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Fuzzy Membership Function- Different Views, Context Dependence and Number of Occurrence - A Theoretical Overview

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Abstract: Fuzzy Membership function is the most important part in the context of uncertainty. The concept of ambiguity or uncertainty is inherent in the fuzzy sets in the form of their respective membership functions. A lot of approaches towards the evaluation, interpretation and application of Fuzzy Membership Function has been enriched the literature. This particular work deals with different aspects of Fuzzy Membership Functions. It accumulates the views from different perspectives, its context dependence and occurrence cardinality in respective problems.
Keywords: Fuzzy Membership Function, Fuzzy Operator, Context Dependence, Likelihood View

I. INTRODUCTION

Fuzzy logic was introduced in 1965 by Lotfi Zadeh, which exposes a mathematical tool for modelling. It provides the access to definite techniques to overcome the imprecision and information granularity.

Research shows through a wealth of examples the ways in which the theory can be applied to the solutions of realistic problems, particularly in the realm of decision analysis, and motivates the theory by applications in which Fuzzy sets play an essential role. So we should approach by Fuzzy logic to those cases where intuitive knowledge about behavior is present while no deterministic algorithm is available. More research on Fuzzy entropy, human behavior, and proper language interpretation can lead to a new system of understanding, where perhaps causality can be remodeled properly. Zadeh (1965, 1971, 1981) tried to accumulate different predicates as Fuzzy functions whereas Kosko emphasized on getting the hypercube whose vertices represent those predicates. According to him the association rule of Fuzzy set theoretic results with the properties of hypercube constructs the causal model. Now Fuzzy logic and probabilistic logic are mathematically similar- both giving truth values in [0, 1], but philosophically distinct. Probabilistic logic corresponds to probability, likelihood where the former corresponds to "degrees of truth". Since the difference is there, they construct different models for real life situations. Two different kinds of vagueness are there: one-dimensional and multidimensional. The first type may result in artificiality, but unable to remove complexity. Now complexities arise in the collation of multidimensional vagueness. For example, the persons above the age of 60 can be stated 'old', whereas in multidimensional case, a Fuzzy set of 'older persons' can be assigned. The most important thing to notice is that the implementation of Fuzzy logic cannot avoid regimentation of multidimensional vagueness and for that reason this type of collation may improvise new kinds of complexities. So complexities are there in huge mass in the framework of Fuzzy logic.

In literature mostly the operators proposed by Zadeh (1971, 1981) have been a topic of interest. Few of the researchers considered MFs as the research area. Others always suppose that somehow the membership grades of the points are obtained. But it should also be noticed that majority of the problems arising in Fuzzy set theory are due to the lack of knowledge of the word "Fuzzy". In this respect there is a strong ground of not neglecting the origin of membership functions. Dombi (1990), very properly stated, "Working without membership functions can be compared with dealing with probability theory where we have a calculus (or, algebra) without probability density functions."

The rest of the paper is modelled as follows. Section 2 throws some light on the basics of fuzzy sets.

II. BASICS OF FUZZY SETS

A ***Fuzzy set*** $\tilde{A}$ of a universal set X is defined by its membership function $\mu_{\tilde{A}}: X \to [0,1]$.

Given a Fuzzy set $\tilde{A}$ defined on X and any number $\alpha \in$ [0, 1], the ***α-cut*** A_α is the crisp set $A_\alpha = \{x: \mu_{\tilde{A}}(x) \geq \alpha\}$ and the ***strong α-cut*** $A_{\alpha+}$ is the crisp set $A_{\alpha+} = \{x: \mu_{\tilde{A}}(x) > \alpha\}$.

That is the α-cut for a Fuzzy set $\tilde{A}$ is the crisp set A_α that contains all the members of the universal set X whose membership grades in $\tilde{A}$ are greater than or equal to the specified values of α.

The support of a Fuzzy set $\tilde{A}$ within a universal set X is the crisp set that contains all the elements of the universal set X that have non-zero membership grades in $\tilde{A}$ Clearly the support of $\tilde{A}$ is the same as the strong α-cut of $\tilde{A}$ for α = 0, i.e., support($\tilde{A}$) = A_{0+}.

The 1-cut A_1 is called the *core* of $\tilde{A}$.

The height $h(\tilde{A})$ of a Fuzzy set $\tilde{A}$ is the largest membership grade obtained by any element in that set. Formally, $h(\tilde{A}) = \sup_{x \in X} \mu_{\tilde{A}}(x)$.

If X is a collection of objects denoted by x, then a Fuzzy set $\tilde{A}$ on X is the set of ordered pairs $\tilde{A} = (x, \mu_{\tilde{A}}(x))$ such that $x \in X$ where $\mu_{\tilde{A}}(x)$ is a membership function and $0 \le \mu_{\tilde{A}}(x) \le 1$.

If $\max_{x \in X} \mu_{\tilde{A}}(x) = 1$, then Fuzzy set $\tilde{A}$ is called normal.

Thus a sub-normal Fuzzy set that contains only partial members, but no full members. Actually the notion of these sets introduces the grey area between two extremes: non empty sets and empty sets, in the classical sense.

An example of a meaningful sub-normal Fuzzy Set is "the set of perfect people". These Sets are normally generated during the Fuzzy rule base reasoning process.

Among the various types of Fuzzy sets, of special significance are Fuzzy sets that are defined on the set $\mathbb{R}$ of real numbers. Membership function of this form $\mu_A : \mathbb{R} \to [0,1]$ clearly has a quantitative meaning and may, under certain conditions, be viewed as Fuzzy numbers. To qualify as a Fuzzy number, a Fuzzy set $\tilde{A}$ on $\mathbb{R}$ must possess at least the following three properties:

1) $\tilde{A}$ must be a normal Fuzzy set; *i.e.*, there should exist $x \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x) = 1$.
2) A_α must be a closed interval for every $\alpha \in (0,1]$,
3) The support of $\tilde{A}$, A_{0+} must be bounded.

Since α-cuts of any Fuzzy number are required to be closed intervals for all $\alpha \in (0,1]$, every Fuzzy number is a convex Fuzzy set. The inverse is not necessarily true.

$\tilde{A}$ is a Fuzzy number if and only if there exists closed interval $[a,b] \ne \Phi$ such that

$$\mu_{\tilde{A}}(x) = \begin{cases} \ell(x) \text{ for } x \in (-\infty, a) \\ 1 \quad \text{for } x \in [a, b] \\ r(x) \text{ for } x \in (b, \infty). \end{cases}$$

where [a, b] is the core of $\tilde{A}$, ℓ is a function from $(-\infty, a)$ to [0, 1], that is monotonic increasing, continuous from the right, and is such that $\ell(x)=0$ for $x \in (-\infty, a - w_1)$, r is a function from (b, ∞) to [0, 1], that is monotonic decreasing, continuous from the left, and such that r(x) = 0 for $x \in (b + w_2, \infty)$. Basically this form allows us to define Fuzzy numbers in a piecewise manner. We call this Fuzzy number of the type LR and denote it by the notation $\tilde{A} = (a, b, w_1, w_2)_{LR}$.

A Fuzzy set $\tilde{A}$ is called trapezoidal Fuzzy number with tolerance interval [a, b], left width w_1 and right width w_2 if its membership function has the following form:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \dfrac{a - x}{w_1}, \text{ if } a - w_1 < x < a \\ 1, \text{ if } a \le x \le b \\ 1 - \dfrac{x - b}{w_2}, \text{ if } b < x < b + w_2 \\ 0, \text{ otherwise.} \end{cases}$$

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Let us use the same notation $\tilde{A} = (a, b, w_1, w_2)$ to denote Fuzzy trapezoidal number.

Now by condition (2) one can write $\tilde{A}_\alpha = [\tilde{A}^L_\alpha, \tilde{A}^U_\alpha]$.

A. *Fuzzy arithmetic*

Fuzzy arithmetic is based on two properties of Fuzzy numbers:

1) each Fuzzy set, and thus also each Fuzzy number, can fully and uniquely be represented by it's α- cuts and
2) α- cuts of each Fuzzy number are closed intervals of real numbers for all $\alpha \in (0, 1]$.

These properties enable us to define arithmetic operations on Fuzzy numbers in terms of arithmetic operations on their α- cuts (i.e., arithmetic operations on closed intervals).

Let * denote any of the four arithmetic operations and let $\tilde{A}$, $\tilde{B}$ denote Fuzzy numbers.

Then a Fuzzy set is defined on $\mathbb{R}$, $\tilde{A} * \tilde{B}$ by the membership function

$$\mu_{\tilde{A}*\tilde{B}}(z) = \sup_{z=x*y} \min[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)] \ \forall z \in \mathbb{R}.$$

More specifically, for all $z \in \mathbb{R}$ define,

$$\mu_{\tilde{A}+\tilde{B}}(z) = \sup_{z=x+y} \min[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)], \mu_{\tilde{A}-\tilde{B}}(z) = \sup_{z=x-y} \min[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)],$$

$$\mu_{\tilde{A}.\tilde{B}}(z) = \sup_{z=x.y} \min[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)], \mu_{\tilde{A}/\tilde{B}}(z) = \sup_{z=x/y} \min[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)].$$

If $\tilde{A}$ and $\tilde{B}$ are continuous Fuzzy numbers, then $\tilde{A} * \tilde{B}$ is also a continuous Fuzzy number. The above results can also be written in the form:

$$(\tilde{A}+\tilde{B})_\alpha = [\tilde{A}^L_\alpha + \tilde{B}^L_\alpha, \tilde{A}^U_\alpha + \tilde{B}^U_\alpha],$$

$$(\tilde{A}-\tilde{B})_\alpha = [\tilde{A}^L_\alpha - \tilde{B}^L_\alpha, \tilde{A}^U_\alpha - \tilde{B}^U_\alpha],$$

$$(\tilde{A}\tilde{B})_\alpha = [\min\{\tilde{A}^L_\alpha.\tilde{B}^L_\alpha, \tilde{A}^L_\alpha.\tilde{B}^U_\alpha, \tilde{A}^U_\alpha.\tilde{B}^L_\alpha, \tilde{A}^U_\alpha.\tilde{B}^U_\alpha\}, \max\{\{\tilde{A}^L_\alpha.\tilde{B}^L_\alpha, \tilde{A}^L_\alpha.\tilde{B}^U_\alpha, \tilde{A}^U_\alpha.\tilde{B}^L_\alpha, \tilde{A}^U_\alpha.\tilde{B}^U_\alpha\}\}],$$

$$(\tilde{A}/\tilde{B})_\alpha = [\min\{\frac{\tilde{A}^L_\alpha}{\tilde{B}^L_\alpha}, \frac{\tilde{A}^L_\alpha}{\tilde{B}^U_\alpha}, \frac{\tilde{A}^U_\alpha}{\tilde{B}^L_\alpha}, \frac{\tilde{A}^U_\alpha}{\tilde{B}^U_\alpha}\}, \max\{\frac{\tilde{A}^L_\alpha}{\tilde{B}^L_\alpha}, \frac{\tilde{A}^L_\alpha}{\tilde{B}^U_\alpha}, \frac{\tilde{A}^U_\alpha}{\tilde{B}^L_\alpha}, \frac{\tilde{A}^U_\alpha}{\tilde{B}^U_\alpha}\}].$$

We know that $\tilde{A}$ is a crisp number with value m if its membership function,

$$\mu_{\tilde{A}}(r) = \begin{cases} 1 \text{ if } r = m \\ 0 \text{ if } r \neq m. \end{cases}$$

The notation $\tilde{1}\{m\}$ is used to represent the crisp number with value m. Clearly,

$$[\tilde{1}\{m\}]^L_\alpha = [\tilde{1}\{m\}]^U_\alpha = m \ \forall \alpha \in [0,1].$$

III. DIFFERENT ASPECTS OF FUZZY MEMBERSHIP FUNCTIONS

In this section some unfold aspects of Fuzzy Membership Functions are discussed. Few researches emerged during recent days on these theoretical components. The demonstration is theoretical and the philosophy behind the approach is just to enhance the provision of setting or fitting or interpolating better Fuzzy Membership Functions at the time of applying to real life situations.

A. *Description of different views of Membership Functions*

In science and technology many problems related to reality are of great importance. To clear the vagueness in these problems, the membership grades are considered from different views (likelihood theory, random set theory, similarity theory, utility theory, measurement theory) by different authors.

1) *The Likelihood View:* Hisdal (1985, 1988) proposed a mathematical model or construction of Membership Functions on the basis of Likelihood concept of the graded membership. Later Mabuchi (1992) and Thomas (1979, 1995) followed and developed this concept to some extent. This model is known as TEE (Threshold, Error, assumption of Equivalence). Let us describe the model by considering a linguistic term $\tilde{A}$ and an element x in the universe of discourse. Now, comments on three subjects are asked from the experts:

a) A linguistic scale is formed for $\tilde{A}$. If the scale is {$LT_i(\tilde{A})$}, where *i*= 1, 2, …, n; then the experts are asked to give response in terms of any one of the linguistic terms of the scale.

b) Is x exactly $\tilde{A}$?

c) What is the degree of belongingness of x to $\tilde{A}$?

The modified and aggregated combination of these three responses constructs the desired membership function. To be more explicit, this model deals with several sources of uncertainty, e.g., Measurement Error, Incomplete Information, Contradicting Opinion, etc.

2) *Interval Sum View:* Zadeh (1971) defined the membership function as $\mu_{\tilde{A}}(x)$ = sup { $\alpha \in (0,1]: x \in \tilde{A}_\alpha$ }, where $\tilde{A}_\alpha$ are the level cuts. Later Dubois and Prade (1989) defined the same as $\mu_{\tilde{A}}(x) = \int_0^1 \mu_{\tilde{A}_\alpha}(x)d\alpha$, where

$$\tilde{A}_\alpha = 1, \text{ if } x \in \tilde{A}_\alpha$$
$$= 0, \text{ otherwise.}$$

Since this definition involves integral, it must satisfy the continuity and measurability conditions. Thus, if $\mu_{\tilde{A}}(x) = p \in [0,1]$, according to the interval sum view, it can be said that *p* % of the mass defines a certain interval [a , b] on the universe of discourse X as an interval containing *x* on the basis of evaluating $\tilde{A}$.

So, in this view, the Membership Function is interpreted as a random set that

a) is uniformly distributed over X,

b) consists of the Lebesgue measure on [0, 1].

3) *Prototype Similarity View:* The prototype similarity view (Rosch and Mervis (1975), Lakoff (1987)) states that the membership degree of an element is its normalized similarity to an ideal element of the set. The ideal element is assumed to belong to the set to full membership degree, i.e., 1. In contrary to the subjective preference of this theory, Osherson and Smith (1981, 1982) argued that subjective probability is sufficient to rate the similarity degree of an element of the set.

On the other hand, Kempton (1981) and Zysno (1981) supported the similarity view. Kempton (1981) extended the cognitive anthropological folk classification methods using Fuzzy set theory operations. Whereas Zysno (1981) configures an alternative as the measurement device and it is assumed that the relative fuzziness stems from the cognitive inabilities of the alternative in response to compare the object with the prototype or imaginary ideal. This idea was later expanded by Zimmermann and Zysno (1985). Another interesting and worthy work was presented by Zwick, Carlstein and Budescu (1987). They experimentally verified 19 similarity measures and their performance. Metric measure is better when we try to distinguish between degrees of different similarities. Later Ruspini (1991) considered another similarity semantics where a notion of similarity is constructed on a modal logic and its accessibility relation is certainly the relation in a metric space.

4) *Utility View:* If the Fuzzy set is considered as a property, then the membership function can be viewed by considering it together with the problem of Fuzzy reasoning (Giles (1988)). Here 'Membership' is nothing but a graded truth value acquired from the logical base. Dealing with the Fuzzy sentences Giles considered asserted sentences with more importance than the merely uttered ones. The assertion emulates a pay-off function that offers more, if the statement is closer to the truth. The Utility theory approach elicits Membership Functions on interval scales.

5) *Uncertainty measurement View:* In this theory, dealing with uncertain situation in real life problems is viewed as a measurement in real line. The modeling of the problem is at first structured and then it is mapped to the real value or real interval. The imposed numeric structure allows us to go through the representation of a qualitative structure and its necessity. Now 'measurement' can be applied in two distinct areas here:

a) Measurement of Membership Functions (later it will be discussed elaborately), and

b) Measurement of degree of choosing the Fuzzy term for a single element.

In criticism to this theory, Michell (1990), H.E. Kyburg (1992) argue that the representation view of measurement, i.e., RTM should not be applied as:

i) it is based on axioms,
ii) it is unable to deliver satisfactory account of actual scientific measurement practice,
iii) there occurs confusion whether the obtained real numbers/intervals are primitive or they have come through a measurement process.
iv) errors in the measurement process cannot be incorporated.

On the other hand, Luce (1996) defends the measurement theory and denies the criticism. He strongly mentions that RTM is a well-established way to enrich the behavioural and social sciences by its full range of measurement possibilities.

B. Context Dependence of Membership Functions

Another important part, attached to the construction of Membership Functions is its context dependence. Hersh et al. (1979) showed the effects of variable context in evaluating the Membership Functions. It is claimed that the variable frequency of occurrence of the elements cannot influence the location and form of the Membership Functions. But the unique number of elements does affect this. Let us illustrate this by considering a simple example.

Consider the question: "Is 50 Km/hr is a risky speed?" while determining the Membership Function of the risk factor of a motor bike in average crowd. Now, if the question is asked repeatedly to more than one person, the output membership is always of the same form. But if the question is asked to one more person, the output varies. That is, the context changes. Thus Bilgick and Turksen (1995) claims that the Membership Function is not only a function of the object from the universe of discourse, but of the discourse as well. To be more specific, $\mu_{\tilde{A}}(x) = f_{\tilde{A}}(x, X)$.

C. The number of Membership Functions

The number of Membership Functions for a particular Fuzzy set has been acknowledged as application dependent. It means that the number (n) certainly counts on the relation of the concerned Fuzzy set or the linguistic term with the type of the application. According to Zadeh (1965), the principle of incompatibility states that if n increases in the positive direction, the exactness of the whole system will definitely increase, but it will lose the relevancy. Let m is the total number of statistical data provided in the system. Now, as soon as n approaches to m, the system starts to change its nature and in the limit, the Fuzzy system turns into a numeric system. So, it is understandable that there should be an upper bound in this case.

Cognitive psychology reveals that 7 ± 2 is the typical number for handling several entities efficiently at the short time. Statistically, it has been proved that above this number, the errors generated by the practitioner while dealing with any system, rise in a non linear graph. Another interesting thing is that most of the successful corporate applications have adopted this distinctive number as the upper bound.

So, the number of Membership Functions should not exceed this limit.

Now after taking a gentle look at the literature for the above views of Membership Functions, it can be stated that the measurement theory is more widely and rapidly accepted among these views. It may deliver week measurement procedure, but also it should not be forgotten that it deals with uncertain information. The phenomenon is different but the procedure is expected to be similar! On the other hand, in measurement theory, numbers are philosophically viewed to represent information. So, according to this theory, numbers are not 'real', a mere 'abstraction'.

IV. CONCLUSION

The present area of the research is a contribution to the development of the philosophical aspect of developing Fuzzy Membership Functions. It will help the researchers to overcome the initial formation stage of modelling and solution. More research in this particular field will develop new horizon to the membership evaluation and modelling.

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