iJRASET
International Journal For Research in Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: ETPAM-2018 **Issue:** Conference **Month of publication:** March 2018

DOI:

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Contra (i,j) $(gsp)^*$ - continuous Function in Bitopological Space

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Abstract: In this paper we have introduced a new function of contra (i,j) $(gsp)^*$ ***-continuous in bitopological spaces which is properly placed in between the class of closed sets and gsp-closed sets.***
Key Words: contra (i,j) g-continuous, contra (i,j) gs-continuous,αcontinuous, contra (i,j) gsp-continuous.

I. INTRODUCTION

A triple (X, τ_i, τ_j) where X is a non-empty set and τ_i and τ_j are topologies on X is called a bitopological space." Kelly [20] introduced study of such spaces.In 1985, Fukutake [19] introduced the concepts of g-closed sets[10] in bitopological spaces. In the year 1994 , Maki.et.al [12] defined αg -closed sets in topological space.S.P. Arya and N. Tour [3] defined gs -closed sets in 1990. Dontchev [8], Gnanambal [9] and Palaniappan and Rao [17] introduced gsp-closed sets. J.Dontchev [8] introduced gsp-closed sets in 1995. Levine [10] Devi.et.al is. In this paper the new function contra (i,j)$(gsp)^*$-continuousfunction is introduced. The concepts contra (i,j) g-continuous,contra (i,j) gs-continuous,contra (i,j) αg- continuous,contra (i,j) gsp-continuous are defined few of their properties are studied.

II. PRELIMINARIES

A. Definition 2.1

"A subset A of topological space (X, τ_i, τ_j) is called

1) a pre-open set[14] if $A \subseteq \text{int}(cl(A))$ and a pre-closed set if cl(int(A)) $\subseteq A$

2) a semi-open set [11] if $A \subseteq cl(\text{int}(A))$ and a semi-closed set if

3) a semi-pre open set[1] if $A \subseteq cl(\text{int}(cl(A))$ and a semi-pre closed set[1] if

4) an α-open set [15]if $A \subseteq \text{int}(cl(\text{int}(A)))$ and an α-closed set[15] if

5) $regular-open$ set[14] if int(cl(A))=A and an $regular-closed$ set[14] if A=int(cl(A))"

B. Definition 2.2

"A subset A of topological space (X, τ_i, τ_j) is called

1) a generalized closed set (briefly (i,j) g-closed) [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in

2) generalized semi-closed set(briefly) (i,j) gs-closed [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i.

3) an α- generalized closed set (briefly (i,j)αg-closed) [12] if $\alpha\, cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i.

4) a generalized semi pre-closed set (briefly (i,j) gsp-closed) [8] if $\text{sp}\, cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i.

C. Definition 2.3

" A function f : (X, τ_i, τ_j) $\rightarrow$ (Y,σ_i,σ_j) is called

1) Contra (i,j) g-continuous [4] if f^{-1}(V) is a g-closed set of (X, τ_i, τ_j) for every closed set V of (Y,σ_i,σ_j).

2) Contra (i,j) αg-continuous[9] if f^{-1}(V) is a αg-closed set of (X, τ_i, τ_j) for every closed set V of (Y,σ_i,σ_j).

3) Contra (i,j) gs-continuous [7] if f^{-1}(V) is a gs-closed set of (X, τ_i, τ_j) for every closed set V of (Y,σ_i,σ_j).

4) contra (i,j) gsp-continuous [8] if f^{-1}(V) is a gsp-closed set of (X, τ_i, τ_j) for every closed set V of (Y,σ_i,σ_j).

D. Definition 3

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A function $f:(X,\tau_i,\tau_j)\to(Y,\sigma_i,\sigma_j)$ is called contra (i,j) (gsp)*-continuous if $f^{-1}(v)$ is (i,j) (gsp)*-closed in (X,τ_i,τ_j) for each open set v of (Y,σ_i,σ_j) .

E. Theorem 3.1

Every contra continuous function is contra (i,j) (gsp)*-continuous

Let $f:(X,\tau_i,\tau_j)\to(Y,\sigma_i,\sigma_j)$ be a contra continuous map

Let v be any open set in (Y,σ_i,σ_j).

Then the inverse image of $f^{-1}(v)$ is (i,j) (gsp)*- closed in (X,τ_i,τ_j).

Since every closed set is (i,j) (gsp)*- closed.

$f^{-1}(v)$ is (i,j) (gsp)*- closed in (X,τ_i,τ_j).

Therefore f is contra (i,j) (gsp)*-continuous.

Converse is not true.

F. Theorem 3.2

Every contra (i,j) (gsp)*-continuous map is contra (i,j) g-continuous.But the converse is not true.

1) Proof: Let $f:(X,\tau_i,\tau_j)\to(Y,\sigma_i,\sigma_j)$ be a contra (i,j) (gsp)* continuous map.

Let v be any open set in (Y,σ_i,σ_j).

Then the inverse image of $f^{-1}(v)$ is (i,j) (gsp)*- closed in (X,τ_i,τ_j).

Since every (i,j) (gsp)*- closed set is (i,j) g-closed.

$f^{-1}(v)$ is (i,j) g-closed in (X,τ_i,τ_j).

Therefore f is contra (i,j) g-continuous.

G .Example 3.3

Let X= {a, b, c}= Y, $\tau_i=\{\varphi,X,\{c\},\{a,c\}\}$, $\tau_j=\{\varphi,X,\{b\},\{a,b\}\}$, $\sigma_i=\varphi,Y,\{a,c\}$,

$\sigma_j=\varphi,Y,\{b,c\}$.

Let $f:(X,\tau_i,\tau_j)\to(Y,\sigma_i,\sigma_j)$ be the identity map.

Let us prove that f is contra (i,j) g- continuous.But not contra (i,j) (gsp)*-continuous.

We have proved that the (i,j) g-closed sets are all the subsets of X.

And the (i,j) (gsp)*- closed sets are φ, X, $\{c\},\{a,b\},\{a,c\}$.

$f^{-1}\{a\}=\{a\}$ is (i,j) g-closed in (X,τ_i,τ_j).

But it is not (i,j) (gsp)*- closed in (X,τ_i,τ_j).

Hence f is contra (i,j) g- continuous but not (i,j) (gsp)*-continuous.

G. Theorem 3.4

Every contra (i,j) (gsp)*-continuous map is contra (i,j) gs-continuous

1) Proof: Let $f:(X,\tau_i,\tau_j)\to(Y,\sigma_i,\sigma_j)$ be a contra (i,j) (gsp)* continuous map.

Let v be any open set in (Y,σ_i,σ_j).

Then the inverse image of $f^{-1}(v)$ is (i,j) (gsp)*- closed in (X,τ_i,τ_j).

Since every (i,j) (gsp)*- closed set is (i,j) gs-closed.

$f^{-1}(v)$ is (i,j) gs-closed in (X,τ_i,τ_j).

Therefore f is contra (i,j) gs-continuous.

H. Example 3.5

Let X= {a, b, c}= Y, $\tau_i=\{\varphi,X,\{c\},\{a,c\}\}$, $\tau_j=\{\varphi,X,\{b\},\{a,b\}\}$, $\sigma_i=\varphi,Y,\{a,c\}$,

$\sigma_j=\varphi,Y,\{b,c\}$.

Let $f:(X,\tau_i,\tau_j)\to(Y,\sigma_i,\sigma_j)$ be the identity map.

Let us prove that f is contra (i,j) gs- continuous.But not contra (i,j) (gsp)*-continuous.

We have proved that the (i,j) gs-closed sets are all the subsets of X.

And the (i,j) (gsp)*- closed sets are φ, X, $\{c\},\{a,b\},\{a,c\}$.

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f^{-1}\{a\} =\{a\} is (i,j) gs-closed in (X, τ_i, τ_j).

But it is not (i,j) (gsp)*- closed in (X, τ_i, τ_j).

Hence f is contra (i,j) gs- continuous but not (i,j) (gsp)*-continuous.

I. Theorem 3.6

Every contra (i,j) $(gsp)^*$-continuous map is contra (i,j) αg-continuous.

1) Proof: Let $f: (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a contra (i,j) $(gsp)^*$ continuous map.

Let v be any open set in (Y, σ_i, σ_j).

Then the inverse image of f^{-1}(v) is (i,j) (gsp)*- closed in (X, τ_i, τ_j).

Since every (i,j) (gsp)*- closed set is (i,j) αg-closed.

f^{-1}(v) is (i,j) αg-closed in (X, τ_i, τ_j).

Therefore f is contra (i,j) αg-continuous.

J. Example 3.7

Let X= {a, b, c}= Y, $\tau_i = \{\varphi, X, \{c\}, \{a, c\}\}$, $\tau_j = \{\varphi, X, \{b\}, \{a, b\}\}$, $\sigma_i = \varphi, Y, \{a,c\}$,

$\sigma_j = \varphi, Y, \{a,c\}$.

Let $f: (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be the identity map.

Let us prove that f is contra (i,j) αg- continuous.But not contra (i,j) (gsp)*-continuous.

We have proved that the (i,j) αg-closed sets are all the subsets of X.

And the (i,j) (gsp)*- closed sets are φ, X, $\{c\}, \{a, b\}, \{a,c\}$.

f^{-1}\{b\} =\{b\} is (i,j) αg-closed in (X, τ_i, τ_j).

But it is not (i,j) (gsp)*- closed in (X, τ_i, τ_j).

Hence f is contra (i,j) αg- continuous but not (i,j) (gsp)*-continuous.

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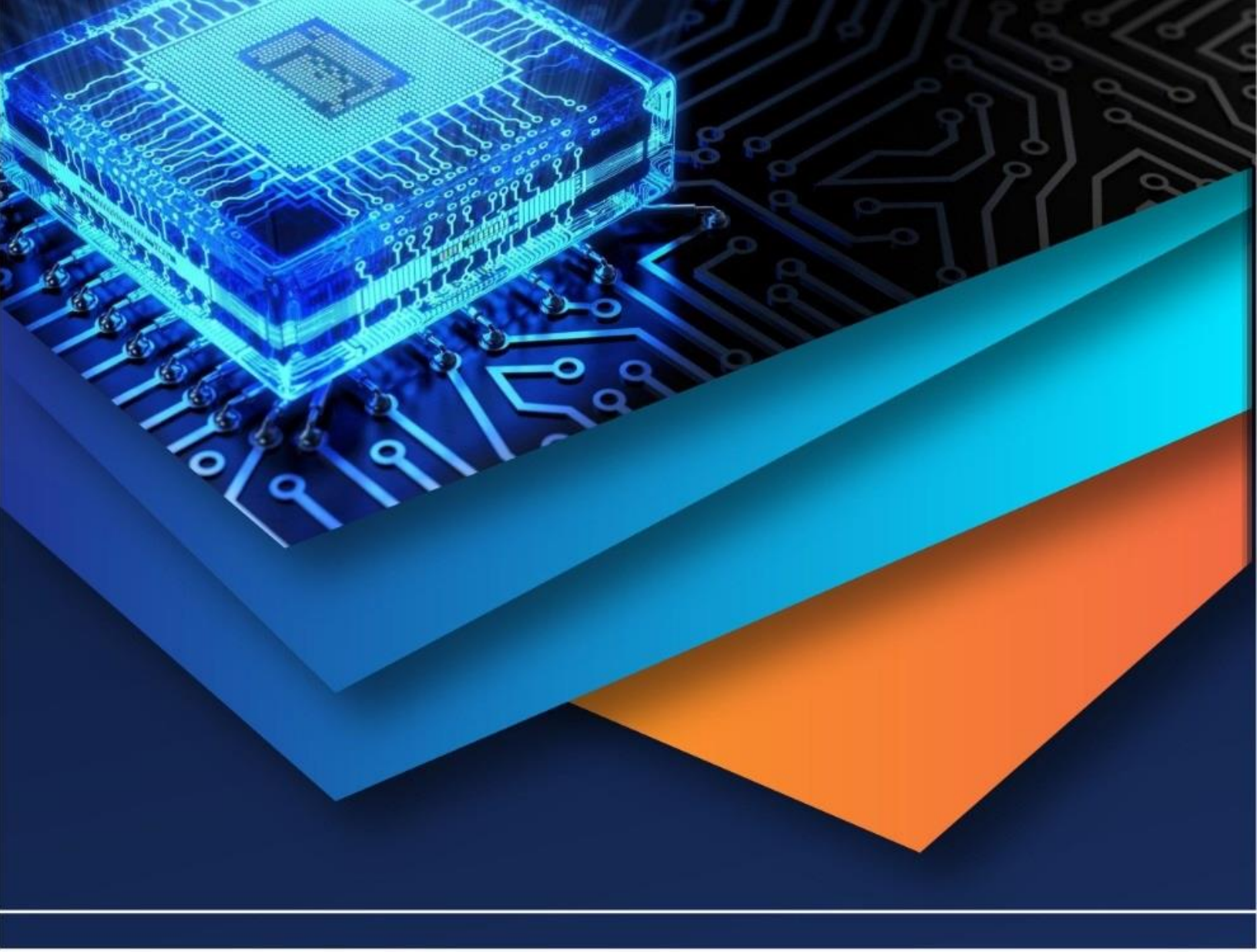
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