

Effect of Coriolis Force and Internal Heat Generation on Bénard-Marangoni Convection in a Micropolar Fluid

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Abstract: *The effects of Internal heat generation and Coriolis force on the onset of Bénard-Marangoni convection in a horizontal layer of micropolar fluid confined between upper free/adiabatic and a lower rigid/isothermal boundary is considered and is investigated by using a linear stability analysis. The single term Galerkin technique is used to obtain the eigen values. The microrotation is assumed to vanish at the boundaries. The influence of various micropolar fluid parameters, internal Rayleigh number, Taylor number and the Marangoni number on the onset of convection is analyzed.*

Keywords: *Bénard-Marangoni convection, Micropolar fluid, Internal Rayleigh number, Taylor number, Coriolis Force.*

I. INTRODUCTION

When a fluid confined between two plates flowing horizontally is heated from below, natural convection sets in. This convection can be due to the combined effect of buoyancy and surface tension. The buoyancy driven convection was extensively studied by Chandrasekhar[1]. The convection driven by surface tension alone was studied by Pearson[2]. The former problem is called as the Rayleigh-Bénard problem, the latter is called as the Marangoni problem. The combined effect was investigated by Nield[3]. This problem is called as the Bénard-Marangoni problem. Bénard-Marangoni effect has several applications in the field of Chemistry[7-9]. It is used for drying silicon wafers during the manufacture of integrated circuits. It is also used in the fields of welding and crystal growth.

Micropolar fluids are those fluids which have micro structured particles in it. Each particle has a finite size and has a micro-structure in it. These structures tend to rotate and move independently in system. When the micropolar theory is formulated, we have additional degree of freedom and gyration (rotation of the micro structure). Hence we have the balance law of angular momentum in the basic governing equations. This equation introduces a mechanism that takes into account the molecular spin. The pioneer work in micropolar fluids is by Eringen [4]. Later many authors investigated instabilities in micropolar fluid under different conditions [5,6,10,17-21].

Convection can happen even in rotating system which is mainly encountered in geophysical processes. When a rotating frame of reference is considered, the system exhibits a force called as Coriolis force. Many studies are conducted on investigating the effect of Coriolis force in Bénard-Marangoni convection[11-13]. The results have shown that this effect stabilizes the system. In several important practical situations, it is observed that the elements have their own source of heat, and due to this there set a different convective flow within a layer of fluid through generation of local heat. This type of condition takes place through decay of radioactive materials or through the comparatively weak exothermic reactions which occurs within the materials [14-16]. Therefore, the internal heating has great importance and various application in the area of development of waste form for nuclear fuel, geophysics, studies of combustion and fire, analysis of reactor safety and radioactive elements storage. In literature very less studies are available where effect of internal heat generation on the onset of Bénard-Marangoni convection in a micropolar fluid is considered. The results have shown that this effect has a destabilizing effect on the system. The aim of this paper is to investigate the effect of internal heat generation and coriolis force on the onset of Bénard-Marangoni convection in a micropolar fluid. A linear stability analysis is performed to study the onset of convection and the graphs are obtained for the problem for a steady case and the conclusions on the stability of the system are made.

II. MATHEMATICAL FORMULATION AND SOLUTION

A. Physical Configuration

Consider an infinite horizontal layer of Boussinesquian, micropolar fluid of depth 'd'. A Cartesian coordinate system is taken with the origin in the lower boundary and z-axis vertically upwards. The x-axis is along the lower plate as shown in the Fig.1. Let ΔT be

the temperature difference between lower and upper boundaries of the fluid. The layer is kept rotating uniformly around the vertical z-axis with a constant angular velocity Ω_0 . A heat source is placed within the flow to allow possible heat generation effects.

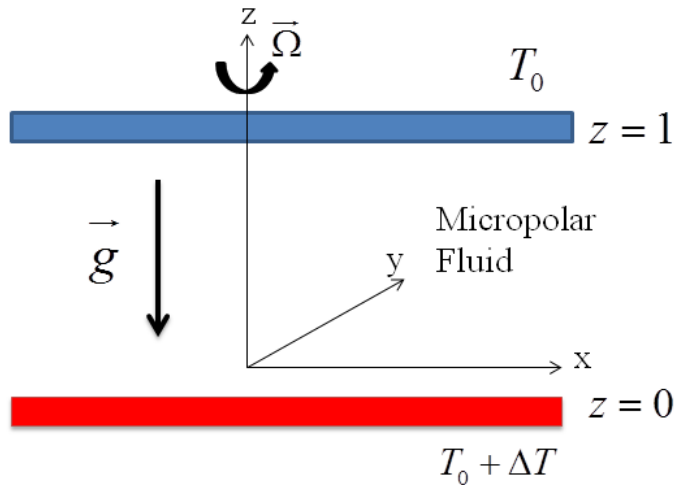


Fig 1 Physical Configuration

B. Basic Governing Equations

The governing equations for the problem are: Continuity Equation

$$\nabla \cdot \vec{q} = 0, \quad (1)$$

Conservation of Linear Momentum

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} + 2\vec{\Omega} \times \vec{q} \right] = -\nabla P - \rho g \hat{k} + (2\zeta + \eta) \nabla^2 \vec{q} + \zeta \nabla \times \vec{\omega}, \quad (2)$$

Conservation of Angular Momentum

$$\rho_0 I \left[\frac{\partial \vec{\omega}}{\partial t} + (\vec{q} \cdot \nabla) \vec{\omega} - \vec{\omega} \times \vec{\Omega} \right] = (\lambda' + \eta') \nabla (\nabla \cdot \vec{\omega}) + \eta' \nabla^2 \vec{\omega} + \zeta (\nabla \times \vec{q} - 2\vec{\omega}), \quad (3)$$

Conservation of Energy

$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = \frac{\beta}{\rho_0 C_v} (\nabla \times \vec{\omega}) \cdot \nabla T + \chi \nabla^2 T + Q(T - T_0), \quad (4)$$

Equation of State

$$\rho = \rho_0 [1 - \alpha_1 (T - T_0)], \quad (5)$$

where, \vec{q} is the velocity, $\vec{\omega}$ is the spin, T is the temperature, $P = p + \frac{1}{2} |\vec{\Omega} \times \vec{r}|^2$, p is the pressure, ρ is the density, ρ_0 is the density of the fluid at a reference temperature $T = T_0$, \vec{g} is the acceleration due to gravity, ζ is the coupling viscosity coefficient, η is the shear kinematic viscosity coefficient, I is the moment of inertia, λ' and η' are the bulk and shear spin viscosity coefficient, β is the micropolar heat conduction coefficient, C_v is the specific heat, χ is the thermal conductivity, α_1 is the coefficient of thermal expansion, Q is the heat flux, $\vec{\Omega}$ is the angular velocity and t is the time.

Eqs. (1)—(5) are solved subject to containment conditions appropriate for a rigid and thermally perfect conducting wall on the underside and by a free surface on the upper side. This free surface is adjacent to a non-conducting medium and subject to constant heat flux (i.e., adiabatic). Further no spin boundary condition is assumed for microrotation. In the reference steady-state the fluid is assumed to be at rest and is given by:

$$\begin{aligned} \bar{q}_b = 0, \quad \bar{\omega}_b = 0, \quad \bar{\Omega} = \Omega_0 \hat{k}, \quad P = P_b(z), \\ \rho = \rho_b(z), \quad - \frac{d}{\Delta T} \frac{dT_b}{dz} = f(z). \end{aligned} \tag{6}$$

where, $f(z) = \frac{\sqrt{\frac{Q}{\chi}} d \cos \left[\sqrt{\frac{Q}{\chi}} d \left(1 - \frac{z}{d} \right) \right]}{\sin \left[\sqrt{\frac{Q}{\chi}} d \right]}$.

C. Linear Stability Analysis

Let the reference steady-state be disturbed by an infinitesimal perturbation in velocity, microrotation, angular velocity, density, temperature and pressure. We now have

$$\begin{aligned} \bar{q} = \bar{q}_b + \bar{q}', \quad \bar{\omega} = \bar{\omega}_b + \bar{\omega}', \quad \bar{\Omega} = \bar{\Omega}_b + \bar{\Omega}', \\ \rho = \rho_b + \rho', \quad T = T_b + T', \quad P = P_b + P' \end{aligned} \tag{7}$$

he primes indicate the quantities are infinitesimal perturbations and subscript b indicates the reference steady-state value. Substituting Eq.(7) into Eqs.(1)—(5), we get the linearised equations governing the infinitesimal perturbations in the form:

$$\nabla \cdot \bar{q}' = 0 \tag{8}$$

$$\rho_0 \left[\frac{\partial \bar{q}'}{\partial t} + 2\bar{\Omega}_b \times \bar{q}' \right] = -\nabla P' - \rho' g \hat{k} + (2\zeta + \eta) \nabla^2 \bar{q}' + \zeta \nabla \times \bar{\omega}' \tag{9}$$

$$\rho_0 I \left[\frac{\partial \bar{\omega}'}{\partial t} - \bar{\omega}' \times \bar{\Omega}_b \right] = (\lambda' + \eta') \nabla (\nabla \cdot \bar{\omega}') + \eta' \nabla^2 \bar{\omega}' + \zeta (\nabla \times \bar{q}' - 2\bar{\omega}') \tag{10}$$

$$\frac{\partial T'}{\partial t} - w' \frac{\Delta T}{d} f(z) = \chi \nabla^2 T' + QT' + \frac{\beta}{\rho_0 C_v} \left[(\nabla \times \bar{\omega}') \cdot \left(\frac{-\Delta T}{d} f(z) \hat{k} \right) \right] \tag{11}$$

$$\rho' = -\rho_0 \alpha_1 T' \tag{12}$$

where, w' is the z-component of \bar{q}' . The perturbation equations (8)—(11) are non-dimensionalised using the following definition:

$$\begin{aligned} (x^*, y^*, z^*) = \frac{(x, y, z)}{d}, \quad \bar{q}'^* = \frac{\bar{q}'}{\chi/d^2}, \\ t^* = \frac{t}{d^2/\chi}, \quad T^* = \frac{T'}{\Delta T}, \quad \rho^* = \frac{\rho'}{\rho_0} \end{aligned} \tag{13}$$

In the present problem we assume the principle of exchange of stability to be valid and hence deal with only stationary convection [5]. Using Eq.(12) in Eq.(9), operating curl twice on the resulting equation to eliminate pressure, and operating curl once on Eq.(10) to get the vorticity equation. Also operating curl once on the resulting equation and non-dimensionalising all the resulting equations and Eq.(11) using Eq.(13). Therefore we have the following equations:

$$(1 + N_1)\nabla^2\phi_r + N_1 \left[D\phi_r - \left(\frac{\partial^2\omega_z}{\partial x^2} + \frac{\partial^2\omega_z}{\partial y^2} \right) \right] + Ta^{1/2}Dw = 0, \tag{14}$$

$$R \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + (1 + N_1)\nabla^4 w - Ta^{1/2}D\phi_r + N_1\nabla^2\Omega_z = 0, \tag{15}$$

$$N_3\nabla^2\Omega_z - N_1\nabla^2w - 2N_1\Omega_z - \frac{1}{2}Ta^{1/2}\phi_r N_2 = 0, \tag{16}$$

$$N_3\nabla^2 \left[D\phi_r - \left(\frac{\partial^2\omega_z}{\partial x^2} + \frac{\partial^2\omega_z}{\partial y^2} \right) \right] - N_1\nabla^2\phi_r - 2N_1 \left[D\phi_r - \left(\frac{\partial^2\omega_z}{\partial x^2} + \frac{\partial^2\omega_z}{\partial y^2} \right) \right] + \frac{1}{2}Ta^{1/2}N_2D\Omega_z = 0, \tag{17}$$

$$N_4D\phi_r + N_4D^2\omega_z + N_3\nabla^2\omega_z + N_1\phi_r - 2N_1\omega_z = 0, \tag{18}$$

$$\nabla^2T + R_I T + (w - N_5\Omega_z)f(z) = 0, \tag{19}$$

where, $D \equiv \frac{d}{dz}$, $\phi_r = \frac{\partial\omega_x}{\partial x} + \frac{\partial\omega_y}{\partial y}$, $\phi_r = \nabla \times \vec{q}$ and $\Omega_z = \nabla \times \vec{\omega}$, the asterisks have been dropped for simplicity and

$$N_1 = \frac{\zeta}{\zeta + \eta} \text{ (Coupling Parameter),}$$

$$N_2 = \frac{I}{d^2} \text{ (Inertia Parameter),}$$

$$N_3 = \frac{\eta'}{(\eta + \zeta)d^2} \text{ (Couple Stress Parameter),}$$

$$N_4 = \frac{\lambda' + \eta'}{(\zeta + \eta)d^2} \text{ (Stress Parameter),}$$

$$N_5 = \frac{\beta}{\rho_0 C_v d^2} \text{ (Micropolar Heat Conduction Parameter),}$$

$$R = \frac{\alpha_1 g \Delta T d^3 \rho_0}{\chi(\zeta + \eta)} \text{ (Rayleigh Number),}$$

$$R_I = \frac{Qd^2}{\chi} \text{ (Internal Rayleigh Number),}$$

and
$$Ta^{1/2} = \frac{2\Omega_0\rho_0d^2}{\zeta + \eta} \text{ (Taylor Number).}$$

The infinitesimal perturbations $W, \Omega_z, \phi_r, \varphi_r, \omega_z, T$ are assumed to be periodic and hence these permit a normal mode solution in the form

$$[w, \Omega_z, \phi_r, \varphi_r, \omega_z, T] = [W(z), \Omega_z(z), \phi_r(z), \varphi_r(z), \omega_z(z), T(z)] e^{i(lx+my)}, \tag{20}$$

where, l and m are horizontal components of the wave number, \vec{a} . Substituting Eq. (20) in Eqs. (14)–(19), we get

$$(1 + N_1)(D^2 - a^2)\phi_r + N_1(D\varphi_r + a^2\omega_z) + Ta^{1/2}DW = 0, \tag{21}$$

$$(1 + N_1)(D^2 - a^2)W - Ra^2T - Ta^{1/2}D\phi_r + N_1(D^2 - a^2)\Omega_z = 0, \tag{22}$$

$$N_3(D^2 - a^2)\Omega_z - N_1(D^2 - a^2)W - 2N_1\Omega_z - \frac{1}{2}Ta^{1/2}N_2\varphi_r = 0, \tag{23}$$

$$N_3(D^2 - a^2)(D\varphi_r + a^2\omega_z) - 2N_1(D\varphi_r + a^2\omega_z) - N_1(D^2 - a^2)\phi_r + \frac{1}{2}Ta^{1/2}N_2D\Omega_z = 0, \tag{24}$$

$$N_4D\varphi_r + N_4D^2\omega_z + N_1\phi_r + N_3(D^2 - a^2)\omega_z - 2N_1\omega_z = 0, \tag{25}$$

$$(D^2 - a^2)T + R_1T + (W - N_5\Omega_z)f(z) = 0. \tag{26}$$

Eqs. (21)–(26) are solved subject to the following boundary conditions

$$W = DW = T = \Omega_z = \phi_r = D\varphi_r = D\omega_z = 0 \text{ at } z = 0,$$

$$W = D^2W + a^2MT = DT = \Omega_z = \phi_r = D\varphi_r = D\omega_z = 0 \text{ at } z = 1, \tag{27}$$

where, $M = \frac{\sigma_T \Delta T d}{\eta \chi}$ is the Marangoni number. Eq.(27) indicates the use of rigid, isothermal lower boundary and upper, free, thermally insulating boundary(with respect to the perturbation). The condition on ω is the spin-vanishing boundary condition.

We now use the Galerkin technique to find the critical eigen-value. Multiplying Eq.(21) by ϕ_r , Eq.(22) by W , Eq.(23) by Ω_z , Eq.(24) by φ_r , Eq.(25) by ω_z , Eq.(26) by T , integrating the resulting equations by parts with respect to z from 0 to 1, using the boundary conditions Eq.(27) and taking

$$\begin{aligned} \phi_r &= A\phi_{r_1}, \quad W = BW_1, \quad \Omega_z = C\Omega_{z_1}, \\ \varphi_r &= E\varphi_{r_1}, \quad \omega_z = F\omega_{z_1}, \quad T = GT_1. \end{aligned}$$

where, A, B, C, E, F and G are constants. $\phi_{r_1}, W_1, \Omega_{z_1}, \varphi_{r_1}, \omega_{z_1}$ and T_1 are trial solutions that will satisfy the boundary conditions. Writing the above Eigen value problem in the matrix form as

$$Y_1 S = 0, \tag{28}$$

where,

$$S = [A \ B \ C \ E \ F \ G]^T,$$

$$Y_1 = \begin{bmatrix} (1+N_1)X_1 & Ta^{1/2}X_4 & 0 & N_1X_2 & N_1a^2X_3 & 0 \\ -Ta^{1/2}X_8 & (1+N_1) \begin{bmatrix} X_{28} \\ +2a^2X_{29} \\ +a^4X_{30} \end{bmatrix} & N_1X_7 & 0 & 0 & \begin{bmatrix} -2(1+N_1)a^2M \\ -Ra^2X_6 \end{bmatrix} \\ 0 & -N_1X_{11} & \begin{bmatrix} N_3X_9 \\ -2N_1X_{10} \end{bmatrix} & -\frac{1}{2}Ta^{1/2}N_2X_{12} & 0 & 0 \\ -N_1X_{17} & 0 & \frac{1}{2}Ta^{1/2}N_2X_{18} & N_3X_{13} & N_3a^2X_{14} & 0 \\ N_1X_{23} & 0 & 0 & N_4X_{22} & \begin{bmatrix} N_4X_{19} + \\ N_3X_{20} - \\ N_1X_{21} \end{bmatrix} & 0 \\ 0 & X_{25} & -N_5X_{26} & 0 & 0 & X_{24} + R_1X_{27} \end{bmatrix},$$

where,

$$\begin{aligned} X_1 &= \langle \phi_{r_1} (D^2 - a^2) \phi_{r_1} \rangle, X_2 = \langle \phi_{r_1} D\phi_{r_1} \rangle, X_3 = \langle \phi_{r_1} \omega_{z_1} \rangle, X_4 = \langle \phi_{r_1} DW_1 \rangle, \\ X_5 &= \langle W_1 (D^2 - a^2) W_1 \rangle, X_6 = \langle T_1 W_1 \rangle, X_7 = \langle W_1 (D^2 - a^2) \Omega_{z_1} \rangle, X_8 = \langle W_1 D\phi_{r_1} \rangle, \\ X_9 &= \langle \Omega_{z_1} (D^2 - a^2) \Omega_{z_1} \rangle, X_{10} = \langle \Omega_{z_1}^2 \rangle, X_{11} = \langle \Omega_{z_1} (D^2 - a^2) W_1 \rangle, X_{12} = \langle \Omega_{z_1} \phi_{r_1} \rangle, \\ X_{13} &= \langle \phi_{r_1} (D^2 - a^2) D\phi_{r_1} \rangle, X_{14} = \langle \phi_{r_1} (D^2 - a^2) \omega_{z_1} \rangle, X_{15} = \langle \phi_{r_1} D\phi_{r_1} \rangle, X_{16} = \langle \phi_{r_1} \omega_{z_1} \rangle, \\ X_{17} &= \langle \phi_{r_1} (D^2 - a^2) \phi_{r_1} \rangle, X_{18} = \langle \phi_{r_1} D\Omega_{z_1} \rangle, X_{19} = \langle \omega_{z_1} D^2 \omega_{z_1} \rangle, X_{20} = \langle \omega_{z_1} (D^2 - a^2) \omega_{z_1} \rangle, \\ X_{21} &= \langle \omega_{z_1}^2 \rangle, X_{22} = \langle \omega_{z_1} D\phi_{r_1} \rangle, X_{23} = \langle \phi_{r_1} \omega_{z_1} \rangle, X_{24} = \langle T_1 (D^2 - a^2) T_1 \rangle, \\ X_{25} &= \langle W_1 f(z) T_1 \rangle, X_{26} = \langle \Omega_{z_1} f(z) T_1 \rangle, X_{27} = \langle T_1^2 \rangle, X_{28} = \langle (D^2 W_1)^2 \rangle \\ X_{29} &= \langle (DW_1)^2 \rangle, X_{30} = \langle W_1^2 \rangle. \end{aligned}$$

In order to obtain a non trivial solution of the homogenous system given by Eq.(28) for the constant A,B,C,E,F and G, the determinant of the coefficient matrix must vanish. Solving the determinant for M using the following trial functions we get an expression for the Marangoni number, M.

$$\begin{aligned}
 W_1 &= z^2(1 - z^2), \\
 T_1 &= z(2 - z), \\
 \Omega_{z_1} &= z(1 - z), \\
 \varphi_{r_1} &= z^2(3 - 2z), \\
 \omega_{z_1} &= z^2(3 - 2z), \\
 \phi_{r_1} &= z(1 - z).
 \end{aligned}$$

III. RESULTS, DISCUSSION AND CONCLUSION

As mentioned in the beginning of the paper, we considered the effects of internal heat generation and coriolisforce on the Bénard-Marangoni convection in a micropolar fluid. These effects are denoted by the Micropolarparameters (N₁, N₂, N₃, N₄, N₅), Internal Rayleigh number(R_i) and Taylor number (Ta). The thermodynamic restriction permits the following range of values for N₁, N₂, N₃, N₄, N₅:

$$0 \leq N_1 \leq 1, 0 \leq N_2, N_3, N_4, N_5 \leq p \text{ where, } p \text{ represents a finite positive quantity}$$

TABLE I

Ta=100, R_i=2, N₂=0.1, N₃=2, N₄=0.1, N₅=1

R	Mc			
	N ₁ =0.1	N ₁ =0.3	N ₁ =0.5	N ₁ =0.7
0	47.2994	51.1286	55.5699	60.7443
100	42.321	46.9162	51.9191	57.523
500	22.4076	30.0664	37.3159	44.6378
950.101	0	11.1061	20.8837	30.1388
1213.7494		0	11.2584	21.6459
1522.1315			0	11.712
1885.7135				0

TABLE II

Ta=100, R_i=-2, N₂=0.1, N₃=2, N₄=0.1, N₅=1

R	Mc			
	N ₁ =0.1	N ₁ =0.3	N ₁ =0.5	N ₁ =0.7
0	119.562	131.527	145.923	163.521
100	114.584	127.314	142.272	160.299
500	94.6703	110.465	127.669	147.414
2401.6373	0	30.3589	58.2442	86.157
3122.332		0	31.9331	62.9413
3997.022			0	34.765
5076.249				0

TABLE III
 $Ta=500, R_1=2, N_2=0.1, N_3=2, N_4=0.1, N_5=1$

R	M_C			
	$N_1=0.1$	$N_1=0.3$	$N_1=0.5$	$N_1=0.7$
0	49.2816	52.5949	56.6825	61.5897
100	44.3032	48.3825	53.0317	56.3685
500	24.3898	31.5327	38.4285	45.4833
989.9165	0	10.8951	20.5427	29.7017
1248.5577		0	11.1002	21.3701
1552.6067			0	11.5758
1911.9596				0

TABLE IV
 $Ta=500, R_1=-2, N_2=0.1, N_3=2, N_4=0.1, N_5=1$

R	M_C			
	$N_1=0.1$	$N_1=0.3$	$N_1=0.5$	$N_1=0.7$
0	122.496	133.757	147.672	164.905
100	117.518	129.544	144.021	161.683
500	97.6046	112.695	129.418	148.798
2460.5798	0	30.1061	57.8414	85.6423
3175.272		0	31.7495	62.62
4044.931			0	34.6058
5119.2159				0

TABLE V
 Values of critical Marangoni number(M_C)for different values Stress parameter(N_4),
 Taylor number(Ta) and internal Rayleigh number (R_1)

N_4	M_C			
	$R_1=2, Ta=100$	$R_1=2, Ta=1000$	$R_1=-2, Ta=100$	$R_1=-2, Ta=1000$
0.5	88.1429061	271.9820047	88.88957556	273.3810898
1	88.14530571	271.9944995	88.91320739	273.5056624
1.5	88.14718404	272.0042831	88.93170116	273.6032599
2	88.1486943	272.0121516	88.94656802	273.6817876
2.5	88.14993503	272.0186171	88.9587795	273.7463368

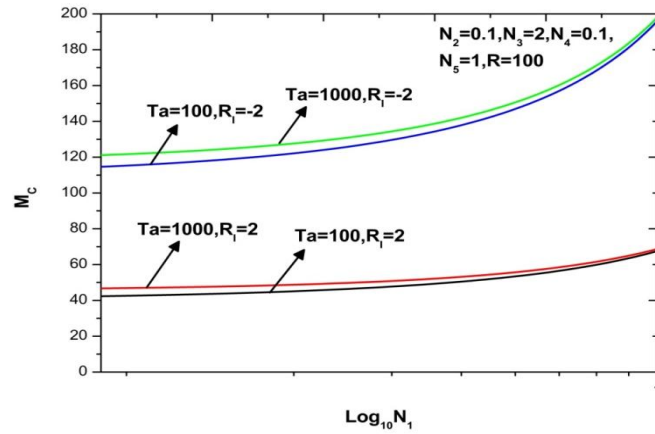


Fig 2 Plot of critical Marangoni Number and Copuling Parameter

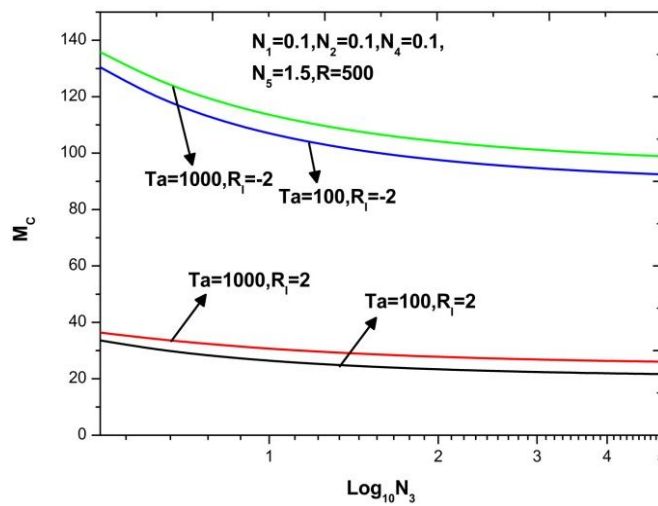


Fig 3 Plot of critical Marangoni Number and Inertia Parameter

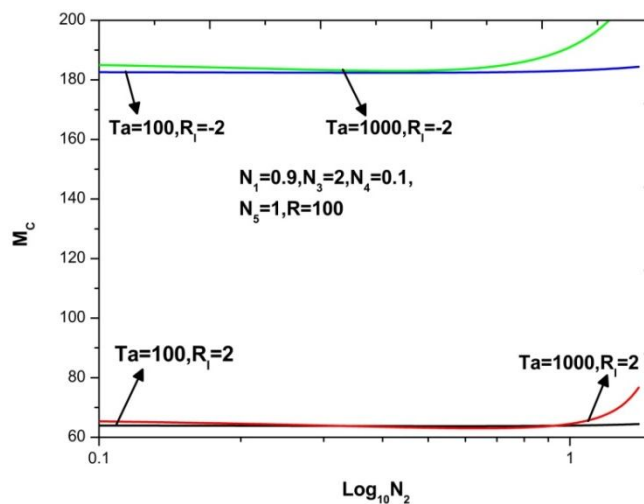


Fig 4 Plot of critical Marangoni Number and Copule stress Parameter

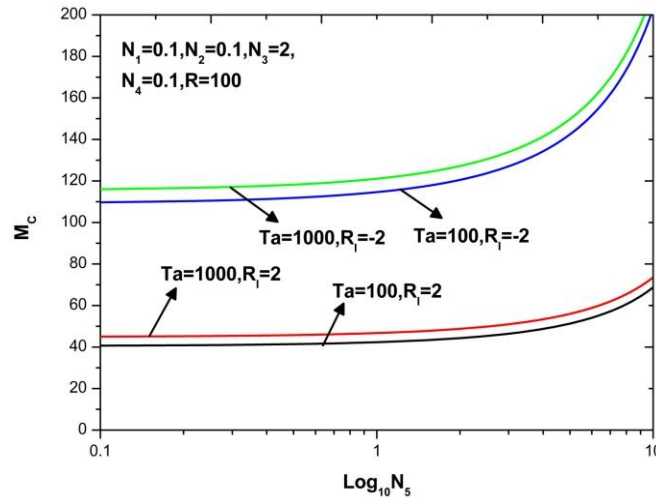


Fig 5 Plot of critical Marangoni Number and Micropolar Heat Conduction Parameter

Fig.2 is the plot of critical Marangoni number (M_C) versus Coupling parameter (N_1) for different values of Taylor number(Ta) and internal Rayleigh number(R_1). It is seen that as N_1 increases the critical Marangoni number increases. This shows that N_1 stabilizes the system. This is because as N_1 increase the concentration of microelements increases and these elements consume a greater part of energy of the system in developing the gyrational velocities of the fluid and hence the onset of convection is delayed. Fig.3 is the plot of critical Marangoni number (M_C) versus inertia parameter (N_2) for different values of Taylor number(Ta) and internal Rayleigh number(R_1). It is seen that as N_2 increases the critical Marangoni number increases. But however the effect is negligible for smaller values of N_2 . This is because as N_2 increases the inertia of the fluid due to the suspended particles increases. Hence the fluid becomes more rigid which in turn delays the onset of convection. Fig.4 is the plot of critical Marangoni number (M_C) versus Couple stress parameter (N_3) for different values of Taylor number(Ta) and internal Rayleigh number(R_1). It is seen that as N_3 increases the critical Marangoni number decreases. This is because as increase in N_3 decreases the couple stress of the fluid which causes a decrease in the microtation and hence advances the onset of convection.

Table V is the table of values of critical Marangoni number(M_C) and stress parameter(N_4) for different values of Taylor number(Ta) and internal Rayleigh number(R_1). It is seen that as N_4 increases M_C increases, but however the effect is negligible which is seen in the table.

Fig.5 is the plot of critical Marangoni number (M_C) versus Micropolar heat conduction parameter (N_5) for different values of Taylor number(Ta) and internal Rayleigh number(R_1). It is seen that as N_5 increases the critical Marangoni number increases indicating that the parameter stabilizes the system. This is because as N_5 increases the heat induced into the fluid due to these microelements also increases which reduce the heat transfer from bottom to top. Also it is observed from Fig.2-5 and table V that increase in internal Rayleigh number(R_1) decreases the critical Marangoni number(M_C). This is because, increase in internal Rayleigh number(R_1) increases the heat supplied to the system, thereby advancing the onset of convection. Also observed that increase in Taylor number(Ta) increases the critical Marangoni number(M_C) and thus stabilizes the system. This is due to the fact that, when rotation acts, it suppress the vertical motion and hence thermal convection by restricting the motion only in the horizontal direction. From tables I to IV, we observe that increase in Coupling parameter(N_1), critical Marangoni number(M_C) becomes zero for higher values of Rayleigh number(R). Thus as Rayleigh number(R) becomes larger and larger, critical Marangoni number(M_C) becomes smaller and ultimately we get a case dominated by buoyancy alone.

IV. CONCLUSION

The following conclusions are drawn from the present study

- A. Coupling parameter(N_1), Inertial parameter(N_2), Stress parameter(N_4), Micropolar heat conduction parameter(N_5), Taylor number(Ta) stabilizes the system.
- B. Couple-stress parameter(N_3), internal Rayleigh number(R_1) destabilizes the system.

- C. Although internal Rayleigh number(R_i) destabilizes the system, the presence of Taylor number(Ta) slows down the destabilizing process.
- D. The system becomes more stable in the presence of suspended particles compared to clean fluid.

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