iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: TPAM-2018 Issue: onference Month of publication: March 2018

DOI:

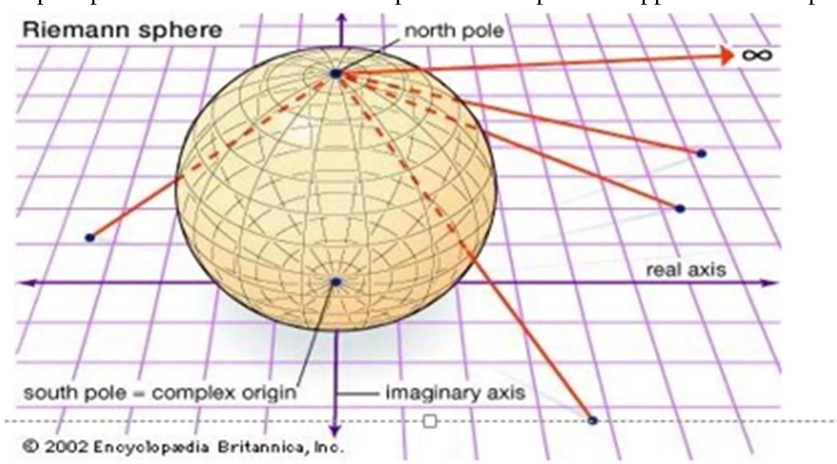
The Geometry of The Night Sky

A. Priya Dharshini[1] T. Mahalakshmi[2]
[1](PG-MATHS) Department of mathematics St. Joseph's College of Arts and science for Women Periyar University
[2]Associate professor Department of mathematics St. Joseph's college of Arts and science for Women Periyar University

Abstract: Practical application of mobius and lorentz transformation which is used by the space navigators to check their headings in the right direction using the stars location. It is done by showing isomorphism between lorentz and mobius group.
Key Words :Riemann sphere, Mobius transformation, Lorentz transformation, Stereographic projection and Spacetime and Light cone.

I. RIEMANN SPHERE

Model of an extended complex plane. Visualized as the complex number plane wrapped around a sphere.



II. MOBIUS TRANSFORMATION [2]

Mobius transformation is a bijective conformal mapping of the extended complex plane. The set of all mobius transformations forms a group under composition called the Mobius group. The general form of a mobius transformation is given by

$$f(z)=\frac{az+b}{cz+d} \qquad \text{……….(1)}$$

where a, b, c, d are any complex number satisfying $ad - bc \neq 0$.

III. LORENTZ TRANSFORMATION [6]

Lorentz transformation (or transformations) are coordinate transformations between two coordinate frames that move at constant velocity relative to each other.

The Lorentz group is the group of all Lorentz transformations of Minkowski space time.

The projection is defined on the entire sphere, except at one point the projection point.

It is conformal but neither isometric nor area-preserving.

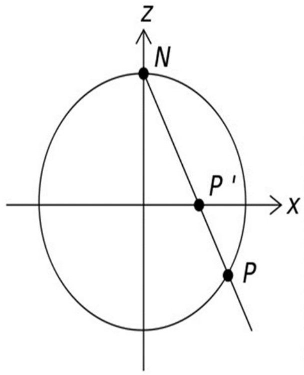
The stereographic projection of a point in (u, v)-space has coordinates

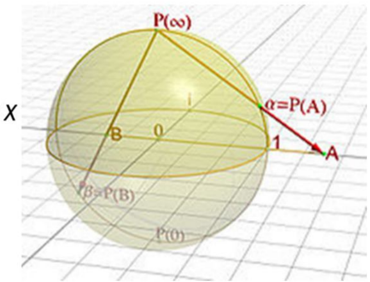
$$(x,y,z)=\left(\frac{2u}{1+u^2+v^2},\frac{2v}{1+u^2+v^2},\frac{u^2+v^2-1}{1+u^2+v^2}\right) \qquad \text{………..(2)}$$

IV. STEREOGRAPHIC PROJECTION IN SPACE

Consider the unit sphere S^2 defined by $x^2+y^2+z^2=1$

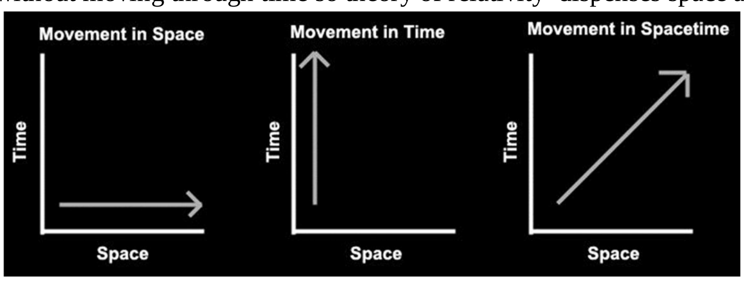
in the Cartesian (x,y,z) – Space. The point N = (0, 0, 1) is the north pole and its antipode S = (0, 0, −1) is the south pole. Each non-horizontal line through the north pole intersects the sphere in a unique point $(x, y, z) \neq N$ and intersects the (x, y)-plane in exactly one point (u, v). We may therefore view the sphere S^2 as the plane.



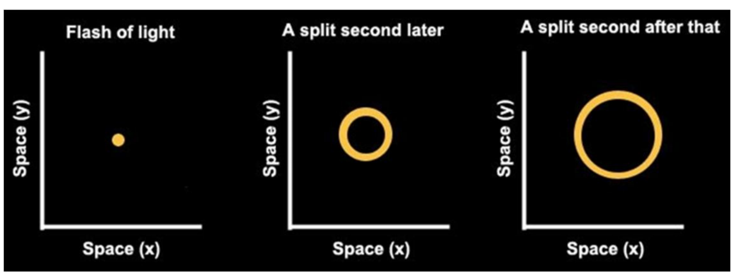


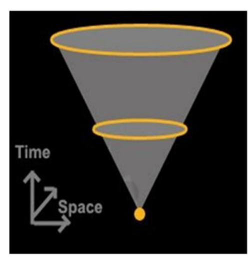
V. SPACETIME AND LIGHTCONE [1]

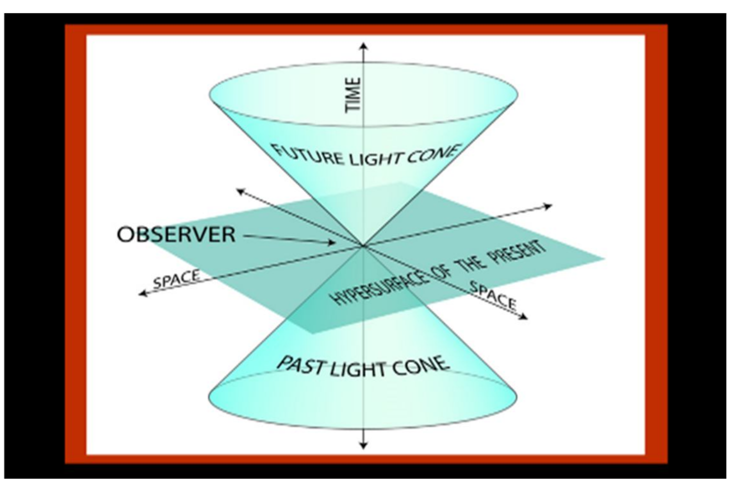
Something cannot move in space without moving through time so theory of relativity dispenses space and time as spacetime.



Light cone is the mathematical way of explaining the universe.

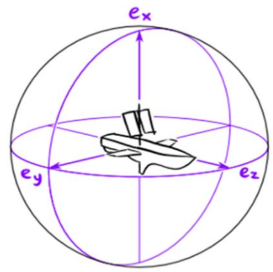






A. Describing points in spacetime [6]

e_t,the vector pointing forward in time along my worldline ,whose orthogonal complement is what I call space
e_x,the space vector pointing up through my canopy
e_y ,the space vector pointing out to my right
e_z ,the space vector pointing forward



Describing points in spacetime as linear combination

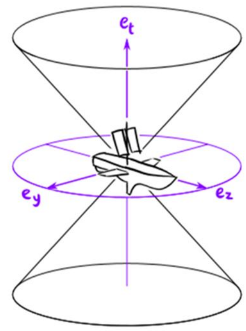
$$te_t + xe_x + ye_y + ze_z \qquad \text{………..(3)}$$

The sky coordinate x,y,z is given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1+|\zeta|^2}\begin{bmatrix} 2\,Im\zeta \\ 2Re\zeta \\ 1-|\zeta|^2 \end{bmatrix} \qquad \text{……….(4)}$$

Where,

If a brief burst is fired from the ship's engines, the orthonormal vectors describing the heading after the boost will be related to the ones from before by equations of the form



$$e_t' + e_z' = \lambda\,(e_t + e_z)$$
$$e_t' - e_z' = 1/\lambda\,(e_t - e_z)$$
$$e_x' = e_x$$
$$e_y' = e_y, \qquad \text{……….(5)}$$

te_t+xe_x+ye_y+ze_z
sub eqn(4)&(5) in (3)

BEFORE BOOST

$$-\left(1+|\tau|^2\right)e_t+(2\operatorname{Im}\tau)e_x+(2\operatorname{Re}\tau)e_y+\left(1-|\tau|^2\right)e_z \qquad \text{………..(6)}$$

AFTER BOOST

$$-\left(1+\left|\frac{1}{\lambda}\tau\right|^2\right)e_t'+(2\operatorname{Im}\frac{1}{\lambda}\tau)\,e_x'+(2\operatorname{Re}\frac{1}{\lambda}\tau)\,e_y'+\left(1-\left|\frac{1}{\lambda}\tau\right|^2\right)e_z' \qquad \text{………………(7)}$$

Are we directed in the right direction?

VI. CONCLUSION

Hence, the Homomorphism is Surjective (which can be realized through a combination of rotations and engine-axis boost) and Injective (as any two rotations as well as engine-axis boost can be distinguished by their effects on the view of monitor) giving an isomorphism between the lorentz group and mobius group which helps to navigate the ship in the right direction by positioning the stars in space.

REFERENCES

[1]. http://aschoonerofscience.com/how-things-work/what-is –a-light-cone
[2]. http://www.thefullwiki.org/M%C3%B6bius_transformation#parabolic_transforms
[3]. http://en.wikipedia.org/wiki/stereographic_projection
[4]. http://johno.dk/mathematics/moebius.pdf
[5]. http://mathfaculty.fullerton.edu/Mathews/c2003/complexgeometryMod.html
[6]. The geometry of the night sky or, An Ape pointing at the stars Aaron Fenyes