But not contra (i,j) (gsp)\*-continuous.

We have proved that the (i,j) g-closed sets are all the subsets of  $X$ .

And the (i,j) (gsp)\*- closed sets are  $\varphi, X, \{c\}, \{a, b\}, \{a, c\}$ .

$$f^{-1}\{a\} = \{a\} \text{ is (i,j) g-closed in } (X, \tau_i, \tau_j).$$

But it is not (i,j) (gsp)\*- closed in  $(X, \tau_i, \tau_j)$ .

Hence  $f$  is contra (i,j) g- continuous but not (i,j) (gsp)\*-continuous.

*G. Theorem 3.4*

Every contra (i,j) (gsp)\*-continuous map is contra (i,j) gs-continuous

*1) Proof:* Let  $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$  be a contra (i,j) (gsp)\* continuous map.

Let  $v$  be any open set in  $(Y, \sigma_i, \sigma_j)$ .

Then the inverse image of  $f^{-1}(v)$  is (i,j) (gsp)\*- closed in  $(X, \tau_i, \tau_j)$ .

Since every (i,j) (gsp)\*- closed set is (i,j) gs-closed.

$$f^{-1}(v) \text{ is (i,j) gs-closed in } (X, \tau_i, \tau_j).$$

Therefore  $f$  is contra (i,j) gs-continuous.

*H. Example 3.5*

Let  $X = \{a, b, c\} = Y$ ,  $\tau_i = \{\varphi, X, \{c\}, \{a, c\}\}$ ,  $\tau_j = \{\varphi, X, \{b\}, \{a, b\}\}$ ,  $\sigma_i = \varphi, Y, \{a, c\}$ ,

$$\sigma_j = \varphi, Y, \{b, c\}.$$

Let  $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$  be the identity map.

Let us prove that  $f$  is contra (i,j) gs- continuous. But not contra (i,j) (gsp)\*-continuous.

We have proved that the (i,j) gs-closed sets are all the subsets of  $X$ .

And the (i,j) (gsp)\*- closed sets are  $\varphi, X, \{c\}, \{a, b\}, \{a, c\}$ .

$f^{-1}\{a\} = \{a\}$  is (i,j) gs-closed in  $(X, \tau_i, \tau_j)$ .

But it is not (i,j) (gsp)\*- closed in  $(X, \tau_i, \tau_j)$ .

Hence  $f$  is contra (i,j) gs- continuous but not (i,j) (gsp)\*-continuous.

### I. Theorem 3.6

Every contra (i,j) (gsp)\*-continuous map is contra (i,j)  $\alpha$ g-continuous.

1) Proof: Let  $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$  be a contra (i,j) (gsp)\* continuous map.

Let  $v$  be any open set in  $(Y, \sigma_i, \sigma_j)$ .

Then the inverse image of  $f^{-1}(v)$  is (i,j) (gsp)\*- closed in  $(X, \tau_i, \tau_j)$ .

Since every (i,j) (gsp)\*- closed set is (i,j)  $\alpha$ g-closed.

$f^{-1}(v)$  is (i,j)  $\alpha$ g-closed in  $(X, \tau_i, \tau_j)$ .

Therefore  $f$  is contra (i,j)  $\alpha$ g-continuous.

### J. Example 3.7

Let  $X = \{a, b, c\} = Y$ ,  $\tau_i = \{\varphi, X, \{c\}, \{a, c\}\}$ ,  $\tau_j = \{\varphi, X, \{b\}, \{a, b\}\}$ ,  $\sigma_i = \varphi, Y, \{a, c\}$ ,

$\sigma_j = \varphi, Y, \{a, c\}$ .

Let  $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$  be the identity map.

Let us prove that  $f$  is contra (i,j)  $\alpha$ g- continuous. But not contra (i,j) (gsp)\*-continuous.

We have proved that the (i,j)  $\alpha$ g-closed sets are all the subsets of  $X$ .

And the (i,j) (gsp)\*- closed sets are  $\varphi, X, \{c\}, \{a, b\}, \{a, c\}$ .

$f^{-1}\{b\} = \{b\}$  is (i,j)  $\alpha$ g-closed in  $(X, \tau_i, \tau_j)$ .

But it is not (i,j) (gsp)\*- closed in  $(X, \tau_i, \tau_j)$ .

Hence  $f$  is contra (i,j)  $\alpha$ g- continuous but not (i,j) (gsp)\*-continuous.

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