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Mathematical Modeling & Theoretical Analysis of Vibration Control Using Shape Memory Alloy

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Abstract: The main purpose of this study is to evaluate the dynamic performance characteristics of a stiffness controlled Adaptive tuned vibration absorber (ATVA). A base excited single degree of freedom structure coupled with an ATVA model is adopted as the baseline model for our analysis. Based on the material characterization of SMA ATVA parameters are optimized. Using mathematical model and dynamic results of excitation model by varying the Temperature of SMA wire i.e. stiffness, the dynamic performance of ATVA evaluated by peak transmissibility. The results showed that the peak transmissibility of ATVA is nearly 48.7 % lower than low temp. Results further showed that increase in stiffness reduces the vibration levels at higher temperature.
Keywords: Shape memory Alloy, ATVA, transmissibility stiffness, Resonance etc.

I. INTRODUCTION

A passive vibration absorber generally acts to minimize structural vibration at a specific frequency associated with a disturbance or a structural vibration. In real life application the frequency is not constant it changes with the time so the passive vibration absorber fails as it has only single frequency to be controlled. In order to adjust with the varying applied frequency, the vibration absorber has to be tuned with the externally applied frequency & frequency of vibration has to be change with respect to the external frequency. So, this spring-mass system should be accurately tuned with the system this can be achieved by mass tuning or stiffness tuning Thus, an actively tuned vibration absorber should perform better than a passive one and could be made lighter.

Attempts have been made to develop the adaptive tuned vibration absorber (ATVA) using the multifunctional materials, sometimes called as active materials such as Electro-Rheological Fluid (ERF), Magneto-Rheological Fluid (MRF). Shape Memory Alloys (SMAs) are a unique class of shape memory materials with the ability to recover their shape when the temperature is increased. An increase in temperature can result in shape recovery even under high applied loads therefore resulting in high actuation energy densities. So, when this shape changes occurring there is change in the Elasticity as well as in the stiffness of the various phases. This changing stiffness property utilized in the tuned vibration absorber ,as the continues fluctuation in the stiffness on the virtue of the heating occurring the SMA wires such as NITINOL, tuned with the range of frequencies with which the system vibrates in this ways the continues adaption in the frequencies can be achieved.

The adaptive tuned vibration absorber (ATVA) is an adaptive passive vibration control device similar to a TVA but with adaptive elements that can be used to change the ATVA tuned condition. Most commonly, adaptive stiffness elements are used to vary the device natural frequency such that an ATVA may be tuned to track uncertain or time varying excitation frequencies. At the same time, the ATVA is generally simpler than completely active approaches due to the less stringent actuator demands. For an ATVA, actuator bandwidth is related to the rate of change of excitation frequency.

II. MATHEMATICAL MODEL FOR BASE EXCITATION MODEL

A. *Mathematical Model for 2 DOF Systems:-*

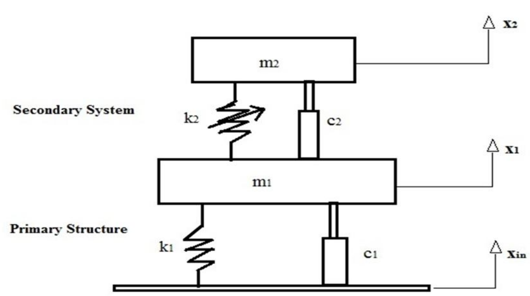

Fig. 1: Conventional Passive TVA Model (Base Excited Model)

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As shown in Fig. 1 above system consists of Base Excitation model of Two degree freedom system consists of, *[3]*

m_1= mass of primary system
m_2= mass of secondary system
k_1=stiffness of primary spring
k_2=stiffness of secondary system
c_1, c_2= damping factor of primary and secondary system.
Fig.1 is used to derive the dynamic equations of motion for semi active model. The equations of motion that describes the system are,

$$\begin{bmatrix} m1 & 0 \\ 0 & m2 \end{bmatrix}\begin{pmatrix} \ddot{x1} \\ \ddot{x2} \end{pmatrix} + \begin{bmatrix} c1+c2 & -c2 \\ -c2 & c2 \end{bmatrix}\begin{pmatrix} \dot{x1} \\ \dot{x2} \end{pmatrix} + \begin{bmatrix} k1+k2 & -k2 \\ -k2 & k2 \end{bmatrix}\begin{pmatrix} x1 \\ x2 \end{pmatrix} = \begin{bmatrix} c1\dot{x}in + k1xin \\ 0 \end{bmatrix} \quad (1)$$

Assume $$X1 = X1e^{st} \quad (2)$$
$$X2 = X2e^{st} \quad (3)$$
$$Xin = xine^{st} \quad (4)$$

Where, s=jw and w is the driving frequency substituting Eqns. (2)-(4) into Eqn. (1) yields,

$$\begin{bmatrix} m1s^2 + (c1+c2)s + k1 + k2 & -c2s - k2 \\ -c2s - k2 & m2s^2 + c2s + k2 \end{bmatrix}\begin{pmatrix} x1 \\ x2 \end{pmatrix} = \begin{bmatrix} c1\dot{x}in + k1xin \\ 0 \end{bmatrix} \quad (5)$$

Using Cramer's Rule, The amplitudes X1 and X2 can be solved as,

$$X1 = \frac{\begin{vmatrix} (c1s+k1)Xin & -c2s-k2 \\ 0 & m2s^2+c2s+k2 \end{vmatrix}}{detA} = \frac{(c1s+k1)(m2s^2+c2s+k2)}{detA} \quad (6)$$

$$X2 = \frac{\begin{vmatrix} m1s^2+c2s+k2+k2 & (c1s+k1)Xin \\ -c2s-k2 & 0 \end{vmatrix}}{detA} = \frac{(c1s+k1)(c2s+k2)}{detA} \quad (7)$$

Where det (A) = $(m1s^2+c1s+c2s+k1+k2)(m2s^2+c2s+k2)-(c2s+k2)^2$

Therefore the Transmissibility Equations Become,

$$\frac{X1}{xin} = \frac{(c1s+k1)(m2s^2+c2s+k2)}{(m1s^2+(c1+c2)s+k1+k2)(m2s^2+c2s+k2)-(c2s+k2)^2} \quad (8)$$

$$\frac{X2}{xin} = \frac{(c1s+k1)(c2s+k2)}{(m1s^2+(c1+c2)s+k1+k2)(m2s^2+c2s+k2)-(c2s+k2)^2} \quad (9)$$

Here, K_2 is the equivalent stiffness of the secondary system which is varying as shown in Fig.1 as there is SMA wire of which stiffness is varying so in Fig.1 an arrow has shown to indicate the varying stiffness therefore,

$$K_2 = Ks + K_{sma} \quad (10)$$

K_s= stiffness of spring.
Ksma= stiffness of SMA wire.
As, stiffness of SMA changes with the temperature i.e. Elastic modulus of SMA changes with the Temperature so, there are many mathematical models explained but most reliable model as far as material properties of NiTi Alloy concerned is Lagoudas model, [7]
So, by Lagoudas model,

$$K_{sma} = \frac{A \times Ef}{l} \quad \text{and} \quad K_{sma} = \frac{A \times Er}{l} \quad (11)$$

Ef=Elastic modulus during forward transformation.
E_r=Elastic modulus during reverse transformation.

A= Area of SMA wire
l= length of SMA wire

$$Ef = \left(\frac{Em-Ea}{2}\right)\cos\left(\pi\frac{T-As}{Af-As}\right) + \left(\frac{Em+Ea}{2}\right) \qquad \text{For, As} < T < Af \qquad (12)$$

$K_{sma,}$ during forward transformation is,

$$K_{sma}=\frac{A\times\left(\frac{Em-Ea}{2}\right)\cos\left(\pi\frac{T-As}{Af-As}\right)+\left(\frac{Em+Ea}{2}\right)}{l} \qquad (13)$$

$$Er = \left(\frac{Em-Ea}{2}\right)\cos\left(\pi\frac{T-Mf}{Ms-Mf}\right) + \left(\frac{Em+Ea}{2}\right) \qquad \text{For, } Mf < T < Ms \qquad (14)$$

Now, K_{sma} for reverse transformation is,

$$K_{sma}=\frac{A\times\left[\left(\frac{Em-Ea}{2}\right)\cos\left(\pi\frac{T-Mf}{Ms-Mf}\right)+\left(\frac{Em+Ea}{2}\right)\right]}{l} \qquad (15)$$

$$K_2 = K_s + K_{SMA}$$

So, Absorber frequency is, $\omega 2=\sqrt{\frac{K2}{m2}}$ (16)

so, in order to determine the stiffness different material properties noted below should be known,

B. Material characterization of SMA

Em= Elastic modulus at low temperature i.e. Low temperature
Ea= Elastic modulus at High temperature i.e. High Temperature
Ms= Martensitic start temperature Mf= Martensitic finish temperature
As= Austenitic start temperature Af = Austenitic finish temperature

By using DeweFRF software natural frequency of wire and damping factor noted for temperature T= 55º C. There is resonance point at 65.79 Hz and corresponding damping factor is 0.23895. From Table 1, Avg. Natural frequency = 62.42 Hz, Avg. damping factor = 0.1324.

Table 1: Summary of FRF method

Sr. No.	Temp. T(ºc)	Natural Frequency (Hz)	Damping Factor	Stiffness K(KN/m)	Elastic Modulus (Gpa)
1	30	56.5524	0.2785	198.88	25.36
2	35	57.8241	0.26873	207.93	26.48
3	40	62.448	0.1998	242.51	30.88
4	45	66.66	0.17957	276.33	35.19
5	50	63.61	0.14526	248.08	31.59
6	55	62.42	0.1324	251.14	31.86

From Table 1 It is noted that there is increase in stiffness from 198.88 (KN/m) to 251.14 (KN/m) at low temperature and High temperature respectively i.e. by 21 % and average damping factor is 0.1900. There is repeatability in the stiffness values with respect to Temperature so by using curve fitting relation between Temperature (T) and stiffness (k) evaluated as follows

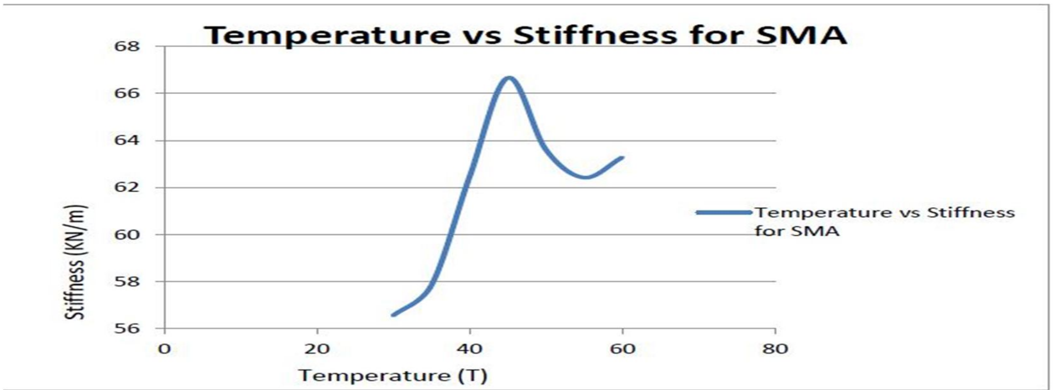

Fig. 2: Temperature vs. Stiffness for SMA

As shown in Fig.2 there is increase in stiffness (K) with temperature (T) from 30º C to 45º C but after that there is not linear relation between temperatures and stiffness. So while tuning SMA should be operated between 30º C to 45º C temperature so the relation for this plotted as shown in Fig.3.

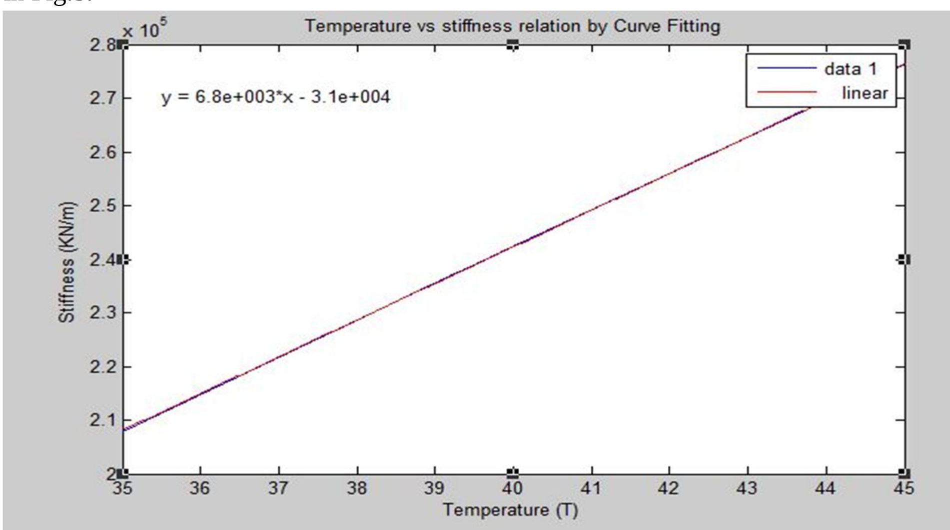

Fig.3 Curve fitting for Temperature Vs Stiffness plot

The best fit by curve fitting is linear and the equation is,

$$\text{Stiffness} \quad K_{SMA} = 6.8\text{x}10^3(T) - (31\text{x}10^3) \qquad (17)$$

So, while testing for required stiffness this relation will be used and by varying the temperature stiffness will be varied. Final mathematical model used for base excitation model will be,

Therefore from the Equation 8 and 9,Transmissibility Equations Becomes,

$$\frac{X1}{xin} = \frac{(c1s+k1)(m2s^2+c2s+k2)}{(m1s^2+(c1+c2)s+k1+k2)(m2s^2+c2s+k2)-(c2s+k2)^2}$$

$$\frac{X2}{xin} = \frac{(c1s+k1)(c2s+k2)}{(m1s^2+(c1+c2)s+k1+k2)(m2s^2+c2s+k2)-(c2s+k2)^2}$$

Here,

$$K_2 = Ks + K_{sma} \qquad (18)$$

$$K_2 = Ks + (6.8\text{x}10^3(T) - (31\text{x}10^3)) \qquad (19)$$

Ks= stiffness of secondary springs

T= temperature of SMA wire

Therefore by varying the temperature, required stiffness achieved and tuning of frequency occurs

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Table 2: Material properties of SMA

Sr. No.	Property	Value
1	Elastic Constant For Martensite Phase(Em)	25.36× $10^3 Mpa$
2	Elastic Constant For Austenite Phase (Ea)	31.98× $10^3 Mpa$
3	Austenite Start Temp.(As)	38ºc
4	Austenite Finish Temp.(Af)	55ºc
5	Martensite Start Temp.(Ms)	38ºc
6	Austenite Finish Temp.(Mf)	30ºc

Table 2, Shows the material properties of SMA wire using FRF method.

III. THEOROTICAL DYNAMIC PERFORMANCE ANALYSIS Of ATVA

This section contains the results of the parametric studies performed on the baseline mathematical model. This parametric studies the understanding of the dynamics of the TVAs as their parameter changes. It uses displacement using transmissibility equations to evaluate TVA performance.

A. Performance Analysis of the Passive TVA

Fig 4 shows the frequency response X1 of primary system using displacement transmissibility or the ratio between the output (X_1) and the input displacement (X_{in}) of the primary structure without TVA in frequency domain.

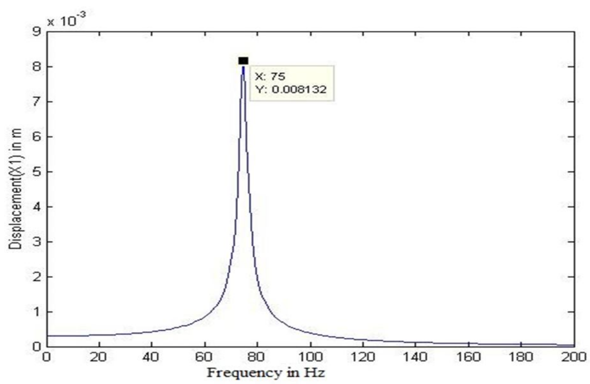

Fig. 4: Frequency Response of X1 for SDOF system

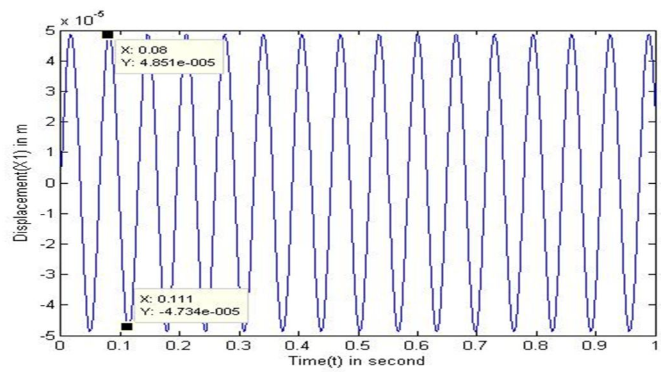

Fig. 5: Time Response of X1 for SDOF system

As shown in Fig. 4 (see appendix 2) there is single peak at 75 Hz i.e. there is resonance of Single degree of freedom system at 75 Hz frequency. For this value of frequency the Displacement of primary system i.e. X1 is 8.132 mm when Xin =0.3 mm. Fig.5 shows the time response of primary system.

B. Performance Analysis of the SMA ATVA

As shown in Fig.4 there is resonance at 75 Hz frequency so, ATVA designed such that it suppresses the extreme vibrations of the primary system. Fig.6. shows the displacement (X1) of primary system when ATVA attached with primary system. (See Appendix 4)

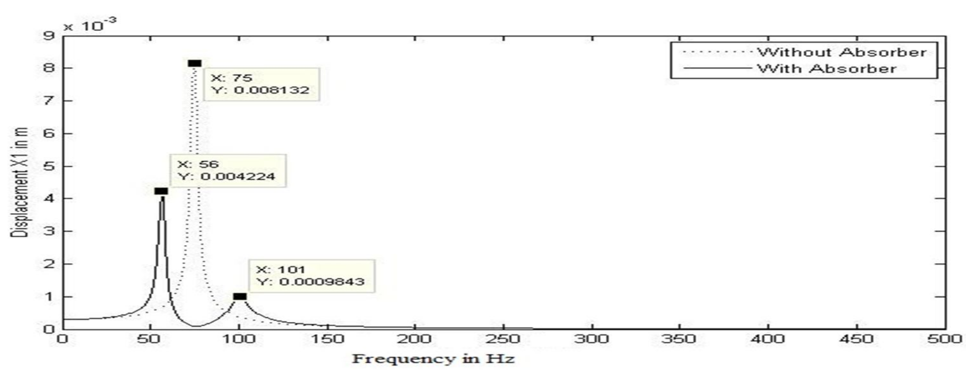

Fig. 6: Frequency Response of primary system (X1) of two DOF and SDOF system

In order to reduce the vibrations of the primary system SMA wire along with the two secondary springs of stiffness 10 kN/m in parallel with SMA wire temperature of 35 ºC used. The required temperature recorded and performance of the system with SMA TVA noted. While tuning as shown in fig.6, we can see there are two peaks one at 56 Hz and another at 101 Hz but the amplitude is less compared with single DOF system. Here, displacement of X1 is 4.2 mm at 56 Hz and 0.98 mm at 101 Hz. There is frequency shift from 75 Hz to 56 Hz and 101 Hz. The valley in between the frequency 56 and 101 Hz is the feasible range to operate the ATVA as there will not be resonance and vibration amplitude will be less. Again as temperature increases from 35ºC to 45ºC the valley between the peaks widens and vibration amplitude decreases by 50 %.

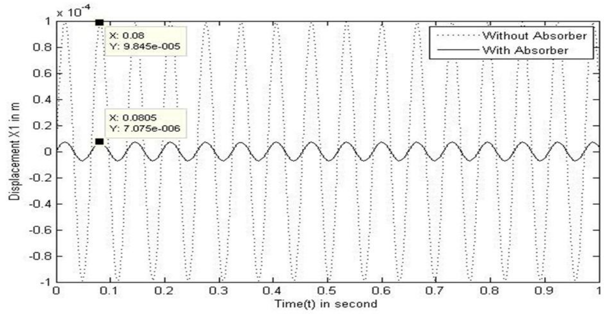

Fig.7 Time Response of primary system of two DOF and SDOF systems

Similarly, Fig. 7 shows the time response of X1 for primary system with ATVA and without ATVA. As shown in Fig.7firstly the displacement X1 of the primary system with ATVA is less as compared with the SDOF system.

IV. CONCLUSION

A Base excited tuned vibration absorber is modeled & analyzed. The material characterization of shape memory alloy has shown that there is shift of 25.36 % in elasticity. This in term changes the stiffness of the shape memory alloy, so the total change in stiffness is 25.36 %. So natural frequency of secondary system changes by 26 %. It has seen that by numerical study the total reduction in the deflection of the primary system is48.7 %. So by attaching the secondary system which itself has SMA wire the tuning can be done of the frequencies & vibration levels can be reduced.

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