Sensitivity Analysis Technique in Minimizing Death Rate

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Abstract: This paper deals with the Sensitivity Analysis Technique in Minimizing Death Rate. The Preliminaries need are discussed in detail. The data required for death rate from 2010-2017 are collected from newspapers. Next the linear programming problem is framed for the data and Two-phase method is applied in general. In sensitivity analysis, the Rank Correlation Coefficient (RCC) method is used to correlate to identify the variable to change in the LPP for better reduction. The results thus obtained are discussed.
Keywords: Sensitivity, LPP, Variables, Minimization, Rank Correlation

I. INTRODUCTION

Employing techniques from other mathematical sciences, such as mathematical modeling, mathematical optimization and statistical analysis, operations research arrives at optimal or near-optimal solutions to complex decision-making problems.

Operational research (OR) encompasses a wide range of problem solving techniques and methods in the pursuit of improved decision making and efficiency. The computational nature of OR had a strong tie to computer science and analysis. It faces a problem to determine which techniques are most appropriate based on nature of the system, improvement and constraints on time and computing power.

Sensitivity Analysis is defined as the technique used to determine how different values of an independent variable will impact a particular dependent variable under a given set of assumptions. It is used within specific boundaries that will depend on one or more input variables like effect which changes in interest rates have on a bond's price.

Sensitivity Analysis is also known as the what- if analysis. It can be used for any activity analysis or system. It helps in analyzing how sensitive the output is by the changes in one input while keeping the other inputs constant. It works on the simple principle change the model and observes the behavior. It is a way to predict the outcome of a decision if a situation turns out to be complicated compared to the key predictions.

The current paper is a comparative assessment of several methods and is intended to demonstrate calculation rigor and compare parameter sensitivity ranking resulting from various sensitivity analysis techniques.

II. LINEAR PROGRAMMING

It is a technique for determining an optimal value of interdependent activities. A particular plan of action from several alternations can be done based on the available resources. Linear indicates that all relationships involved are linear.

A. *Mathematical Formulation Of Lpp*

Many problems deal with the allocation of resources-money, materials, machines, space, time.., in limited order to maximize profit or minimize cost. In such cases, applying the linear programming technique helps in finding the optimal value.

The objective function is to
Max/Min $Z = c_1X_1+c_2X_2+\ldots+c_nX_n$
Subject to the constraints
$a_{11}X_1+a_{12}X_2+\ldots+a_{1n}X_n \leq b_1$
$a_{21}X_1+a_{22}X_2+\ldots+a_{2n}X_n \leq b_2$
……………………………
$a_{m1}X_1+a_{m2}X_2+\ldots+a_{mn}X_n \leq b_m$
Where $X_1, X_2, \ldots X_n \geq 0$.

This is called the canonical form of a linear programming problem.

III. MINIMIZATION OF DEATH RATE USING BY LINEAR PROGRAMMING PROBLEM SOLVING IN TWO-PHASE METHOD

Given data are collected from Newspaper 'Daily Thanthi' which are collected from various durations. Then by using those data, the linear programming problems are formed to optimize and hence to produce the minimum death for the peoples, operation research simplex method is used to solve problems and the results are obtained.

The following table gives a detail of dengue effect and accidental cases in Tamilnadu.

A. Detail of data collected in dengue effect

Year	No. of people Dengue effect	No. of people Dengue death	Total dengue effect
2010	2051	8	25500
2011	2501	9	
2012	12826	66	
2013	8122	0	
2014	2804	3	23870
2015	4535	2	
2016	2531	5	
2017	14000	45	
Total	49370	148	49370

Table 2.1

B. Detail of data collected in accident case

Year	Accident	Accident death	Total accident
2010	14241	15409	58176
2011	14359	15422	
2012	15072	16175	
2013	14504	15563	
2014	15176	26068	64792
2015	15642	17666	
2016	17218	17343	
2017(up to march)	16756	4148	
Total	122968	127794	122968

Table 2.2

1) *Forming Of The Linear Programming Problem:* The entire state of Tamilnadu is considered together as dengue effect in first 4 years (2010-2013) and remaining 4 years (2014-2017; other accident case in first 4 years (2010-2013) and remaining 4 years(2014-2017). The suffering of people in these year are divided into two categories: dengue death and other death and the total percentage of people who have died due to these factors are given below

   Solution

Year wise	Factors for sufferings(in Percentage)		No. of people effected Dengue and accident case
	Dengue Effect	Accident case	
2010-2013	25.5	58.2	83.7
2014-2017	23.9	64.8	88.7

Table 2.3

Let X_1 denote the number of peoples who effect in dengue and X_2 denote the number of peoples accident case The objective function and the complete LPP can be represented as follows:

$$\text{Min } Z=0.148x_1+127.8x$$

Subject to

$$25.5x_1+58.2x_2 \geq 83.7$$

$$23.9x_1+64.8x_2 \geq 88.7$$

$$\text{With } X_1.X_2 \geq 0.$$

IV. TWO PHASE SIMPLEX METHOD

Steps taken to construct LPP for this model are discussed below

A. *Phase-I*

In phase I we construct an auxiliary LPP containing a basic feasible solution to the original problem.

1) *Step: 1* Assign cost -1 to each artificial variable and zero to all other variable and hence given a new objective function $Z^* = -A_1, -A_2, -A_3, \ldots$ where A_i's are artificial variables.
2) *Step: 2:* Here, the new objective function is to be maximized for the given set of constraints.
3) *Step: 3:* Solve the auxiliary LPP by simplex method until any of the following three cases arise.

a) Max $Z^* < 0$ and at least one artificial variable appears in the optimum basis at positive Level.
b) Max $Z^* = 0$ and at least one artificial variable appears in the optimum basis at zero level.
c) Max $Z^* = 0$ with no artificial variable appears in the optimum basis. Incase (i) The given LPP does not possess any feasible solution. Where as in case (ii), (iii). We get phase-II.

B. *Phase-II*

The optimum basic feasible solution of phase-I will be used as a starting solution for the original LPP.
Assign the actual costs to the variable in the objective function and a zero cost to every artificial variable. Proceed till an optimal basic feasible solution is obtained or till there is an indication of an unbounded solution.
By the above steps, now we change the previous LPP into required LPP

The objective function is

$$\text{Max } Z^*= -\text{Min } Z$$

$$\text{(i.e) Max } Z^*=--24.7x_1-61.5x_2$$

Assign cost -1 to each artificial variable and cost zero to all other variable, we get a new objective function.

$$\text{Max } Z^{**}=0x_1+0x_2+0s_1+0s_2-A_1-A_2$$

Subject to

$$23.5x_1+58.2x_2-S_1+S_2+A_1=83.7$$

$$23.9x_1+64.8x_2+S_1-S_2+A_2=88.7; \text{ with } X_1, X_2 \geq 0.$$

		C_j	0	0	0	0	-1	-1	
C_B	Y_B	b_i	X_1	X_2	S_1	S_2	A_1	A_2	R_i
-1	A_1	4	4	0	-1	1.2	1	-1.2	1→
0	X_2	1.37	0.37	1	0	-0.02	0	0.02	3.71
		Z_j	-4	0	1	-1.2	-1	1.2	
		Z_j-C_j	-4 ↑	0	1	-1.2	0	2.2	

Phase-I

C. Initial table

		C_j	0	0	0	0	-1	-1	
C_B	Y_B	b_i	X_1	X_2	S_1	S_2	A_1	A_2	R_i
-1	A_1	83.7	25.5	58.2	-1	0	1	0	1.44
-1	A_2	88.7	23.9	64.8	0	-1	0	1	1.37→
		Z_j	-49.4	-123	1	1	-1	-1	
		Z_j-C_j	-49.4	-123 ↑	1	1	0	0	

1) Since all Z_j-$C_j \leq 0$, the current solution is not optimal and now we have to determine the entering variable and the leaving variable.
2) The most negative Z_j-C_j corresponds toX_2, and so X_2 enters into the basis.
3) The leaving variable corresponds to the minimum value of R_j which is A_2.
4) Here the pivot element is 64.8.

D. First iteration

		C_j	0	0	0	0	-1	-1	
C_B	Y_B	b_i	X_1	X_2	S_1	S_2	A_1	A_2	R_i
-1	A_1	4	4	0	-1	1.2	1	-1.2	1→
0	X_2	1.37	0.37	1	0	-0.02	0	0.02	3.71
		Z_j	-4	0	1	-1.2	-1	1.2	
		Z_j-C_j	-4 ↑	0	1	-1.2	0	2.2	

1) Since all $Z_j - C_j \leq 0$, the current solution is not optimal and now we have to determine the entering variable and the leaving variable.
2) The most negative Z_j-C_j corresponds to X_1, and so X_1 enters into the basis.
3) The leaving variable corresponds to the minimum value of R_j which is A_1.
4) Here the pivot element is 4.

E. Second Iteration

Here x_1 enter into the basis

		C_j	0	0	0	0	-1	-1
C_B	Y_B	b_i	X_1	X_2	S_1	S_2	A_1	A_2
0	X_1	1	1	0	-0.25	0.3	0.25	-0.3
0	X_2	1	0	1	0.09	-0.12	-0.09	0.12
		Z_j	0	0	0	0	1	1

Since all Z_j-$C_j \geq 0$

An optimum solution to the auxiliary LPP has been obtained $X_1=1$, $X_2=1$.

Max Z**=0 and no artificial variable appear in the basis we go the phase II.

		C_j	-0.148	-127.8	0	0	-1	-1
C_B	Y_B	b_i	X_1	X_2	S_1	S_2	A_1	A_2
0	X_1	1	1	0	-0.25	0.3	0.25	-0.3
0	X_2	1	0	1	0.09	-0.12	-0.09	0.12
		Z_j	0	0	0	0	1	1

Phase-II

Consider the final simplex table of phase I consider the actual cost associated with the original variables. Delete the artificial variables A_1, A_2.

Max Z**= $-0.148X_1-127.8X_2+0S_1+0S_2$

Subject to constraints

$25.5x_1+58.2x_2-S_1+A_1=83.7$

$23.9x_1+64.8x_2-S_2+A_2=88.7$

Initial table

Since all $Z_j-C_j \geq 0$, optimal solution is reached.

Here, $X_1=1$, $X_2=1$.

Max Z* = -Min Z

Max Z* = $-(0.148X_1+127.8X_2)$

Max Z* = $-0.148X_1-127.8X_2$

Max Z* = -127.95

Min Z = -127.95

Min Z = 127.95

Here both X_1 and X_2 shows the same inference in death rate. So to identify the variable to sensitive we can go with sensitivity analysis.

V. SENSITIVITY ANALYSIS

A. Measurement Of Sensitivity Analysis

Below are mentioned the steps used to conduct sensitivity analysis:

1) Firstly the base case output is defined; say the NPV at a particular base case input value (V1) for which the sensitivity is to be measured. All the other inputs of the model are kept constant.
2) Then the value of the output at a new value of the input (V2) while keeping other inputs constant is calculated.
3) Find the percentage change in the output and the percentage change in the input
4) The sensitivity is calculated by dividing the percentage change in output by the percentage change in input.

This process of testing sensitivity for another input (say cash flows growth rate) while keeping the rest of inputs constant is repeated till the sensitivity figure for each of the inputs is obtained. The conclusion would be that the higher the sensitivity figure, the more sensitive the output is to any change in that input and vice versa.

B. Methods Of Sensitivity Analysis

There are different methods to carry out the sensitivity analysis:

1) Modeling and simulation techniques
2) Scenario management tools through Microsoft excel

There are mainly two approaches to analyzing sensitivity:

1) *Local sensitivity analysis* is derivative based (numerical or analytical). The term local indicates the derivatives which are taken at a single point. Mathematically, the sensitivity of the cost function with respect to certain parameters is equal to the partial derivative of the cost function with respect to those parameters.
2) *Global sensitivity analysis* is the second approach to sensitivity analysis, often implemented using Monte Carlo techniques. This approach uses a global set of samples to explore the design space.

The sensitivity methods include the utilization of the following one-at-time sensitivity measures:

1) partial derivatives (PD),
2) one standard deviation increase and decrease of inputs (±SD), a 20% increase and decrease of inputs (±20%),
3) a sensitivity index (SI).

The sensitivity measures investigated to utilize an array of input and output values generated through random sampling include:

1) an importance index (II),
2) a relative deviation of the output distribution (RD),
3) a relative deviation ratio (RDR),
4) partial rank correlation coefficients (PRCC),
5) Standardized regression coefficients (SRC) and rank regression coefficients (RRC).

Four additional techniques have been used to estimate parameter sensitivity rankings based on the partitioning of input data (Crick et al. 1987).

These methods include

1) The smirnov test (S),
2) The Cramer-von Mises test (CM),
3) The Mann-Whitney test (MW),
4) The square-ranks test (SR).

C. Standardized Regression Coefficients and Rank Regression Coefficients (SRC & RRC).

The use of the regression technique allows the sensitivity ranking to be determined based on the relative magnitude of the regression coefficients. The coefficients are indicative of the amount of influence the Standardization in regression analysis takes place in the form of a transformation by ranks or by the ratio of the parameter’s standard deviation to its mean. The rank regression coefficient (RRC) is calculated by performing regression analysis on rank-transformed data rather than the raw data. The RRC is often referred to as a standardized rank regression coefficient.

D. Rank Correlation Coefficient

$$\rho = rX1X2 = 1 - \frac{6\Sigma D^2}{n(n^2-1)}$$

1) Finding the rank correlation coefficient between Z and X_1

Here, Z=Total number of dengue and accident death rate

X_1=dengue death rate

Z	X_1	R_Z	R_{X2}	D	D^2
15417	8	7	4	3	9
15431	9	6	5	1	1
16241	66	4	8	-4	16
15563	0	5	1	4	16
26071	3	1	2	-1	1
17678	12	2	6	-4	16
17348	5	3	3	0	0
4193	45	8	7	1	1
				$\Sigma D^2=60$	

The rank correlation coefficient is $\mathbf{r_{zx1}=0.29}$

2) Finding The Rank Correlation Coefficient Between Z and X_2

Here, Z=Total number of dengue and accident case rate

X_2=Accident case rate

Z	X_2	R_Z	R_{X2}	D	D^2
15417	15409	7	7	0	0
15431	15422	6	6	0	0
16241	16175	4	4	0	0
15563	15563	5	5	0	0
26071	26068	1	1	0	0
17678	17666	2	2	0	0
17348	17343	3	3	0	0
4193	4148	8	8	0	0
				$\Sigma D^2=0$	

The rank correlation coefficient is $\mathbf{r_{ZX2}=1}$

3) Finding The Rank Correlation Coefficient Between X_1 and X_2

Here, X_1=dengue death rate

X_2=accident case rate

X_1	X_2	R_x	R_{X2}	D	D^2
8	15409	5	7	-2	4
9	15422	4	6	-2	4
66	16175	1	4	-3	9
0	15563	8	5	3	9
3	26068	7	1	6	36
12	17666	3	2	1	1
5	17343	6	3	3	9
45	4148	2	8	-6	36
				$\Sigma D^2=108$	

The rank correlation coefficient is $\mathbf{r_{X1X2}=-0.3}$

VI. SCATTER PLOT

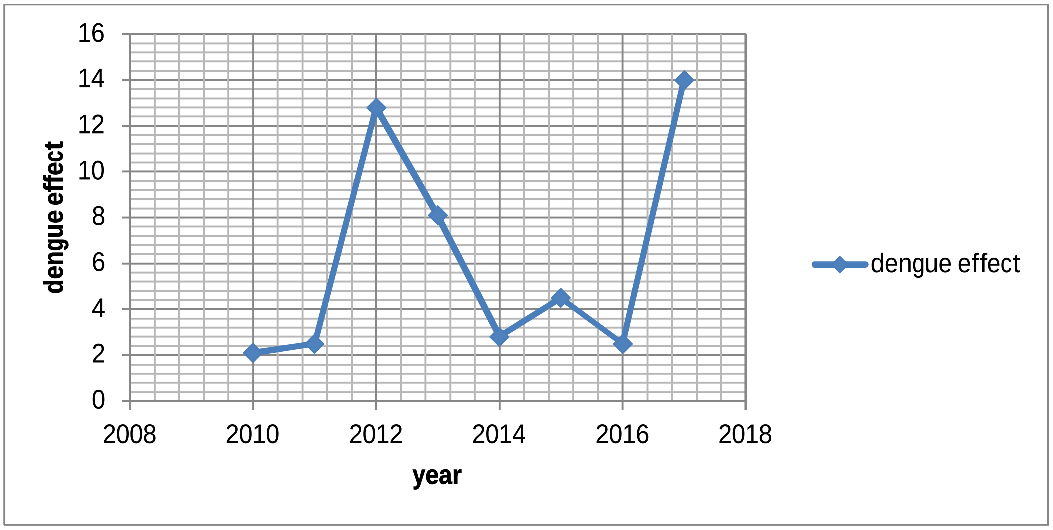
The above correlation of the variables are scattered below.

A. Scatter plot between the dengue effect and accident case rate

Year	Dengue effect	Accident case
2010	2.1	14.2
2011	2.5	14.4
2012	12.8	15.1
2013	8.1	14.5
2014	2.8	15.2
2015	4.5	15.6
2016	2.5	17.2
2017	14	16.8

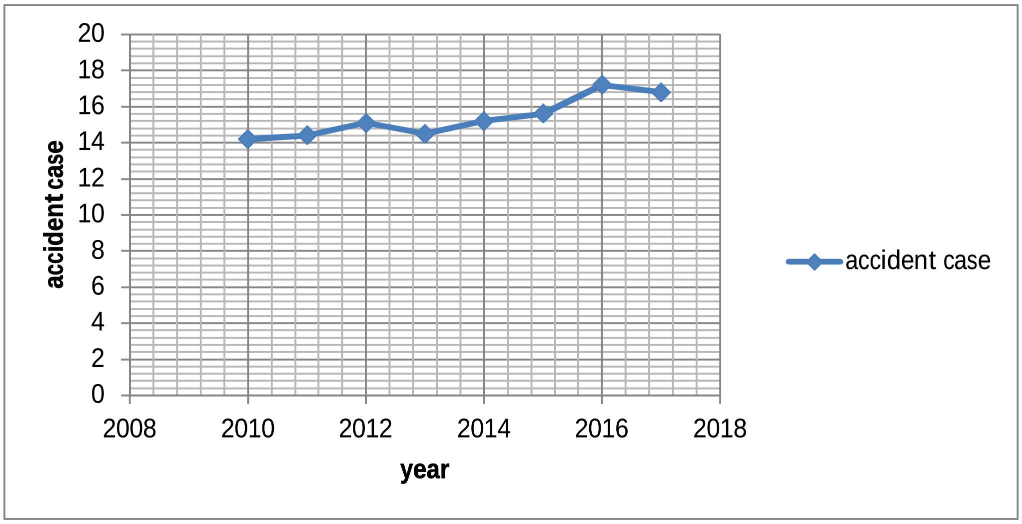
Here, X= Year

Y=dengue effect



Here, X=year

Y=accident case

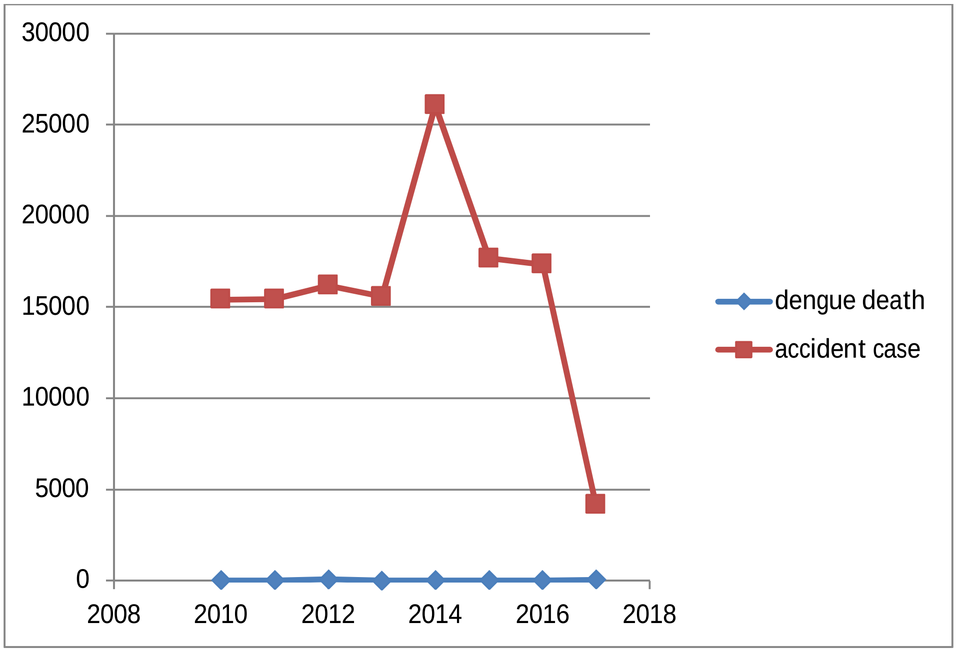


B. Comparison Scatter Plot Between Dengue Death And Accident Case

Here, X=year

Y=dengue effect and accident death rate

Year	Dengue death	Accident death
2010	8	15409
2011	9	15422
2012	66	16175
2013	0	15563
2014	3	26068
2015	12	17666
2016	5	17343
2017	45	4148



VII. RESULT

Comparing three rank correlation coefficient X_1 and X_2 are less related

1) If Z and X_1 are nearly independent variables with $r_{ZX1}=0.29$.
2) If Z and X_2 are next optimum related variables with $r_{ZX2}=1$.
3) If X_1 and X_2 are less related variables with $r_{X1X2}=-0.3$.

To prove minimize the death rate. We need to concentrate on accident case in common according to our data information.

In particular, X_1 and X_2 are independent variables, we cannot apply partial rank correlation coefficient.

VIII. CONCLUSION

Applying Two-phase method by forming the LPP of collected data I concluded that both variables dengue and accident case has same values $X_1=1$ and $X_2=1$. With Min Z=-127.95

(i.e) Minimum death rate obtained in Two-Phase method is 1.3

Next, applied Rank Correlation Coefficient (RCC) between ZX_1, ZX_2, and X_1X_2. As we know X_1 and X_2 are independent variables, we got very low correlation between them. Also comparing Z and X_1 we got the next low correlation. Finally for Z and X_2 the correlation coefficient is $r_{ZX2}=1$, which is optimal.

Hence I conclude my work result as, if we reduce the accidental case rate by considering its factors there will be more minimization in death data rate.

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