iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 **Issue:** IV **Month of publication:** April 2018

DOI: http://doi.org/10.22214/ijraset.2018.4147

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 6 Issue IV, April 2018- Available at www.ijraset.com

K-Super Mean Labeling of Cycle Related Graphs

Tamilselvi M[1], Akilandeswari K[2], Keerthana S[3]
[1,2,3]PG and Research Department of Mathematics, Seethalakshmi Ramaswami College, Tiruchirappalli, - 620 002.

Abstract : ***Let G be a*** (p,q) ***graph and*** $f : V(G) \to \{1, 2, 3 \ldots\ldots p + q + k - 1\}$ ***be an injection. For each edge*** $e = uv$***, let*** $f^*(e) = \frac{f(u)+f(v)}{2}$ ***if*** $f(u) + f(v)$ ***is even and*** $f^*(e) = \frac{f(u)+f(v)+1}{2}$ ***if*** $f(u) + f(v)$ ***is odd ,then*** f ***is called k- Super mean labeling if*** $f(V) \cup \{f^*(e): e \in E(G)\} = \{1, 2, 3, \ldots\ldots p + q + k - 1\}$***. A graph that admits a k-Super mean labeling is called k-Super mean graph.***

In this paper we investigate k – super mean labeling of $< C_m , K_{1,n} >$ ***and*** $< C_m * K_{1,n} >$***.***

Keywords: k–Super mean labeling, k–Super mean graph, $< C_m , K_{1,n} >, < C_m * K_{1,n} >$ ***AMS Subject Classification--- 05C78***

I. INTRODUCTION

All graphs in this paper are finite, simple and undirected. The symbols V(G) and E(G) will denote the vertex set and edge set of a graph G. Terms not defined here are used in the sense of Harary [7]. Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [11]. Labeled graphs serve as useful models for a broad range of applications such as X-ray, crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. The concept of mean labeling was introduced and studied by S. Somasundaram and R.Ponraj [12]. The concept of super mean labeling was introduced and studied by D. Ramya et al. [11]. Futher some results on super mean graphs are discussed in [8, 9, 10, 14, 15]. B. Gayathri and M. Tamilselvi [13] extended super mean labeling to k-super mean labeling. In this paper we investigate k – Super mean labeling of $< C_m , K_{1,n} >$ *and* $< C_m * K_{1,n} >$. For brevity, we use k-SML for k-super mean labeling. Here k denoted as any positive integer greater than or equal to 1.

II. MAIN RESULTS

A. Definition 2.1

Let G be a (p, q) graph and $f: V(G) \to \{1,2,3, \ldots , p + q\}$ be an injection. For each edge e = uv, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$is odd, then f is called Super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2,3, \ldots , p + q\}$. A graph that admits a super mean labeling is called Super mean graph.

B. Definition 2.2

Let G be a (p, q) graph and $f: V(G) \to \{k, k + 1, k + 2, \ldots , p + q + k - 1\}$be an injection. For each edge e = uv, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$is odd, then f is called k-Super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k + 1, \ldots , p + q + k - 1\}$. A graph that admits a k-super mean labeling is called k-Super mean graph.

C. Definition 2.3

$< C_m , K_{1,n} >$ is the graph obtained from C_m and $K_{1,n}$ by identifying the vertex u_1 of C_m with the central vertex v of $K_{1,n}$.

D. Theorem 2.4

The graph $< C_m , K_{1,n} >$ is a k-Super Mean Labeling for $n \leq 2$ and $m \geq 3$.

1) *Proof:* Let G denotes the graph $< C_m , K_{1,n} >$. Let $V(G) = \{u_i ; 1 \leq i \leq m\} \cup \{v_i; 1 \leq i \leq n\}$ and $E(G) = \{e_i = (u_1 v_i); 1 \leq i \leq n\} \cup \{e_i' = (u_i u_{i+1}); 1 \leq i \leq m - 1\} \cup \{e_m' = (u_m u_1)\}$ be the vertices and edges of G respectively. For m=4, the super mean labeling of the graphs $< C_1, K_{1,1} >, < C_2, K_{1,2} >, < C_3, K_{1,3} >$ *and* $< C_1, K_{1,4} >$

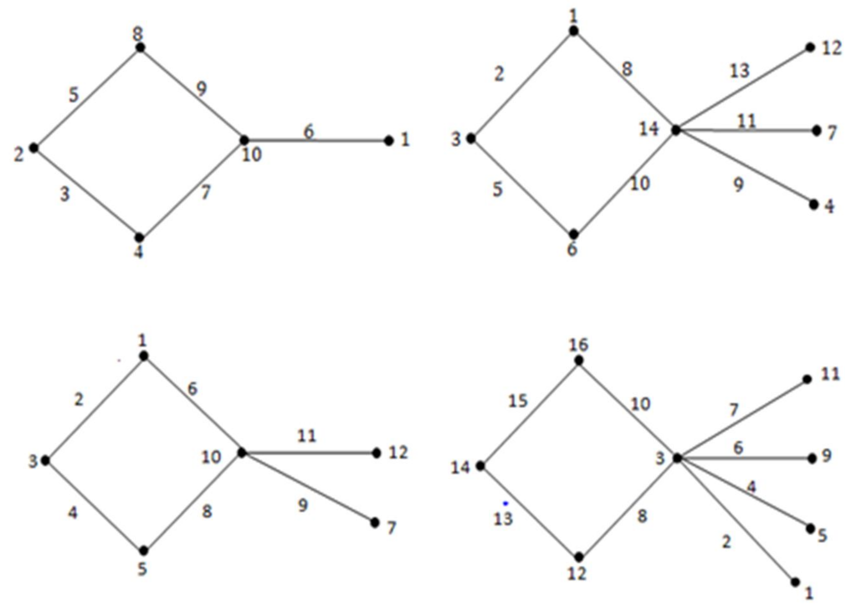


Figure 1

Define $f : V(G) \rightarrow \{k, k+1, \ldots \ldots \ldots . k+2(m+n)-1\}$

2) *Case (1):* m is odd

Let $m = 2l + 1$ for all $l \in Z^+$

For n=1, 2

$f(v_i) = k + i - 1$, $1 \leq i \leq n$

$f(u_1) = k + 2n$

$f(u_j) = k + 2n + 4j - 6$, $2 \leq j \leq l + 1$

$f(u_{l+1+j}) = k + 2n + 4l - 4j + 5$, $1 \leq j \leq l$

For n=3, 4

$f(u_1) = k + 4$

$f(u_j) = k + 2n + 4j - 2$, $2 \leq j \leq l$

$f(u_{l+j}) = k + 2n + 4l - 4j + 5$, $1 \leq j \leq l$

$f(u_{2l+1}) = k + 2n + 3$

3) *Case (2):* m is even

Let $m = 2l$ for all $l \in Z^+$

For n=1, 2

$f(v_i) = k + i$, $1 \leq i \leq n$

$f(u_1) = k + 2n$

$f(u_j) = k + 2n + 4j - 5$, $2 \leq j \leq l$

$f(u_{l+j}) = k + 2n + 4l - 3(j-1)$, $1 \leq j \leq 2$

$f(u_{l+2+j}) = k + 2n + 4l - 4j - 2$, $1 \leq j \leq l - 2$

For n=3, 4

$f(u_1) = k + 4$

$f(u_j) = k + 2n + 4j - 2$, $2 \leq j \leq l - 1$

$f(u_{l-1+j}) = k + 2n + 4l - 3(j-1) - 1$, $1 \leq j \leq 2$

$f(u_{l+1+j}) = k + 2n + 4l - 4j - 3$, $1 \leq j \leq l - 2$

$f(u_{2l}) = 2n + 4$

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 6 Issue IV, April 2018- Available at www.ijraset.com

Clearly, $\boldsymbol{f}$ induces distinct edge labels and it is easy to check that $\boldsymbol{f}$ generates a k −super mean labeling and Hence $< C_m , K_{1,n} >$ is a k −Super Mean Labeling, for all $m \geq 3$ and $n \leq 4$.

E. Example 2.5

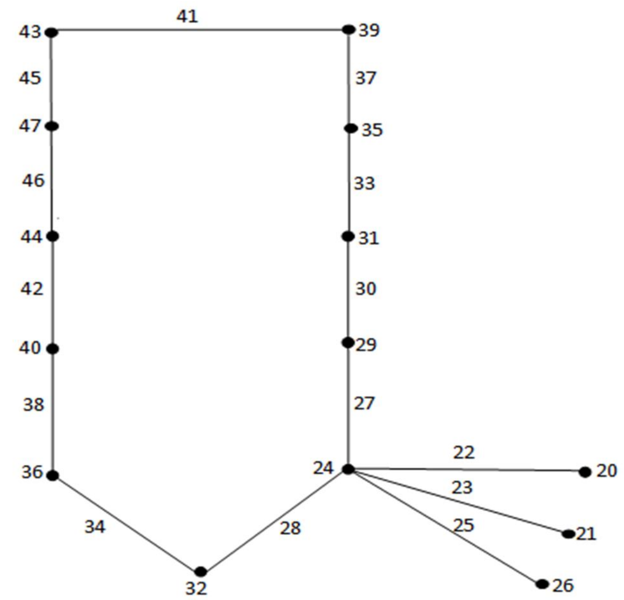


Figure 2: 20-Super mean labeling of $<C_{11}, K_{1,3}>$

F. Definition 2.6

$< C_m * K_{1,n} >$ is the graph obtained from C_m and $K_{1,n}$ by identifying any one of the vertices of C_m with a pendant vertex of $K_{1,n}$ (i.e. a non-central vertex of $K_{1,n}$).

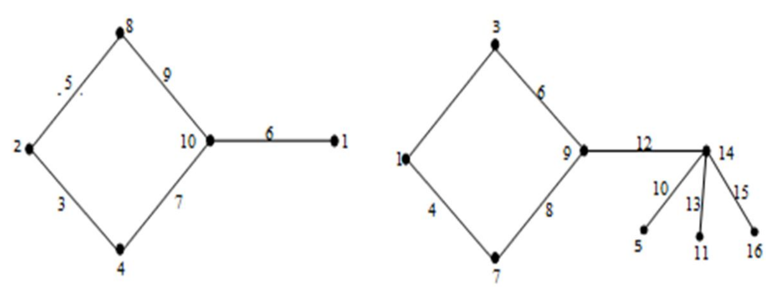
G. Theorem 2.7

The graph $< C_m * K_{1,n} >$ is a k-Super Mean Labeling for $n \leq 6$ and $m \geq 3$.

1) Proof:

Let G denotes the graph $< C_m * K_{1,n} >$. Let $V(G) = \{u_i ; 1 \leq i \leq m\} \cup \{v_i; 1 \leq i \leq n\}$ and $E(G) = \{e_i = (u_1 v_{i+1}); 1 \leq i \leq n-1\} \cup \{e_i' = (u_i u_{i+1}); 1 \leq i \leq m-1\} \cup \{e_m' = (u_m u_1)\} \cup \{e = (v v_1)\}$ be the vertices and edges of G respectively.

For m=4, the super mean labeling of the graphs $< C_m * K_{1,n} >$, n=1, 2, 3,4,5,6 are shown in Figure 3.



$<C_4 * K_{1,1}>$ $<C_4 * K_{1,4}>$

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 6 Issue IV, April 2018- Available at www.ijraset.com

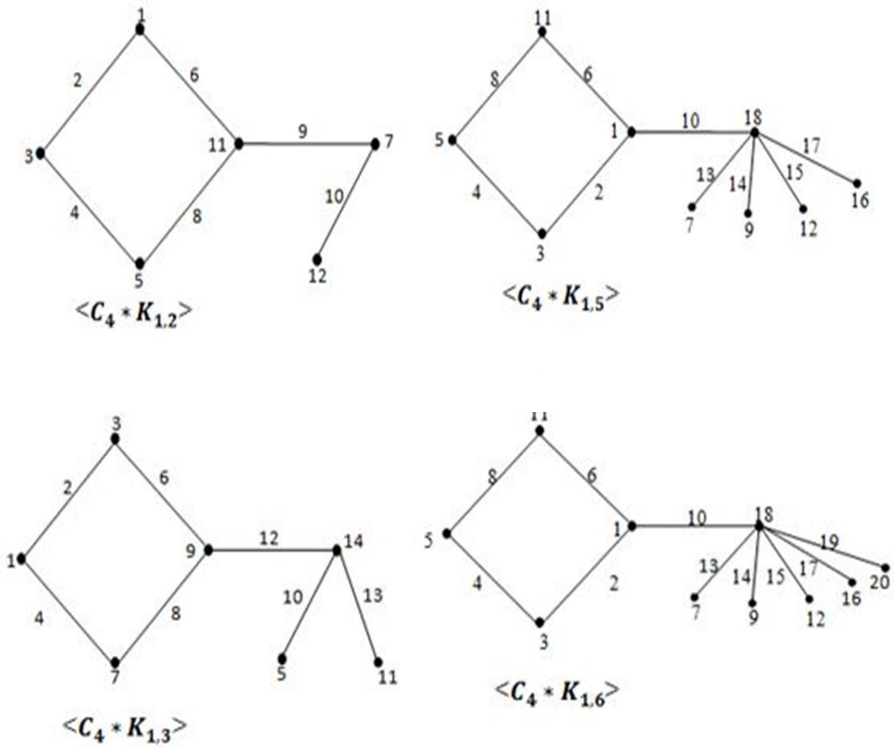


Figure 3

Define $f : V(G) \to \{k, k+1, \dots\dots\dots. k+2(m+n)-1\}$

2) *Case (1):* m is odd

Let $m = 2l + 1$ for all $l \in Z^+$

For n=1,2,3

$f(v) = k + 2n - 2$

$f(u_1 = v_1) = k + 2n$

$f(v_{n+1-i}) = k + i - 1 \qquad ,1 \le i \le n-1 \ ,n = 2,3$

$f(u_j) = k + 2n + 4j - 6 \qquad ,2 \le j \le l+1$

$f(u_{l+1+j}) = k + 2n + 4l - 4j + 5 \qquad ,1 \le j \le l$

For n=4,5

$f(v) = k + 4$

$f(u_1 = v_1) = k + 2n + 2$

$$f(v_{n+1-i}) = \begin{cases} k + i - 1 & ,1 \le i \le 2 \\ k + 6 + 3(i-3) & ,3 \le i \le n-1 \end{cases}$$

For n=6

$f(v) = k + 4$

$f(u_1 = v_1) = k + 14$

$f(v_2) = k + 11$

$f(v_3) = k + 10$

$f(v_4) = k + 6$

$f(v_5) = k + 1$

$f(v_6) = k$

$f(u_1) = k + 2n + 2$

$f(u_2) = k + 2n$

$f(u_j) = k + 2n + 4j - 7 \qquad ,3 \le j \le l+2$

$f(u_{l+2+j}) = k + 2n + 4l - 4j + 2 \quad ,1 \le j \le l - 1$

m is even

Let $m = 2l$ for all $l \in Z^+$

$f(v) = k + 2n - 2$

$f(u_1 = v_1) = k + 2n$

$f(v_{n+1-i}) = k + i - 1 \quad ,1 \le i \le n - 1 \quad ,n = 2,3$

$f(u_j) = k + 2n + 4j - 6 \quad ,2 \le j \le l$

$f(u_{l+j}) = k + 2n + 4l - 3(j - 1) - 1 \quad ,1 \le j \le 2$

$f(u_{l+2+j}) = k + 2n + 4l - 4j - 3 \quad ,1 \le j \le l - 2$

For n=4,5

$f(v) = k + 4$

$f(u_1 = v_1) = k + 2n + 2$

$$f(v_{n+1-i}) = \begin{cases} k + i - 1 & ,1 \le i \le 2 \\ k + 6 + 3(i - 3) & ,3 \le i \le n - 1 \end{cases}$$

For n=6

$f(v) = k + 4$

$f(u_1 = v_1) = k + 14$

$f(v_2) = k + 11$

$f(v_3) = k + 10$

$f(v_4) = k + 6$

$f(v_5) = k + 1$ and

$f(v_6) = 1$

$f(u_1) = k + 2n + 2$

$f(u_2) = k + 2n$

$f(u_j) = k + 2n + 4j - 7 \quad ,3 \le j \le l$

$f(u_{l+j}) = k + 2n + 4l - 4 + 3(j - 1) \quad ,1 \le j \le 2$

$f(u_{l+2+j}) = k + 2n + 4l - 4j - 2 \quad ,1 \le j \le l - 2$

Clearly, the edge labels are distinct. It can be easily verified that f generates a k −super mean labeling and Hence $< C_m * K_{1,n} >$ is a k −Super Mean Labeling, for all $m \ge 3$ and $n \le 6$.

H. Example 2.8

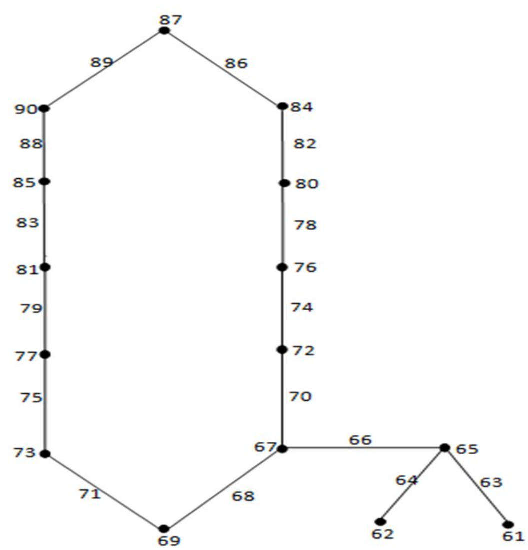


Figure 4: 60-Super mean labeling of $<C_{12} * K_{1,3}>$

III. CONCLUSIONS

Graph Labeling has its own applications in communication network and astronomy, so enormous types of labeling have grown. Towards this, k −Super Mean Labeling is also a kind of mean labeling. In this dissertation we discussed k −Super Mean Labeling of $< C_m, K_{1,n} >$ and $< C_m * K_{1,n} >$ graphs.

IV. ACKNOWLEDGMENT

I owe my heartful thanks to my guide, staff members, family members and my friends who have been the pillars, strength and source constant support.

REFERENCES

[1] G.S. Bloom, S.W. Golomb, Applications of numbered undirected graphs, Proc. IEEE, 65 (1977), 562-570.
[2] G.S. Bloom, S.W. Golomb, Numbered complete graphs unusual rulers and assorted applications, Theory and Applications of Graphs-Lecture notes in Math., Springer Verlag, New York, 642 (1978), 53-65.
[3] G.S. Bloom, D.F. Hsu, On graceful digraphs and a problem in network addressing, Congressus Numerantium, 35 (1982) 91-103.
[4] J.A. Gallian, A dynamic survey of graph labeling, Electronic Journal of Combinatorics, 18 (2015) # DS6.
[5] B. Gayathri, M. Tamilselvi, M. Duraisamy, k-super mean labeling of graphs, In: Proceedings of the International Conference on Mathematicsand Computer Sciences, Loyola College, Chennai (2008), 107-111.
[6] B. Gayathri and M. Tamilselvi, k-super mean labeling of some trees and cycle related graphs, Bulletin of Pure and Applied Sciences, Volume 26E(2) (2007) 303-311.
[7] F. Harary, Graph Theory, Addison Wesley, Massachusetts (1972).
[8] P. Jeyanthi and D. Ramya, Super mean labeling of some classes of graphs, International J. Math. Combin., 1 (2012) 83-91.
[9] P. Jeyanthi, D. Ramya and P. Thangavelu, On super mean graphs, AKCE J. Graphs Combin., 6 No. 1 (2009) 103-112.
[10] D. Ramya, R. Ponraj and P. Jeyanthi, Super mean labeling of graphs, ArsCombin., 112 (2013) 65-72.
[11] Rosa, On certain valuations of the vertices of a graph Theory of Graphs (Internet Symposium, Rome, July (1966), Gordon and Breach, N.Y. and Duhod, Paris (1967) 349-355.
[12] S. Somasundaram and R. Ponraj, Mean labeling of graphs, National Academy Science Letter, 26 (2003), 210-213.
[13] M. Tamilselvi, A study in Graph Theory- Generalization of super mean labeling, Ph.D. Thesis, Vinayaka Mission University, Salem, August (2011).
[14] M. Tamilselvi, Akilandeswari K and N. Revathi, Some Results on k- Super Mean Labeling, International Journal of Scientific Research, Volume 5 Issue 6, June 2016, P. No. 2149-2153.
[15] R. Vasuki and A. Nagarajan, Some results on super mean graphs, International J. Math. Combin., 3 (2009) 82-96.