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International Journal For Research in  
Applied Science and Engineering Technology



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# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

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**Volume:** ETPAM-2018 **Issue:** Conference **Month of publication:** March 2018

**DOI:**

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# Strongly $(\hat{g})^*$ Closed Sets In Topological Space

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**Abstract:** In this paper, we study the concept of strongly  $(\hat{g})^*$ -closed sets and strongly  $(\hat{g})^*$ -continuous functions and check how they deal with the topological spaces and their sub-sets. We also read out how the strongly  $(\hat{g})^*$ -closed sets and maps inter-relates with other sets with a change or transformation in their properties.

**Key words:**  $cl(A)$ ,  $int(A)$ , strongly  $(\hat{g})^*$ -closed set,  $g$ -closed set,  $g^*$ -closed set,  $(\hat{g})^*$ -closed set, strongly  $(\hat{g})^*$ -continuous map,  $g$ -continuous map,  $g^*$ -continuous map,  $(\hat{g})^*$ -continuous map

## I. INTRODUCTION

Levine [4] introduced the class of  $g$ -closed sets in 1970. Veerakumar [5] introduced  $\hat{g}$ -closed sets in 1991. A. Gayathri [8] introduced the class of  $(\hat{g})^*$  sets in 2014. The intention of this paper is to give the basic properties of strongly  $(\hat{g})^*$ -closed set and strongly  $(\hat{g})^*$ -continuous map and how they work in relation with other sets and maps.

## II. PRELIMINARIES

We see the non-empty topological space  $(X, \tau)$ , a subset  $A$  of  $X$  and an open set  $U$  of  $X$ . We also see the terms of closure of  $A$  i.e.  $Cl(A)$  and interior of  $A$  i.e.  $int(A)$ .

- A. *Definition 2.1:* Let  $A$  be a subset of a topological space  $(X, \tau)$ . The interior of  $A$  is defined as the union of all open sets contained in  $A$ . It is denoted by  $int(A)$ .
- B. *Definition 2.2:* Let  $A$  be a subset of a topological space  $(X, \tau)$ . The closure of  $A$  is defined as the intersection of all closed sets containing  $A$ . It is denoted by  $cl(A)$ .
- C. *Definition 2.3:* A subset  $A$  of the topological space  $(X, \tau)$  is called
- 1) a pre-open set [7] if  $A \subseteq int(cl(A))$
  - 2) a pre-closed set [7] if  $cl(int(A)) \subseteq A$
  - 3) a semi-open set [4] if  $A \subseteq cl(int(A))$
  - 4) a semi-closed set [4] if  $int(cl(A)) \subseteq A$
  - 5) a semi-pre open set [1] if  $A \subseteq cl(int(cl(A)))$
  - 6) a semi-pre closed set [1] if  $int(cl(int(A))) \subseteq A$
- D. *Definition 2.4:* A subset  $A$  of a topological space  $(X, \tau)$  is called
- 1)  $g$ -closed or generalized closed set [3] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
  - 2)  $g^*$ -closed set [5] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $(X, \tau)$ .
  - 3)  $\hat{g}$ -closed set [6] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open in  $(X, \tau)$ .
  - 4)  $(\hat{g})^*$ -closed set [8] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\hat{g}$ -open in  $(X, \tau)$ .
- E. *Definition 2.5:* A map  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called
- 1)  $g$ -continuous [2] if  $f^{-1}(V)$  is a  $g$ -closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
  - 2)  $g^*$ -continuous [5] if  $f^{-1}(V)$  is a  $g^*$ -closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
  - 3)  $(\hat{g})^*$ -continuous [8] if  $f^{-1}(V)$  is a  $(\hat{g})^*$ -closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .

## III. BASIC PROPERTIES OF STRONGLY $(\hat{G})^*$ -CLOSED SET

A. *Definition 3.1*

A subset 'A' of a topological space  $(X, \tau)$  is said to be a strongly  $(\hat{g})^*$ -closed set, if  $cl(int(A)) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $\hat{g}$ -open in  $X$ .

Theorem 3.2:

Every closed set is strongly  $(\hat{g})^*$ -closed.

Proof Let  $(X, \tau)$  be a topological space.

And  $A \subseteq (X, \tau)$  is a closed set.

i.e.  $\text{Cl}(A) = A$ .

To prove:  $A$  is strongly  $(\hat{g})^*$ -closed.

Let  $A \subseteq U$  and  $U$  be  $\hat{g}$  open.

Then  $\text{cl}(A) \subseteq U$ .

Also,  $\text{cl}(\text{int}(A)) \subseteq \text{cl}(A)$

We get,  $\text{cl}(\text{int}(A)) \subseteq \text{cl}(A) \subseteq U$

i.e.  $\text{cl}(\text{int}(A)) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $\hat{g}$  open.

$\therefore A$  is strongly  $(\hat{g})^*$ -closed.

*B. Theorem 3.3*

Every  $g$ -closed set is strongly  $(\hat{g})^*$ -closed.

*1) Proof:* Let  $A$  be a  $g$ -closed set.

By the definition 2.4.1,

$\text{Cl}(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .

To prove:  $A$  is strongly  $(\hat{g})^*$ -closed.

Let  $A \subseteq U$  and  $U$  is  $\hat{g}$  open.

We've,  $\text{cl}(A) \subseteq U$

Also,  $\text{cl}(\text{int}(A)) \subseteq \text{cl}(A)$

Then,  $\text{cl}(\text{int}(A)) \subseteq \text{cl}(A) \subseteq U$

i.e.  $\text{cl}(\text{int}(A)) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $\hat{g}$  open in  $(X, \tau)$ .

$\therefore A$  is strongly  $(\hat{g})^*$ -closed.

The converse of the above theorem need not be true as shown in the following example.

*C. Example 3.4*

Let  $X = \{a, b, c\}$  and  $\tau = \{\phi, X, \{a\}, \{a, c\}\}$

Closed sets are  $X, \phi, \{b, c\}, \{b\}$ .

Semi-open sets are  $\{a, c\}, \{a, b\}, \{a\}, \phi, X$ .

$\hat{g}$ -open sets are  $\{a, c\}, \{a\}, \phi, X$ .

Strongly  $(\hat{g})^*$ -closed sets are  $\{b\}, \{c\}, \{a, b\}, \{b, c\}, \phi, X$ .

$g$ -closed sets are  $\{b\}, \{a, b\}, \{b, c\}, \phi, X$ .

$\therefore A = \{c\}$  is a strongly  $(\hat{g})^*$ -closed set but not  $g$ -closed.

Hence, every strongly  $(\hat{g})^*$ -closed set need not be  $g$ -closed.

*D. Theorem 3.5*

Every  $g^*$ -closed set is strongly  $(\hat{g})^*$ -closed.

*1) Proof:* Let  $A$  be a  $g^*$ -closed set.

By the definition 2.4.2,

$\text{Cl}(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $(X, \tau)$ .

To prove:  $A$  is strongly  $(\hat{g})^*$ -closed.

Let  $A \subseteq U$  and  $U$  is  $\hat{g}$ -open.

We've,  $\text{cl}(A) \subseteq U$

Also,  $\text{cl}(\text{int}(A)) \subseteq \text{cl}(A)$

Then,  $\text{cl}(\text{int}(A)) \subseteq \text{cl}(A) \subseteq U$

i.e.  $\text{cl}(\text{int}(A)) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $\hat{g}$ -open in  $(X, \tau)$ .

$\therefore A$  is strongly  $(\hat{g})^*$ -closed.

The converse of the above theorem need not be true as shown in the following example.

*E. Example 3.6*

Let  $X = \{a, b, c\}$  and  $\tau = \{\phi, X, \{a\}, \{a, c\}\}$

Closed sets are  $X, \phi, \{b, c\}, \{b\}$

$g$ -open sets are  $\{a, c\}, \{c\}, \{a\}, \phi, X$ .

$g^*$ -closed sets are  $\{b\}, \{a, b\}, \{b, c\}, \phi, X$ .

Strongly  $(\hat{g})^*$ -closed sets are  $\{b\}, \{c\}, \{a, b\}, \{b, c\}, \phi, X$ .

$\therefore A = \{c\}$  is a strongly  $(\hat{g})^*$ -closed set but not  $g^*$ -closed.

Hence, every strongly  $(\hat{g})^*$ -closed set need not be  $g^*$ -closed.

*F. Theorem 3.7*

Every  $(\hat{g})^*$ -closed set is strongly  $(\hat{g})^*$ -closed.

1) *Proof:* Let  $A$  be a  $(\hat{g})^*$ -closed set.

By the definition 2.4.4,

$Cl(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $\hat{g}$ -open in  $(X, \tau)$ .

To prove:  $A$  is strongly  $(\hat{g})^*$ -closed.

Let  $A \subseteq U$  and  $U$  is  $\hat{g}$ -open.

We've,  $cl(A) \subseteq U$

Also,  $cl(int(A)) \subseteq cl(A)$

Then,  $cl(int(A)) \subseteq cl(A) \subseteq U$

i.e.  $cl(int(A)) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .

$\therefore A$  is strongly  $(\hat{g})^*$ -closed.

The converse of the above theorem need not be true as shown in the following example.

*G. Example 3.8:*

Let  $X = \{a, b, c\}$  and  $\tau = \{\phi, X, \{a\}, \{a, c\}\}$

Closed sets are  $X, \phi, \{b, c\}, \{b\}$

$g$ -open sets are  $\{a, c\}, \{c\}, \{a\}, \phi, X$ .

$(\hat{g})^*$ -closed sets are  $\{b\}, \{a, b\}, \{b, c\}, \phi, X$ .

Strongly  $(\hat{g})^*$ -closed sets are  $\{b\}, \{c\}, \{a, b\}, \{b, c\}, \phi, X$ .

$\therefore A = \{c\}$  is a strongly  $(\hat{g})^*$ -closed set but not  $(\hat{g})^*$ -closed.

Hence, every strongly  $(\hat{g})^*$ -closed set need not be  $(\hat{g})^*$ -closed.

#### IV. BASIC PROPERTIES OF STRONGLY $(\hat{G})^*$ -CLOSED CONTINUOUS MAPS

*A. Definition 4.1*

A map  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called a strongly  $(\hat{g})^*$ -continuous map if  $f^{-1}(V)$  is a strongly  $(\hat{g})^*$ -closed set in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .

*B. Theorem 4.2*

Every continuous map is strongly  $(\hat{g})^*$ -continuous.

1) *Proof:* Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a continuous map.

To prove:  $f$  is strongly  $(\hat{g})^*$ -continuous.

Let  $V$  be a closed set in  $(Y, \sigma)$ .

Since,  $f$  is continuous; there exists a closed set  $f^{-1}(V)$  in  $(X, \tau)$ .

By theorem 3.2,

"Every closed set is a strongly  $(\hat{g})^*$ -closed."

Hence,  $f^{-1}(V)$  is a strongly  $(\hat{g})^*$ -closed set in  $(X, \tau)$ .

$\therefore f$  is strongly  $(\hat{g})^*$ -continuous.

The converse of the above theorem need not be true as shown in the following example.

*C. Example 4.3:*

Let  $X = Y = \{a, b, c\}$

And  $\tau = \{\phi, X, \{a, b\}\}$

Closed sets in  $(X, \tau)$  are  $X, \phi, \{c\}$ .

And  $\sigma = \{\phi, Y, \{b, c\}\}$

Closed sets in  $(Y, \sigma)$  are  $Y, \phi, \{a\}$ .

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an identity map.

Semi-open sets are  $\{a, b\}, \phi, X$ .

$\hat{g}$ -open sets are  $\{a, b\}, \{a\}, \{b\}, \phi, X$ .

Strongly  $(\hat{g})^*$ -closed sets are  $\{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}, \phi, X$ .

$\therefore f^{-1}\{a\} = \{a\}$  is a strongly  $(\hat{g})^*$ -closed set in  $(X, \tau)$  but not a closed set in  $(X, \tau)$ .

Thus, the converse of the above theorem is not true.

Hence, every strongly  $(\hat{g})^*$ -continuous map need not be continuous.

*D. Theorem 4.4*

Every  $g$ -continuous map is strongly  $(\hat{g})^*$ -continuous.

1) *Proof:* Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a continuous map.

By the definition 2.5.1,

$f^{-1}(V)$  is a  $g$ -closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .

To prove:  $f$  is strongly  $(\hat{g})^*$ -continuous.

Let  $V$  be a closed set in  $(Y, \sigma)$ .

Since,  $f$  is  $g$ -continuous; there exists a  $g$ -closed set  $f^{-1}(V)$  in  $(X, \tau)$ .

By theorem 3.3,

“Every  $g$ -closed set is strongly  $(\hat{g})^*$ -closed.”

Hence,  $f^{-1}(V)$  is a strongly  $(\hat{g})^*$ -closed set in  $(X, \tau)$ .

$\therefore f$  is strongly  $(\hat{g})^*$ -continuous.

The converse of the above theorem need not be true as shown in the following example.

*E. Example 4.5*

Let  $X = Y = \{a, b, c\}$

And  $\tau = \{\phi, X, \{a, b\}\}$

Closed sets in  $(X, \tau)$  are  $X, \phi, \{c\}$ .

And  $\sigma = \{\phi, Y, \{b, c\}\}$

Closed sets in  $(Y, \sigma)$  are  $Y, \phi, \{a\}$ .

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an identity map.

$g$ -closed sets are  $\{c\}, \{b, c\}, \{a, c\}, \phi, X$ .

Strongly  $(\hat{g})^*$ -closed sets in  $(X, \tau)$  are  $\{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}, \phi, X$ .

$\therefore f^{-1}\{b\} = \{b\}$  is a strongly  $(\hat{g})^*$ -closed set in  $(X, \tau)$  but not a  $g$ -closed set in  $(X, \tau)$ .

Thus, the converse of the above theorem is not true.

Hence, every strongly  $(\hat{g})^*$ -continuous map need not be  $g$ -continuous.

*F. Theorem 4.6*

Every  $g^*$ -continuous map is strongly  $(\hat{g})^*$ -continuous.

1) *Proof:* Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a continuous map.

By the definition 2.5.2,

$f^{-1}(V)$  is a  $g^*$ -closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .

To prove:  $f$  is strongly  $(\hat{g})^*$ -continuous.

Let  $V$  be a closed set in  $(Y, \sigma)$ .

Since,  $f$  is  $g^*$ -continuous; there exists a  $g^*$ -closed set  $f^{-1}(V)$  in  $(X, \tau)$ .

By theorem 3.5,

“Every  $g^*$ -closed set is strongly  $(\hat{g})^*$ -closed.”

Hence,  $f^{-1}(V)$  is a strongly  $(\hat{g})^*$ -closed set in  $(X, \tau)$ .

$\therefore f$  is strongly  $(\hat{g})^*$ -continuous.

The converse of the above theorem need not be true as shown in the following example.

### G. Example 4.7

Let  $X = Y = \{a, b, c\}$

And  $\tau = \{\phi, X, \{a, b\}\}$

Closed sets in  $(X, \tau)$  are  $X, \phi, \{c\}$ .

And  $\sigma = \{\phi, Y, \{b, c\}\}$

Closed sets in  $(Y, \sigma)$  are  $Y, \phi, \{a\}$ .

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an identity map.

$g$ -open sets in  $(X, \tau)$  are  $\{a, b\}, \{a\}, \{b\}, \phi, X$ .

$g^*$ -closed sets are  $\{c\}, \{b, c\}, \{a, c\}, \phi, X$ .

Strongly  $(\hat{g})^*$ -closed sets in  $(X, \tau)$  are  $\{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}, \phi, X$ .

$\therefore f^{-1}\{b\} = \{b\}$  is a strongly  $(\hat{g})^*$ -closed set in  $(X, \tau)$  but not a  $g^*$ -closed set in  $(X, \tau)$ .

Thus, the converse of the above theorem is not true.

Hence, every strongly  $(\hat{g})^*$ -continuous map need not be  $g^*$ -continuous.

### H. Theorem 4.8

Every  $(\hat{g})^*$ -continuous map is strongly  $(\hat{g})^*$ -continuous.

1) *Proof:* Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a continuous map.

By the definition 2.5.3,

$f^{-1}(V)$  is a  $(\hat{g})^*$ -closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .

To prove:  $f$  is strongly  $(\hat{g})^*$ -continuous.

Let  $V$  be a closed set in  $(Y, \sigma)$ .

Since,  $f$  is  $(\hat{g})^*$ -continuous; there exists a  $(\hat{g})^*$ -closed set  $f^{-1}(V)$  in  $(X, \tau)$ .

By theorem 3.7,

“Every  $(\hat{g})^*$ -closed set is strongly  $(\hat{g})^*$ -closed.”

Hence,  $f^{-1}(V)$  is a strongly  $(\hat{g})^*$ -closed set in  $(X, \tau)$ .

$\therefore f$  is strongly  $(\hat{g})^*$ -continuous.

The converse of the above theorem need not be true as shown in the following example.

### I. Example 4.9

Let  $X = Y = \{a, b, c\}$

And  $\tau = \{\phi, X, \{a, b\}\}$

Closed sets in  $(X, \tau)$  are  $X, \phi, \{c\}$ .

And  $\sigma = \{\phi, Y, \{b, c\}\}$

Closed sets in  $(Y, \sigma)$  are  $Y, \phi, \{a\}$ .

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an identity map.

$\hat{g}$ -closed sets are  $\{c\}, \{b, c\}, \{a, c\}, \phi, X$ .

$\hat{g}$ -open sets are  $\{a, b\}, \{a\}, \{b\}, \phi, X$ .

$(\hat{g})^*$ -closed sets are  $\{c\}, \{b, c\}, \{a, c\}, \phi, X$ .

Strongly  $(\hat{g})^*$ -closed sets in  $(X, \tau)$  are  $\{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}, \phi, X$ .

$\therefore f^{-1}\{b\} = \{b\}$  is a strongly  $(\hat{g})^*$ -closed set in  $(X, \tau)$  but not a  $(\hat{g})^*$ -closed set in  $(X, \tau)$ .

Thus, the converse of the above theorem is not true.

Hence, every strongly  $(\hat{g})^*$ -continuous map need not be  $(\hat{g})^*$ -continuous.

### III. CONCLUSION

Hence, I would like to conclude my paper by giving the properties of strongly  $(\hat{g})^*$ -closed set and strongly  $(\hat{g})^*$ - continuous function. And also with further results and solutions we can bring in the comparison of strongly  $(\hat{g})^*$ -closed set and function with other sets and functions as well in a given topological space.

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