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Thermal Instability of Time-Periodic Oscillation in Maxwell-Cattaneo Dielectric Fluids

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Abstract: Impact of time periodic oscillation of small amplitude in a dielectric fluid subject to vertical ac electric field and a vertical temperature gradient is investigated for linear stability analysis. Perturbation solution in powers of amplitude of applied temperature field is obtained. The effect of Prandtl number, thermal Rayleigh number and Roberts number on the onset of convection is studied. It is found that time -periodic body force leads to delay in convection. The system is most stable with respect to time-periodic body force.
Keywords: Time periodic oscillations, Maxwell-Cattaneo heat flux, Dielectric fluid, Roberts number.

I. INTRODUCTION

The Rayleigh-Bènard convection(RBC) is a type of natural convection, where a layer of fluid confined between two horizontal plates of infinite length, is heated from below and cooled from above. The upward heat transfer can be achieved by conduction, that is, in the absence of motion on the part of the fluid because its viscosity cannot be overcome by the buoyancy forces. If the fluid is heated sufficiently large enough (higher temperature gradient across the layer), then only the top heavy state becomes unstable and convective motion is ensured. Such a thermal instability is known as RBC.

The Cattaneo equation was the first explicit mathematical correction of propagation of speed defect in the Fourier equation. It contains an extra inertial term with respect to the Fourier law. This heat conductivity equation and conservation of energy equation introduce the hyperbolic equation which describes heat propagation with finite speed. Smita and Pranesh [1] studied the problem of the onset of Rayleigh-Bénard convection in a second order Colemann-Noll fluid by replacing the classical Fourier heat flux law with non-classical Maxwell-Cattaneo law. The eigenvalue problem is solved using the general boundary conditions on velocity and third type of boundary conditions on temperature. It is found that the classical Fourier heat flux law overestimates the critical Rayleigh number compared to that predicted by the non-classical law and that the results are noteworthy at short times.

A liquid dielectric is a dielectric material in liquid state. Its main purpose is to prevent or satisfy electric discharges. It is used as electric insulators in high voltage applications such as transformers and capacitors. Liquid dielectrics are self-healing; when an electric breakdown occurs, the discharge channel does not leave a permanent conductive trace in the fluid. The effect of uniform rotation on the onset of convective instability in a dielectric fluid under the simultaneous action of a vertical ac electric field and a vertical temperature gradient was considered by Takashima [2]. It is shown that the principle of exchange of stabilities is valid for most dielectric fluids.

Time-periodic oscillations also known as gravity modulation/Time periodic body force. The effect of gravity modulation acts on the entire volume of the liquid and may have a stabilizing or destabilizing effect depending on the amplitude. The regulation of convection is important from the applications point of view and thermo gravitational vibration(g-jitter) is known to be an effective means of controlling instabilities. It is also of importance in the large-scale convection in atmosphere. Existence of adverse density variations within the fluid and a body force are the necessary conditions to initiate natural convection. The idea of using mechanical vibration as a tool to improve the heat transfer rate has received much attention. In the present paper the effect of time-periodic gravity modulation of the Rayleigh- Bénard convection problem on the heat transport in dielectric liquids is studied by linear analysis.

Gresho and Sani[3] and Greshuni *et al.* [4] were the first to study the effect of time periodic oscillations in a fluid layer and they used small amplitude approximation. They predicted that certain flow parameters may stabilize or destabilize the system. Malashetty and Padmavathy [5] studied the effect of gravity modulation on the onset of convection in fluid and found that low frequency oscillations have a significant effect on the stability of the system. Pau and Li [6] discussed about the mechanism of flow reduction also further they suppress the convection in modulated gravity field. Shu *et al.* [7] examined the effect of gravity modulation and found that for low Prandtl number fluids, the modulation in both gravity and temperature gives the same flow field both in structure and in magnitude. Siddavaram and Homsy [8], [9] studied the effect of stochastic gravity modulation. Siddeshwar and Annamma [10] discussed about the thermal instability of dielectric liquid when it is subjected to small amplitude and results in delay in convection.

Skarda [11] studied the effect of gravity modulation in a Marangoni-Bénard problem. He observed that the instabilities are strongly influenced by the Prandtl number in Marangoni-Bénard problem while it is weakly affected by Prandtl number in the case of Rayleigh- Bénard problem.

The available literature suggests that the problem of the onset of convection with time-periodic oscillations on the Maxwell-Cattneo dielectric fluids has not been explored. In this paper, we have made an attempt to study the effects of fluctuating gravity in a dielectric fluid with a vertical ac-electric field and vertical temperature gradient on the onset of Rayleigh-Bénard convection.

II. MATHEMATICAL FORMULATION

Consider an infinite horizontal layer of a Boussinesq dielectric fluid between two horizontal surfaces z = 0 and z = d under the influence of a uniform vertical ac electric field and a time periodically varying gravity forces $\vec{g} = (0,0,-g(t))$ acting on it, where $\vec{g}(t) = g_0[1+\delta cos\gamma t]\hat{k}$ with g_0 being the mean gravity, δ the small amplitude, γ the frequency and t the time. The lower and upper boundaries are maintained at different temperatures T_0 and T_1 respectively($T_0 > T_1$). A Cartesian co-ordinate system is considered with origin on the lower boundary and the z-axis normal to the fluid layer with these assumptions the governing equations to the problem are,

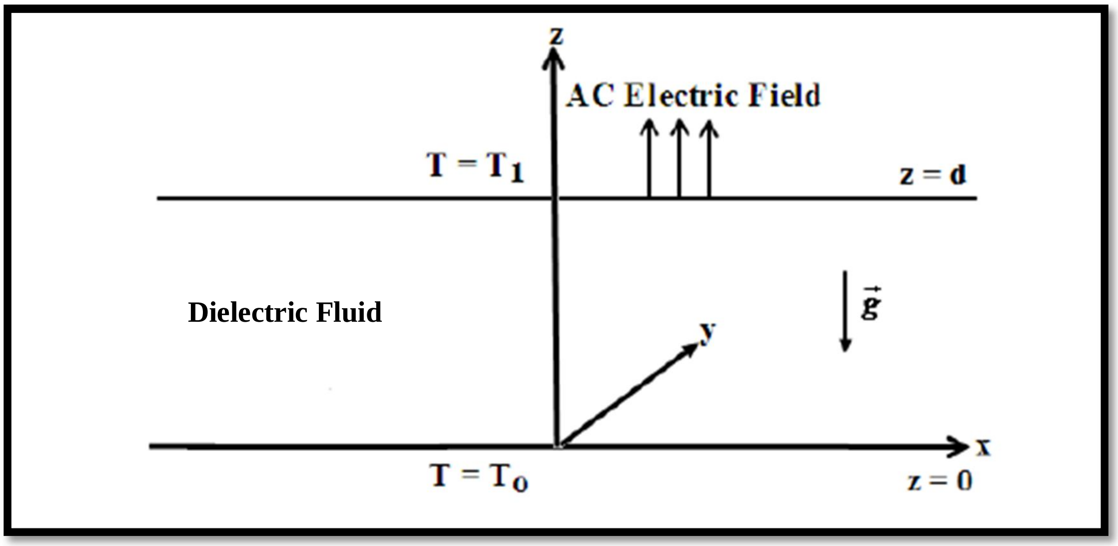

Figure 1: Physical Configuration

The relevant governing equation are given by

Continuity equation

$$\nabla.\vec{q} = 0, \tag{1}$$

Conservation of Linear Momentum

$$\rho_0\left[\frac{\partial\vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla p + \rho\vec{g}(t) + \mu\nabla^2\vec{q} + (\vec{P}.\nabla)\vec{E} \ , \tag{2}$$

$$\vec{g}(t) = g_0[1+\delta cos\gamma t]\hat{k}$$

Conservation of Energy

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = -\nabla.\vec{Q} \ , \tag{3}$$

Maxwell Cattaneo Law

$$\tau\left[\frac{\partial\vec{Q}}{\partial t} + (\vec{q}.\nabla)\vec{Q} + \vec{\omega}\times\vec{Q}\right] = -\vec{Q} - \kappa\nabla T \ , \tag{4}$$

Equation of state

$$\rho = \rho_0[1-\alpha(T-T_0)] \ , \tag{5}$$

Where, $\vec{q}$ is the velocity vector, $\vec{P}$ the dielectric polarization, $\vec{E}$ the electric field, T the temperature, p the pressure, ρ the fluid density, κ the thermal diffusivity, μ the fluid viscosity, α the coefficient of thermal expansion, ρ_0 the density at a reference temperature $T = T_0$, $\vec{Q}$ the heat flux vector, τ the constant relaxation time and $\vec{\omega} = \frac{1}{2}(\nabla \times \vec{q})$ and the electric field gradient $\nabla\vec{E}$.

The relevant Maxwell equations are,

$$\vec{P} = \varepsilon_0[\varepsilon_r - 1]\,\vec{E}\,, \qquad (6)$$

$$\nabla.\left(\varepsilon_0\varepsilon_r\vec{E}\right) = 0\,, \qquad (7)$$

$$\nabla \times \vec{E} = 0 \text{ or } \vec{E}=\nabla\phi, \qquad (8)$$

$$\varepsilon_r = {\varepsilon_r}^0 - e(T - T_0)\,, \qquad (9)$$

where, ε_0 is the electric permittivity, ϕ the electric potential, ε_r the relative permittivity or dielectric constant assumed to be linear function of temperature, $e > 0$ and ${\varepsilon_r}^0 = 1 + \chi_e$ with χ_e being the electric susceptibility.

We investigate the stability of a quiescent state subject to infinitesimal perturbations on the unsteady basic state described by,

$$\left.\begin{aligned} &\vec{q} = \vec{q}_b = (0,0,0)\,, p = p_b(z)\,, \rho = \rho_b(z), T = T_b(z)\,, \vec{Q} = \vec{Q}_b = (0,0,\kappa\beta), \vec{E} = \vec{E}_b = \left(0,0,E_b(z)\right),\\ &\vec{P} = \vec{P}_b = \left(0,0,P_b(z)\right), \varepsilon_r = \varepsilon_{rb}(z)\,, \phi = \phi_b(z)\,, \beta = \frac{T_0 - T_1}{d}, \end{aligned}\right\} \qquad (10)$$

where, the subscript b denotes the basic state. In the undisturbed basic state, the temperature T_b, density ρ_b, permitivitty ε_{rb}, the dielectric polarization $\vec{P}_b$, the electric potential ϕ_b satisfies the following equations,

$$T_b = T_0 - \beta z, \qquad (11)$$

$$\rho_b = \rho_0[1 + \alpha\beta z], \qquad (12)$$

$$\varepsilon_{rb} = (1 + \chi_e)\left[1 + \frac{e\beta z}{1+\chi_e}\right], \qquad (13)$$

$$E_b = \frac{E_0(1+\chi_e)}{1+\chi_e+e\beta z}\,, \qquad (14)$$

$$\vec{P}_b = \varepsilon_0 E_0\left[(1 + \chi_e) - \frac{1}{\left(1+\frac{e\beta z}{1+\chi_e}\right)}\right]\hat{k}, \qquad (15)$$

$$\phi_b = \frac{(1+\chi_e)E_0}{e\beta}\log\left[1 + \frac{e\beta z}{1+\chi_e}\right], \qquad (16)$$

where, E_0 is the value of the electric field at z = 0. We examine the stability of the equilibrium state by means of linear stability analysis.

A. Linear Stability Analysis

Let the basis state be disturbed by an infinitesimal thermal perturbation, so that

$$\left.\begin{aligned} &\vec{q} = \vec{q}_b + \vec{q}' = (u',v',w'),\ p = p_b + p', \rho = \rho_b + \rho', T = T_b + T', \vec{E} = \vec{E}_b + \vec{E}', \vec{P} = \vec{P}_b + \vec{P}',\\ &\varepsilon_r = \varepsilon_{rb} + \varepsilon_r', \phi = \phi_b + \phi', \vec{Q} = \vec{Q}_b + \vec{Q}', \end{aligned}\right\} \qquad (17)$$

where prime indicates that the quantities are infinitesimal perturbations. Substituting (17) in (1) to (9) and using basic state solutions we get,

$$\rho_0\frac{\partial}{\partial t}(\nabla^2\omega') = \mu(\nabla^4\omega') + \alpha\rho_0 g_0[1 + \delta\cos\gamma t]\nabla_1^2 T' + \frac{\varepsilon_0 e^2{E_0}^2\beta}{1+\chi_e}\nabla_1^2 T' - \varepsilon_0 e E_0\beta\frac{\partial}{\partial z}(\nabla_1^2\phi'), \qquad (18)$$

$$\left(1 + \tau\frac{\partial}{\partial t}\right)\left(\frac{\partial T'}{\partial t} - \beta w'\right) = \kappa\nabla^2 T' - \frac{\tau\kappa\beta}{2}\nabla^2 w'\,, \qquad (19)$$

$$(1 + \chi_e)\nabla^2\phi' - eE_0\frac{\partial T'}{\partial z} = 0\,, \qquad (20)$$

where, $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

Non-dimensionalzing (18), (19) and (20) using the length, time, velocity, temperature and electric potential scales $d, \frac{d^2}{\kappa}, \frac{\kappa}{d}, \beta d, \frac{eE_0\beta d^2}{1+\chi_e}$, we obtain

$$\frac{1}{Pr}\frac{\partial}{\partial t}(\nabla^2 w) = \nabla^4 w + (R[1+\delta cos\gamma t] + L)\nabla_1^2 \mathrm{T} - \mathrm{L}\frac{\partial}{\partial z}(\nabla_1^2 \phi)\,, \qquad (21)$$

$$\left(1 + 2C\frac{\partial}{\partial t}\right)\left(\frac{\partial T}{\partial t} - w\right) = \nabla^2 T - \mathrm{C}\nabla^2 w, \qquad (22)$$

$$\nabla^2 \phi - \frac{\partial T}{\partial z} = 0, \qquad (23)$$

where,

$Pr = \frac{\mu}{\rho_0 \kappa}$ (Prandtl number)

$R = \frac{\alpha \rho_0 g_0 \beta d^4}{\mu \kappa}$ (Thermal Rayleigh number)

$L = \frac{\varepsilon_0 (\mathrm{eE_0 \beta d^2})^2}{\mu \kappa (1+\chi_e)}$ (Roberts number)

$C = \frac{\tau \kappa}{2d^2}$ (Cattaneo number)

Equations (20) and (21) are solved subject to the conditions,

$$w = D^2 w = T = D\phi = 0 \; at \; z = 0,1. \qquad (24)$$

Eliminating w from equations (20) and (21), we get an equation for T in the form,

$$\frac{1}{\left(1 - C\nabla^2 + 2C\frac{\partial}{\partial t}\right)}\left[\frac{1}{Pr}\frac{\partial^2}{\partial t^2}\nabla^4 + 2\mathrm{C}\frac{1}{Pr}\frac{\partial^3}{\partial t^3}\nabla^4 + \frac{1}{Pr}\frac{\partial}{\partial t}\nabla^6 - \nabla^6\frac{\partial}{\partial t} - 2C\frac{\partial^2}{\partial t^2}\nabla^6 - \nabla^8\right]T$$

$$= [R(1+\delta\cos\Omega t)\nabla^2\nabla_1^2]T \qquad (25$$

B. Method of Solution

We now seek the eigen-function T and eigen-values R of the equation (25) in the form

$$(\mathrm{R},\mathrm{T}) = (R_0, T_0) + \delta(R_1, T_1) + \delta^2(R_2, T_2) + \cdots \qquad (26)$$

where, R_0 is the Rayleigh number for the unmodulated RBC in a Maxwell cattaneo dielectric fluid .

The expansion of (26) is substituted into equation (25) and the coefficients of like powers of δ are equated on either side of the equation. The resulting system of equations upto the order of O(δ^2) is,

$$\mathrm{L}T_0 = 0, \qquad (27)$$

$$LT_1 = (R_1 + R_0 f)\nabla_1^2 T_0, \qquad (28)$$

$$LT_2 = (R_1 + R_0 f)\nabla_1^2 T_1 + (R_2 + R_1 f)\nabla_1^2 T_0\,, \qquad (29)$$

where,

$$\mathrm{L} = \left[\frac{1}{Pr}\frac{\partial^2}{\partial t^2}\nabla^4 + 2\mathrm{C}\frac{1}{Pr}\frac{\partial^3}{\partial t^3}\nabla^4 + \frac{1}{Pr}\frac{\partial}{\partial t}\nabla^6 - \nabla^6\frac{\partial}{\partial t} - 2C\frac{\partial^2}{\partial t^2}\nabla^6 - \nabla^8\right] - \left(1 - C\nabla^2 + 2C\frac{\partial}{\partial t}\right)L\nabla^2\nabla_1^2 + \left(1 - C\nabla^2 + 2C\frac{\partial}{\partial t}\right)L\frac{\partial^2}{\partial z^2}\nabla_1^2 - \left(1 - C\nabla^2 + 2C\frac{\partial}{\partial t}\right)\nabla^2\nabla_1^2 R_0 \qquad (30)$$

The zero order problem is equivalent to the problem of RBC in a Maxwell-Cattaneo dielectric fluid layer in the absence of thermal modulation.

The marginally stable solution of the unmodulated problem is

$$T_0 = \sin \pi z \, e^{i(lx+my)}, \qquad (31)$$

where, l and m are wave numbers in x and y direction.

The corresponding eigen value is given by,

$$R_0 = \frac{k^6}{a^2(1+Ck^2)} - \frac{L(k^2-\pi^2)}{k^2}\ , \tag{32}$$

where, $k^2 = \pi^2 + a^2$ and $a^2 = l^2 + m^2$ is the overall horizontal wave numbers of the convective disturbances.
(28) becomes,

$$LT_1 = (R_0 T_0 f + R_1 T_0)\nabla^2\nabla_1^2\ , \tag{33}$$

If the above equation is to have a solution, the right hand side must be orthogonal to the null-space of the operator L. This implies that the time independent part of the RHS of the equation (33) must be orthogonal to $\sin\pi z$ Since f varies sinusoidaly with time, the only steady term on the RHS of equation (33) is $R_0 a^2 f\ \sin\pi z$ so that $R_1 = 0$. From (26) odd coefficients are made zero.
Using (30), we find that,

$$L\left[\sin\pi z\ e^{i(lx+my-\Omega t)}\right] = L(\Omega)\sin\pi z\left[\sin\pi z\ e^{i(lx+my-\Omega t)}\right], \tag{34}$$

where,

$$L(\Omega) = Y_1 + i\,Y_2$$

$$Y_1 = -\frac{1}{Pr}k^4\Omega^2 - 2Ck^6\Omega^2 - k^8 - (R_0 + L)k^2a^2 - k^4a^2C(R_0 + L) + L\pi^2a^2(1 + Ck^2)$$

$$Y_2 = -2C\frac{1}{Pr}k^4\Omega^3 + \frac{1}{Pr}\Omega k^6 - \Omega k^6 + 2C\Omega a^2(Lk^2 - L\pi^2 + k^2R_0)$$

The particular solution of (33) is given by

$$T_1 = \frac{1}{|L(\Omega)|^2}(Y_1\ \cos\Omega t - Y_2\sin\Omega t)(R_0 a^2\ \sin\pi z), \tag{35}$$

The solution of the homogeneous equation corresponding to (33) involves a term proportional to $\sin\pi z$. As $R_1 = 0$ from (29) we obtain

$$LT_2 = (R_0 T_1 f + R_2 T_0)\nabla^2\nabla_1^2, \tag{36}$$

To determine R_2 we orthoganalise RHS of (36) to $\sin\pi z$.

$$\int_0^1 R_2 T_0 \sin\pi z\ = -\int_0^1 R_0 T_1 f\sin\pi z\ ,$$

Taking time average we get,

$$R_2 = -\frac{{R_0}^2k^2a^2Y_1}{2|L(\Omega)|^2}\ . \tag{37}$$

This value of R_2 is negative. It is called the critical Rayleigh number.

III. RESULTS AND DISCUSSION

An analytical solution is obtained from this study. The regular perturbation method based on small amplitude of modulation is employed to compute the value of Rayleigh number and corresponding wave number. The expression for critical Rayleigh number R_2 is computed as a function of frequency of modulation Ω, Roberts number L and Cattaneo number C. The effect of these parameters on the stability of the system is analyzed graphically. The sign of R_2 characterizes the stabilizing or destabilizing the effect of modulation. The negative value of R_2 shows the destabilized system. From the graphs it is observed that there is delay in onset of convection by the effect of imposed time periodic oscillations.

The variation of correction Rayleigh number R_2 with frequency modulation Ω for different values of Roberts number L and fixed value of the Cattaneo number C is shown in figures (2) ,(3) and (4). We observe that with increase in frequency modulation and Roberts number, the correction Rayleigh number R_2 decreases. This destabilizes the system. The increase in frequency of modulation and Roberts number there is decrease in R_2.

The variation of correction Rayleigh number $R_2$with frequency modulation Ω for different values of Cattaneo number C and fixed value of the Roberts number L is shown in figures (5) ,(6) and (7). We observe that with increase in frequency modulation and Cattaneo number, the correction Rayleigh number R_2 decreases. This destabilizes the system. The increase in frequency of modulation and Roberts number there is decrease in R_2.

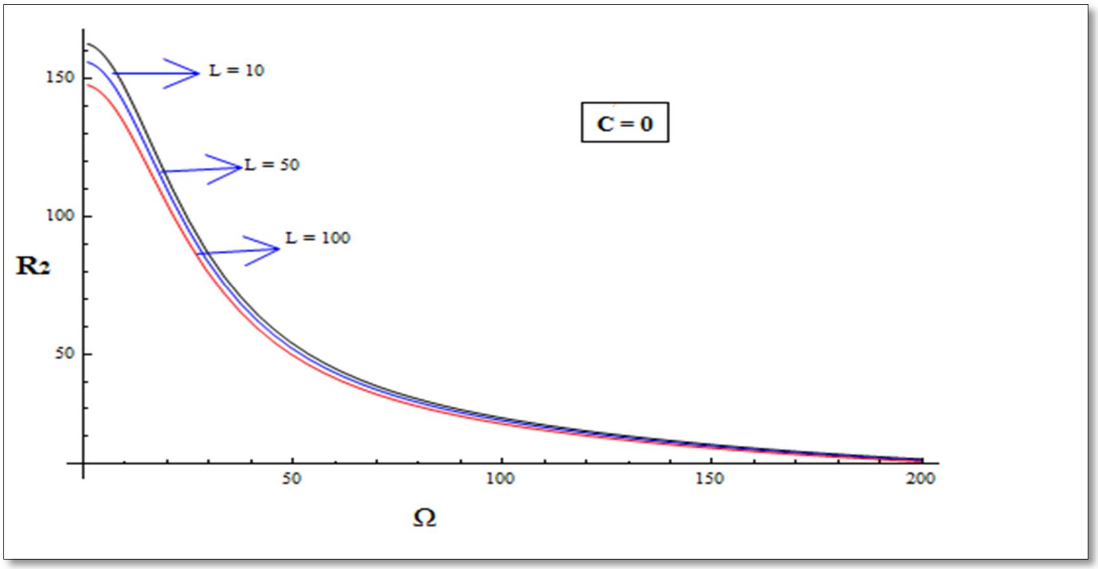

Figure 2: The plot of correction Rayleigh number R_2 vs frequency of Modulation Ω for different values of Roberts number L by taking Cattaneo number as C = 0.

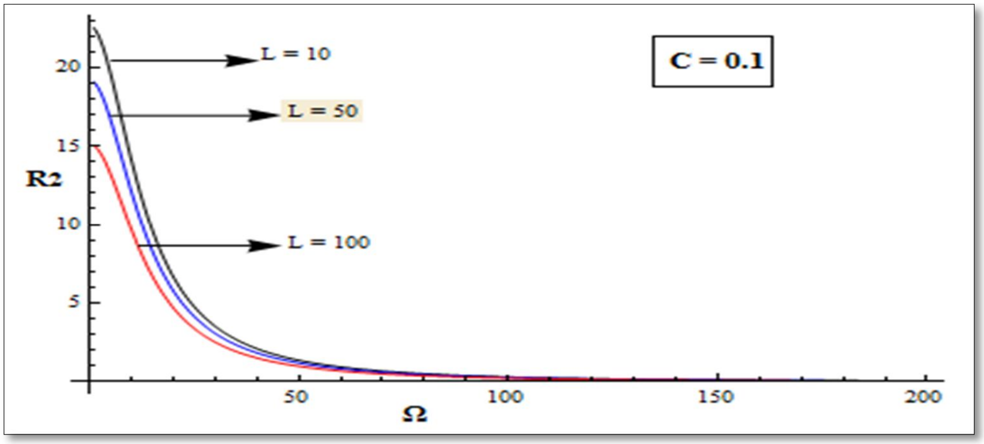

Figure 3:The plot of correction Rayleigh number R2 vs frequency of Modulation Ω for different values of Roberts number L by taking Cattaneo number as C = 0.1.

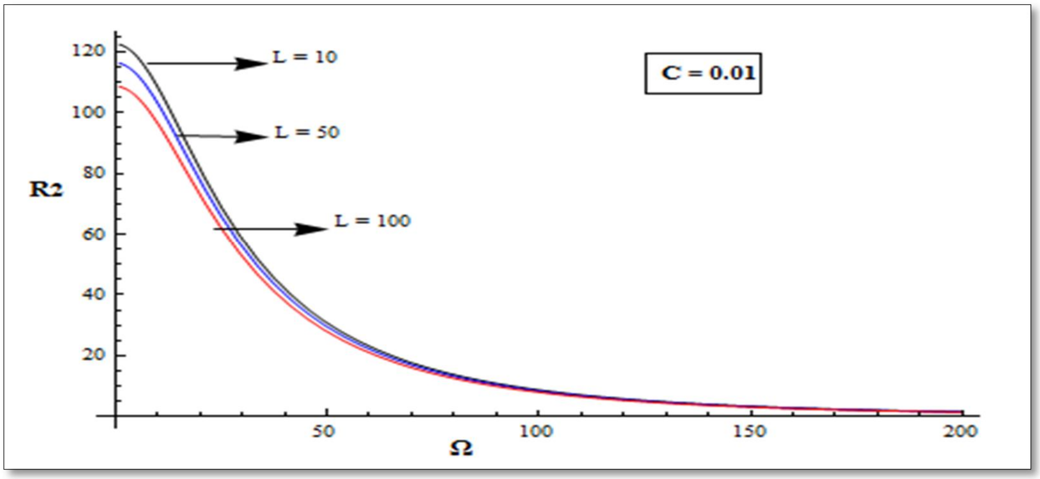

Figure 4:The plot of correction Rayleigh number R2 vs frequency of Modulation Ω for different values of Roberts number L by taking Cattaneo number as C = 0.01.

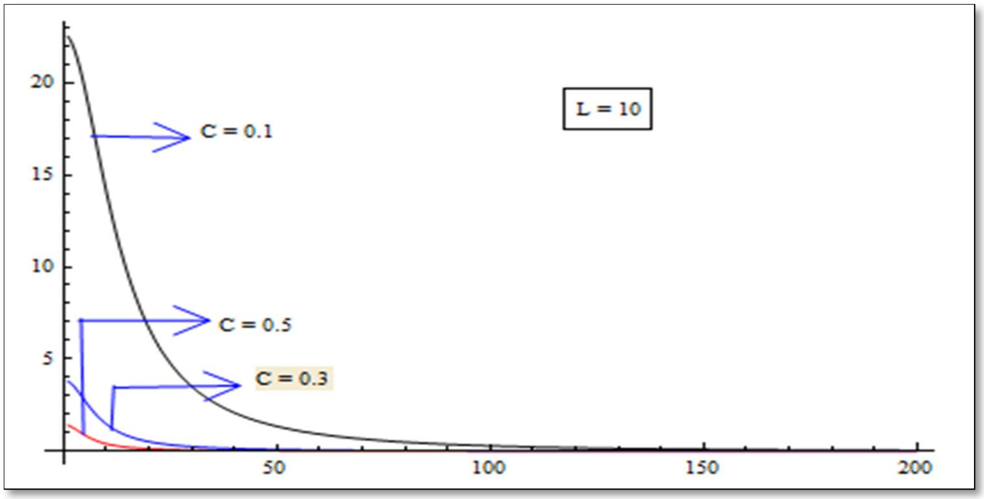

Figure 5:The plot of correction Rayleigh number R2 vs frequency of modulation Ω for different values of cattaneo number C with L = 10.

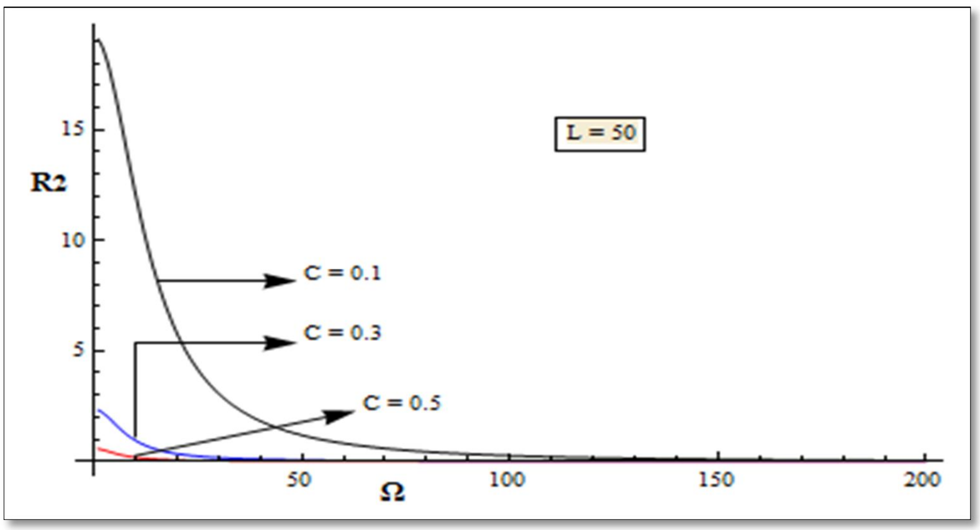

Figure 6:The plot of correction Rayleigh number R2 vs frequency of modulation Ω for different values of cattaneo number C with L = 50.

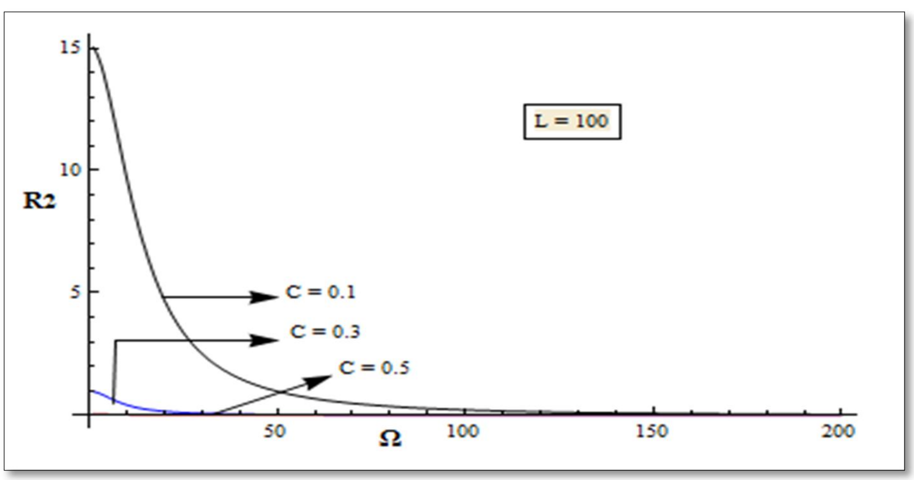

Figure 7:The plot of correction Rayleigh number R2 vs frequency of modulation Ω for different values of cattaneo number C with L = 100.

4. CONCLUSION

From the study we conclude that the gravity modulation leads to delay in convection. Thus, it is also possible to regulate heat transfer with the help of time-periodic vertical oscillations and applied electric field. As the value of C increases the critical Rayleigh number becomes stable.

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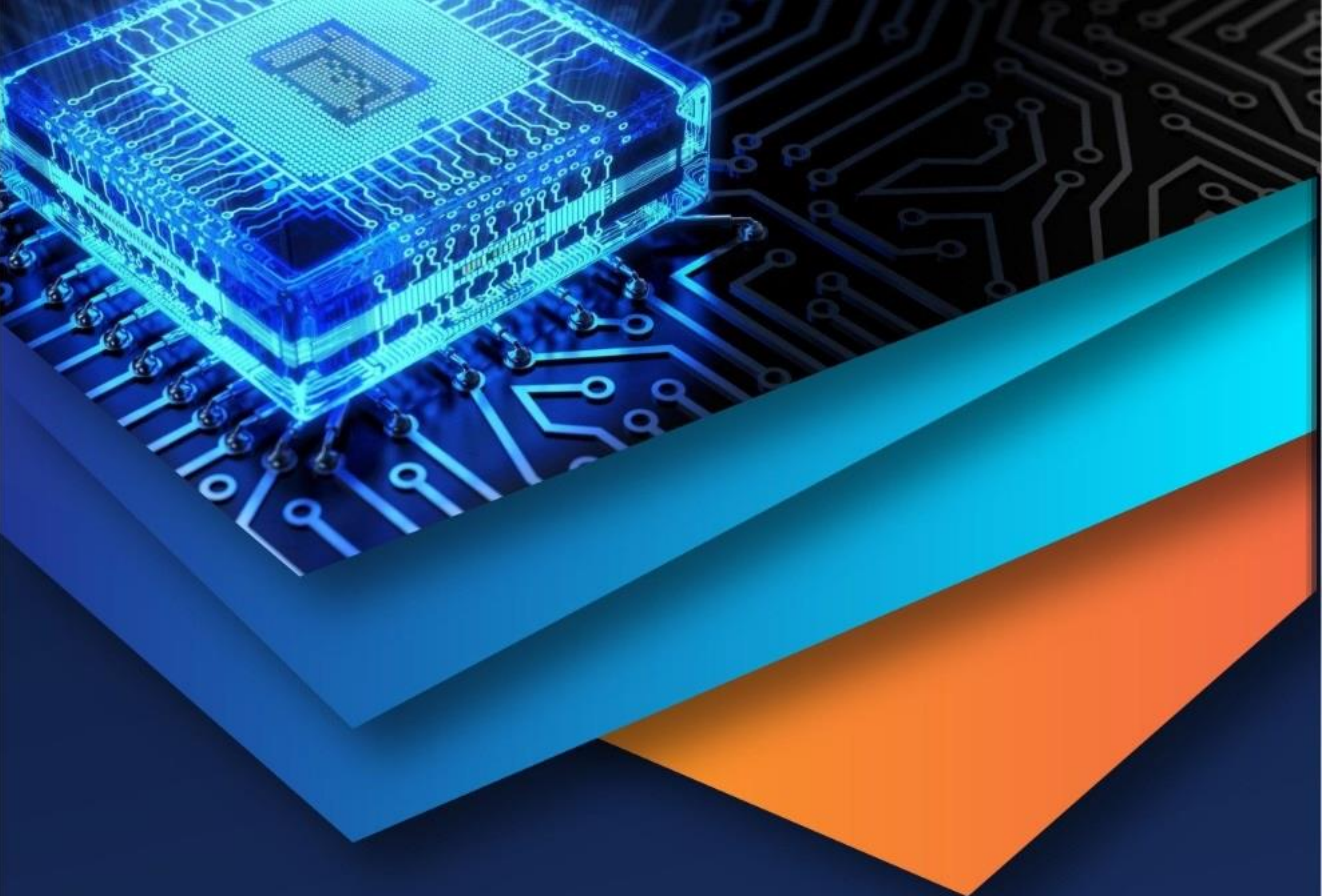

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