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Test of Hypothesis: In Two Way Unbalanced Random Model

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Abstract: Consider the two way nested unbalanced random model,
$Y_{ijk} = \mu + \alpha_i+\beta_{ij} +e_{ijk}$; $i = 1,2...p$, $j = 1, 2.. q$ and $k=1,2,...n_{ij}$
where μ, is a constant and the random variables α_i, β_{ij} and e_{iik} are independently and normally distributed with means zero and variances σ_α^2 , σ_β^2 and σ_e^2 respectively. In this paper the tests of hypotheses about $\frac{\sigma_\beta^2}{\sigma_e^2}$ and $\frac{\sigma_\alpha^2}{\sigma_e^2}$ have been discussed.
Keywords: unbalanced random model, Variance Component, Covariance matrix, orthogonal matrix, optimality.

I. INTRODUCTION

The variance components of a two way nested balanced random model are being estimated by equating the mean sum of squares of analysis of variance to their expected values. The expected values of mean sum of squares suggest the approximate denominator for testing the hypotheses concerning the variance components. However, with the unbalanced data no unique set of sum of squares of observations can be optimally used for estimating the variance components.

In this paper we shall develop some exact tests concerning the variance components in two way nested unbalanced random model by using an orthogonal transformation suggested by Graybill and Haultiquist (1961).

II. TWO WAY UNBALANCED NESTED RANDOM MODEL

A. Consider The Model

$$Y_{ijk} = \mu + \alpha_i+\beta_{ij} +e_{ijk};$$
$$i = 1,2...p,\ j = 1,\ 2..\ q \text{ and}$$
$$k=1,2,...n_{ij} \quad \textstyle\sum_i \sum_j n_{ij}=n$$

Here μ *is* a constant while α_i, β_{ij} and e_{iik} are independently and normally distributed with means zero and variances σ_α^2 , σ_β^2 and σ_e^2 respectively, let us define,

$$\bar{y}_{ij}=(1/n_{ij})\textstyle\sum_{f_{n=1}}^{n_{ij}} y_{ijk} \quad i = 1,2...p,\ j = 1,\ 2..\ q$$

Then

$$\bar{y}_{ij} =\mu+ \alpha_i + \beta_{ij} + \bar{e}_{ij} \qquad (2.1)$$

with $\bar{e}_{ij}= (1/n_{ij}) \sum_{n=1}^{n_{ij}} e_{ijk}$. Here $\bar{e}$ is multivariate normally distributed with mean 0 and covariance matrix $\sum(\bar{e})=K\sigma^2$ where $K = diag(1/n_{11}\ 1/n_{12}\ 1/n_{pq})$ (2.2)

The model (2.1) may be written in the matrix notation as

$$\bar{y} = J_{pq}\ \mu + B_1\alpha +I\beta + \bar{e}$$

where $J_{pq}^{-1/2}$ is a unit Vector of order pq and B_1 = Diag $(J_q,...J_q)$ (p-times) and J_q is a (q x 1) vector with all elements equal to unity. The covariance matrix for $\bar{y}$ turns out to be $\sum(\bar{Y}) = B_1 B_1'\sigma_\alpha^2 + I_{pq}\ \sigma_\beta^2 + K\sigma_e^2$

Consider λ an orthogonal matrix P with the property that $PB_1 B_1'P'$ is a diagonal matrix with eigen values on the diagonal (Herbach, 1959), The first row of p may be taken as

$(pq)^{-1/2}$ [1, 1, … .1]. if Z=Py the covariance matrix for Z is

$$\sum(Z) = PB_1 B_1'\sigma_\alpha^2 + I_{pq}\ \sigma_\beta^2 + PKP'\sigma_e^2$$

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III. TEST OF VARIANCE COMPONENTS

We now partition Z in the following way :-

A. $Z_1 = (pq)^{1/2}$ with the first element in Z.

B. Z_A consists of (p-1) elements whose covariance matrix is

$$I_{p-1}\sigma_\alpha^2 + I_{p-1}\sigma_\beta^2 + K_1 I_{p-1}\sigma_e^2$$

Where K_1 is a sub matrix of order (p-1).(q-1) of PKP'.

C. Z_B consists of p(q-1) elements whose covariance matrix is

$$I_{p(q-1)}\sigma_\beta^2 + K_2\sigma_e^2$$

Where K_2 is a submatrix of order p(q-1) . p(q -1) of PKP' (3.2). Since P is an orthogonal matrix with first row as $(pq)^{-1/2}[1\ldots 1]$, $EZ_A=EZ_B=0$.

Now Z_A and Z_B will be used in testing the hypotheses concerning $\frac{\sigma_\beta^2}{\sigma_e^2}$ and $\frac{\sigma_\alpha^2}{\sigma_e^2}$.

Test for $\frac{\sigma_\beta^2}{\sigma_e^2}$

The covariance matrix $\sum(Z_B)$ can be written as

$$[I_{p(q-1)}\Delta_\beta + K_2]\sigma_e^2 \quad \text{where} \quad \Delta_{\beta=}\frac{\sigma_\beta^2}{\sigma_e^2}$$

Then $Q_B/\sigma_e^2 = Z_B'[I_{p(q-1)}\Delta_\beta + K_2]^{-1}Z_B/\sigma_e^2$has a chi-squre distribution with p(q-1) degrees of freedom. Let us introduce another orthogonal matrix A such that $AK_2A' = D_2$ is a diagonal matrix. Consider $Z_B^* = AZ_{\mathbf{B}}$. The Covariance matrix of Z_B^* is $[I_{p(q-1)}\Delta_\beta + D_2]$ and therefore

$$Z_B'[I_{p(q-1)}\Delta_\beta + K_2]^{-1}Z_B = Z_B'[I_{p(q-1)}\Delta_\beta + D_2]^{-1}Z_B^*$$

Let us define $Q = \sum_i\sum_j\sum_k(y_{ijk} - y_{ij})^2$, then Q/σ_e^2 has a chi-square distribution with (n - pq) degrees of freedom. Q is independent of Q_B and thus $[(n - pq)\,Q_B/\,Qp(q - 1)]$ has an F-distribution with p(q-1) and (n - pq) degrees of freedom respectively.

For testing the hypotheses H_0: $\Delta_\beta \le \Delta_0$ We reject H_0 if $F(\Delta_0)$ if larger than upper (1 –α) quantile of the corresponding F-distribution with p(q-1) and (n-pq) degrees of freedom. The power function is

$$P(\Delta_\beta) = P\ \{(n\text{-}pq)\ [\textstyle\sum_{i=1}^{p(q-1)} Z_{iB}'/(\Delta_0 + d_i)]/[p(q\text{-}1)Q] \ge F_{1-\alpha}\}$$

$$P(\Delta_\beta) = P\ \{(n\text{-}pq)\ [\textstyle\sum_{i=1}^{q}(\Delta_\beta + d_i)R_i/(\Delta_0 + d_i)]/[p(q\text{-}1)Q] \ge F_{1-\alpha}\}$$

This is an unbiassed size α-test.

(3b) Teat of $\frac{\sigma_\alpha^2}{\sigma_e^2}$assuming σ_β^2=0

For test of hypothesis H_0: $\frac{\sigma_\alpha^2}{\sigma_e^2} \le \Delta_0$ against H_1: $\frac{\sigma_\alpha^2}{\sigma_e^2} > \Delta_0$

Consider the covariance matrix of $\begin{bmatrix} Z_A \\ Z_B \end{bmatrix}$ which is given by

$$\sum\begin{pmatrix} Z_A \\ Z_B \end{pmatrix} = \begin{pmatrix} q\,I_{p-1} & 0 \\ 0 & 0 \end{pmatrix}\sigma_\alpha^2 + \begin{pmatrix} k_1 & k_3 \\ k_3 & k_2 \end{pmatrix}\sigma_e^2$$

Where $\begin{pmatrix} k_1 & k_3 \\ k_3 & k_2 \end{pmatrix}$ is a positive definite marix.

Let us introduce a non-singular matrix H such that

$$H\begin{pmatrix} k_1 & k_3 \\ k_3 & k_2 \end{pmatrix}H' = I \text{ and}$$

$$H\begin{pmatrix} q\,I_{p-1} & 0 \\ 0 & 0 \end{pmatrix}H' = \lambda = \text{diag}\{\lambda_1, \ldots \lambda_{p-1}, 0, \ldots .0\}$$

$$\text{Define } U = \begin{pmatrix} U_A \\ U_B \end{pmatrix} = H\begin{pmatrix} Z_A \\ Z_B \end{pmatrix} \text{ and let } \Delta_\alpha = \frac{\sigma_\alpha^2}{\sigma_e^2}$$

Then $Q_A/\ \sigma_e^2 = U_A'(\lambda\Delta_\alpha + I_{p-1})^{-1}\ U_A/\ \sigma_s^2$ has a chi-square distribution with (p-1) degrees of freedom. Similarly $Q_B^* = U_B' I_{p(q-1)}\ U_B/\ \sigma_e^2$ has a chi – square distribution with p(q-1) degrees of freedom. Further Q_A, Q_B^* and Q are independently distributed and therefore (n - p)Q_A /[(p - 1) (Q + Q_B)] is distributed as Snedecor's F with (p - 1, n-p) degrees of freedom. This is also an unbiased size α-test.

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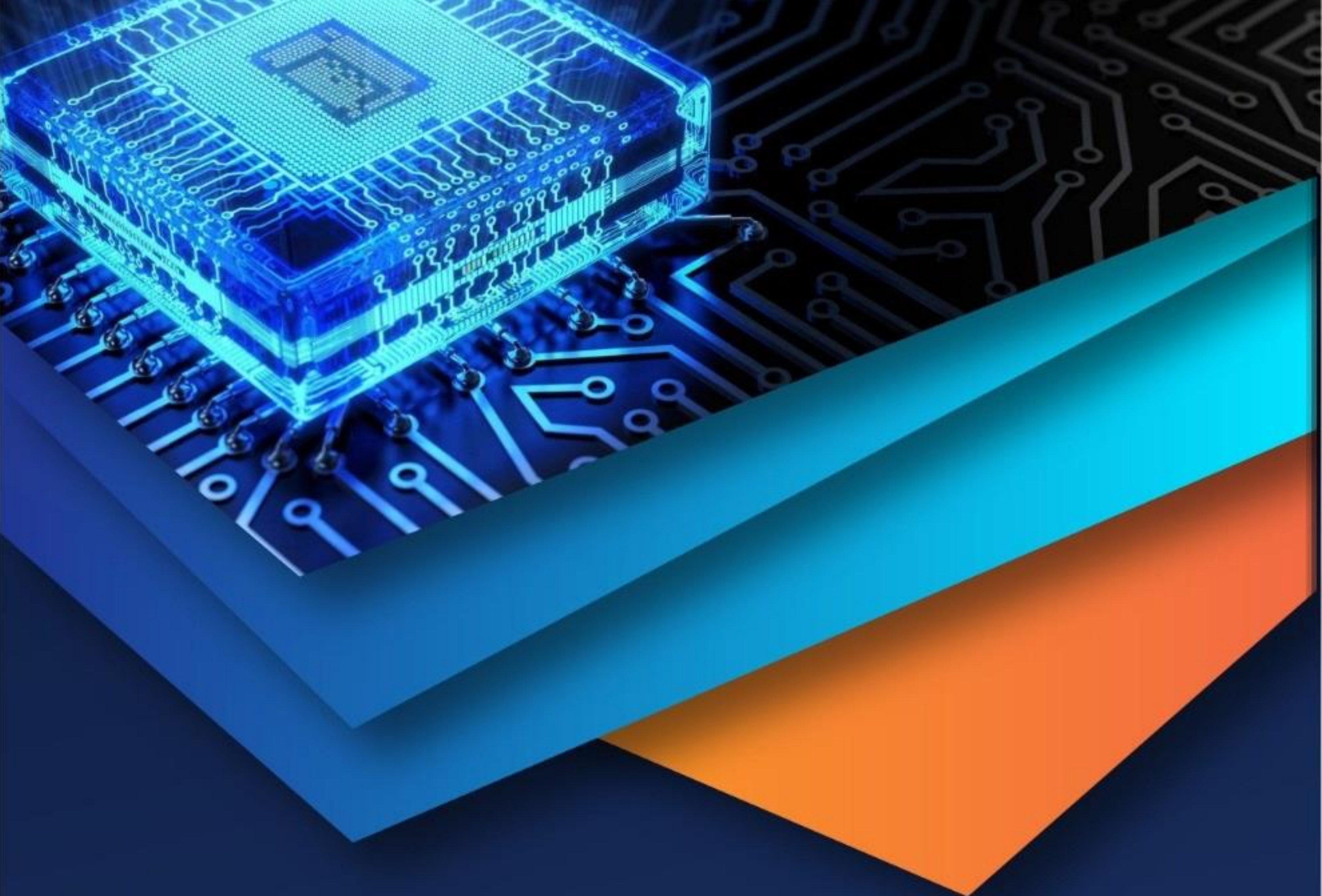

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