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Near Mean Labeling of Path Related Splitted Graphs

A. Esakkimuthu
Department of Mathematics, Thiyagi Dharmakkan Amirtham College of Arts and Science, Kannirajapuram Post – 623 135, Ramanathapuram District, Tamilnadu, India

Abstract: ***The concept of near mean Graph was introduced in [9]. A function f is called a near mean Labeling of graph G if $f: V(G) \rightarrow \{0, 1, 2, \ldots\ldots q-1, q+1\}$ is injective and the induced function $f^*: E(G) \rightarrow \{1, 2, \ldots\ldots q\}$ defined as***

$$f^*(e = uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

is bijective. The graph which admits near mean labeling is called a near mean Graph. In this paper, we proved that $S(H_n)$: $(n: odd)$, $S(H_n)$ $(n: even)$, $S(P_n^+)$ are near mean graphs.

Keywords: Near mean Labeling, Near mean Graph.
2000 mathematics Subject classification: 05C78

I. INTRODUCTION

We begin with simple, finite, connected and undirected graph $G = (V(G), E(G))$ with p vertices and q edges. For standard terminology and notations we follow (Harary, F., 1972). We will provide brief summary of definitions and other information which are prerequisites for the present investigations.

II. PRELIMINARIES

A function f is called a *near mean labeling* of graph G if $f: V(G) \rightarrow \{0, 1, 2, \ldots\ldots q\}$ is injective and the induced function $f^*: E(G) \rightarrow \{1, 2, \ldots\ldots q\}$ defined as

$$f^*(e = uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

is bijective. The graph which admits near mean labeling is called a *near mean graph.*

1) *Definition 2.1.* Let G be a graph. For each vertex u of a graph G, take a new vertex v. Join v to those vertices of G adjacent to u. The graph thus obtained is called the *splitting graph* of G. It is denoted by $S(G)$. For a graph G, the splitting graph S of G is obtained by adding a new vertex v corresponding to each vertex u of G such that $N(u) = N(v)$ and it is denoted by $S(G)$.
2) *Definition 2.2.* Let H_n –graph of a path P_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, v_3, \ldots\ldots\ldots, v_n$ and $u_1, u_2, u_3, \ldots\ldots\ldots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ by means of an edge if n is odd and vertices $v_{\frac{n}{2}+1}$ and $u_{n/2}$ if n is even.
3) *Definition 2.3.* $G_1 \ominus G_2$ of two graphs G_1 and G_2 is obtained by taking one copy of G_1 (with p vertices) and p copies of G_2 and then joining the ith vertex of G_1 to every vertex in the ith copy of G_2. When $G_1 = P_n$ and $G_2 = mK_1$ we obtain $P_n \ominus mk_1$.

III. MAIN RESULTS

Theorem 3.1: $S(H_n)$: (n: odd) is Mean Graph.

Proof: Let $V[S(H_n)] = \{(u_i, v_i, u_i^1, v_i^1) : 1 \le i \le n\}$

$$E[S(H_n)] = \{[(u_iu_{i+1}) \cup (u_i^1u_{i+1}) : 1 \le i \le n-1] \cup [(v_iu_{i+1}) \cup (u_iv_{i+1}) \cup (v_i^1u^1_{i+1}) \cup (u_i^1v^1_{i+1}) : 1 \le i \le n-1] \cup (u_{(n+1)/2}u^1_{(n+1)/2}) \cup (v_{(n+1)/2}u^1_{(n+1)/2}) \cup (u_{(n+1)/2}v^1_{(n+1)/2})\}$$

Let $f : V[S(H_n)] \rightarrow \{0, 1, 2, \ldots\ldots, q\}$ by

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$f(u_{2i-1})$	$= 2(i-1)$	$1 \le i \le [n+1]/2$
$f(u_{2i})$	$= 4n-1+2(i-1)$	$1 \le i \le [n-1]/2$
$f(v_{2i-1})$	$= 4n-2+2(i-1)$	$1 \le i \le [n+1]/2$
$f(v_{2i})$	$= 2i-1$	$1 \le i \le [n-1]/2$
$f(v'_{n+2-2i})$	$= n+2(i-1)$	$1 \le i \le [n+1]/2$
$f(v'_{n+1-2i})$	$= 5n-1+2(i-1)$	$1 \le i \le [n-1]/2$
$f(u'_{n+1-2i})$	$= n-1+2i$	$1 \le i \le [n-1]/2$
$f(u'_{n+2-2i})$	$= 5n-2+2(i-1)$	$1 \le i \le [n-1]/2$

$f(u'_1) = 6n-2$

The induced edge labeling are

$f^*(u_iu_{i+1})$	$= 2n+i-1$	$1 \le i \le n-1$
$f^*(u_{2i-1}v_{2i})$	$= 2i-1$	$1 \le i \le [n-1]/2$
$f^*(u_{2i}v_{2i+1})$	$= 4n+2(i-1)$	$1 \le i \le [n-1]/2$
$f^*(v_{2i-1}u_{2i})$	$= 4n-1+2(i-1)$	$1 \le i \le [n-1]/2$
$f^*(v_{2i}u_{2i+1})$	$= 2i$	$1 \le i \le [n-1]/2$
$f^*(u^1_{n+1-i}u^1_{n-i})$	$= 3n+i-1$	$1 \le i \le n-1$
$f^*(v^1_{n+2-2i}u^1_{n+1-2i})$	$= n+2i-1$	$1 \le i \le [n-1]/2$
$f^*(v^1_{n+1-2i}u^1_{n-2i})$	$= 5n+2(i-1)$	$1 \le i \le [n-1]/2$
$f^*(v^1_{n+1-2i}u^1_{n+2-2i})$	$= 5n-1+2(i-1)$	$1 \le i \le [n-1]/2$
$f^*(v^1_{n-2i}u^1_{n+1-2i})$	$= n+2i$	$1 \le i \le [n-1]/2$
$f^*(u_{(n+1)/2}u^1_{(n+1)/2})$	$= 3n-1$	
$f^*(v_{(n+1)/2}u^1_{(n+1)/2})$	$= n$	if $n \equiv 3 \pmod 4$
$f^*(v_{(n+1)/2}u^1_{(n+1)/2})$	$= 5n-2$	if $n \equiv 1 \pmod 4$
$f^*(v^1_{(n+1)/2}u_{(n+1)/2})$	$= 5n-2$	if $n \equiv 3 \pmod 4$
$f^*(v^1_{(n+1)/2}u_{(n+1)/2})$	$= n$	if $n \equiv 1 \pmod 4$

Hence, distinct induced edge labels shows that $S(H_n)$ (n : odd) is a Mean graph. For example, *$S(H_5)$* and *$S(H_7)$* are Mean Graph an shown in the figure 3.2 and 3.3 respectively.

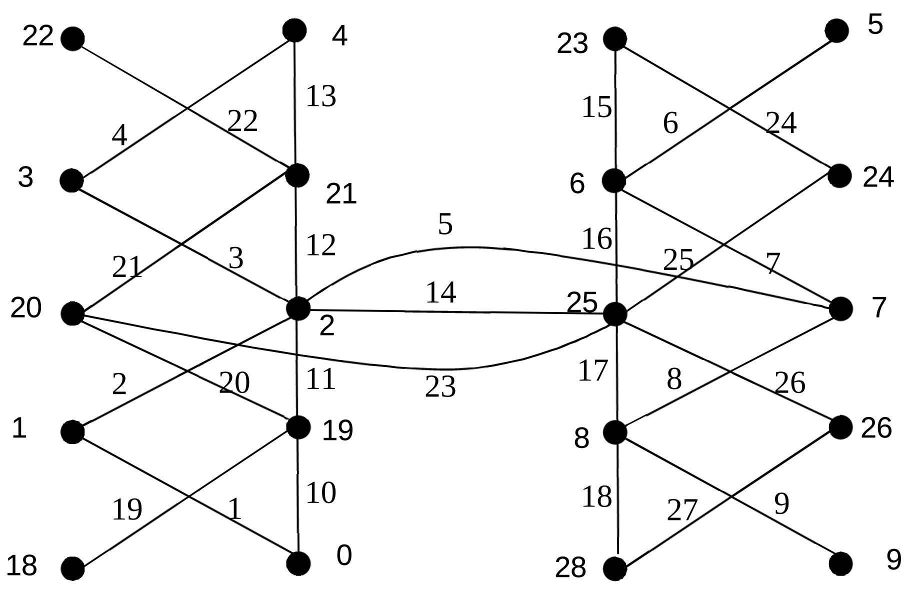

Figure 3.2 $S(H_5)$: $n \equiv 1 \pmod 4$

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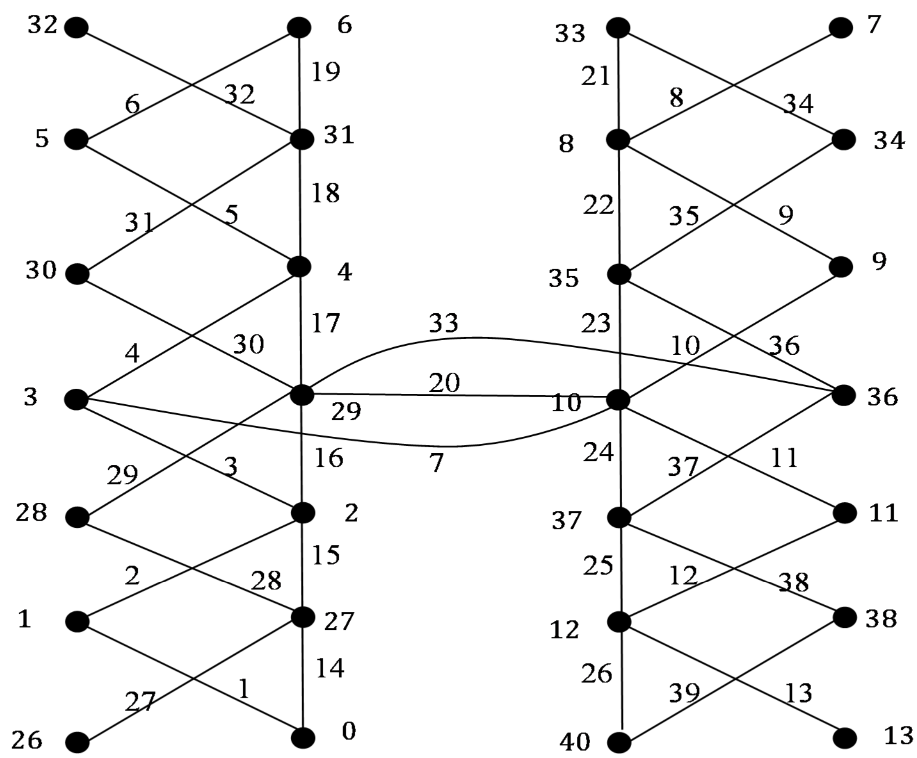

Figure 3.3 $S(H_7)$: $n \equiv 3 \pmod 4$

Theorem 3.4 : $S(H_n)$ (n : even) is Mean Graph.

Proof: Let $V[S(H_n)] = \{u_i, v_i, u_i^1, v_i^1 : 1 \le i \le n\}$

$$E[S(H_n)] = \{[(u_iu_{i+1}) \cup (u_i^1u^1_{i+1}) : 1 \le i \le n-1] \cup (u_{n/2}u^1_{[n/2]+1}) \cup [(v_iu_{i+1}) \cup (u_iv_{i+1}) : 1 \le i \le n-1] \cup [(v_i^1u^1_{i+1}) \cup (u_i^1v^1_{i+1}) : 1 \le i \le n-1] \cup (v_{n/2}u^1_{(n/2)+1}) \cup (u_{n/2}v^1_{(n/2)+1})\}$$

Let $f : V[S(H_n)] \rightarrow \{0,1,2,\ldots\ldots\ldots,q\}$

$f(u_{2i-1})$	$= 2(i-1)$	$1 \le i \le n/2$
$f(u_{2i})$	$= 4n-1+2(i-1)$	$1 \le i \le n/2$
$f(v_{2i-1})$	$= 4n-2+2(i-1)$	$1 \le i \le n/2$
$f(v_{2i})$	$= 2i-1$	$1 \le i \le n/2$
$f(u^1_{2i-1})$	$= n+2(i-1)$	$1 \le i \le n/2$
$f(u^1_{2i})$	$= 5n-1+2(i-1)$	$1 \le i \le [n/2]-1$
$f(u_n')$	$= 6n-2$	
$f(v^1_{2i-1})$	$= 5n-2+2(i-1)$	$1 \le i \le n/2$
$f(v^1_{2i})$	$= n+1+2(i-1)$	$1 \le i \le n/2$

The induced edge labeling are

$f^*(u_iu_{i+1})$	$= 2n+i-1$	$1 \le i \le n-1$
$f^*(u_{2i-1}v_{2i})$	$= 2i-1$	$1 \le i \le n/2$
$f^*(u_{2i}v_{2i+1})$	$= 4n+2(i-1)$	$1 \le i \le [n/2]-1$

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$f^*(v_{2i-1}u_{2i}) = 4n-1+2(i-1) \quad 1 \le i \le n/2$
$f^*(v_{2i}u_{2i+1}) = 2i \quad 1 \le i \le [n/2]-1$
$f^*(u_i^1 u^1_{i+1}) = 3n+i-1 \quad 1 \le i \le n-1$
$f^*(u^1_{2i-1}v^1_{2i}) = n+2i-1 \quad 1 \le i \le n/2$
$f^*(u^1_{2i}v^1_{2i+1}) = 5n+2(i-1) \quad 1 \le i \le [n/2]-1$
$f^*(v^1_{2i-1}u^1_{2i}) = 5n-1+2(i-1) \quad 1 \le i \le n/2$
$f^*(v^1_{2i}u^1_{2i+1}) = n+2i \quad 1 \le i \le [n/2]-1$
$f^*(u_{n/2}u^1_{(n/2)+1}) = 3n-1$
$f^*(v_{n/2}u^1_{(n/2)+1}) = n \quad \text{if } n \equiv 0 \bmod 4$
$f^*(v_{n/2}u^1_{(n/2)+1}) = 5n-2 \quad \text{if } n \equiv 2 \bmod 4$
$f^*(u_{n/2}v^1_{(n/2)+1}) = 5n-2 \quad \text{if } n \equiv 0 \bmod 4$
$f^*(u_{n/2}v^1_{(n/2)+1}) = n \quad \text{if } n \equiv 2 \bmod 4$

Hence, distinct induced edge labels shows that $S(H_n)$ (n : even) is a Mean graph. For example, $S(H_4)$ and $S(H_6)$ are Mean Graphs as shown in the figure 3.5 and 3.6 respectively.

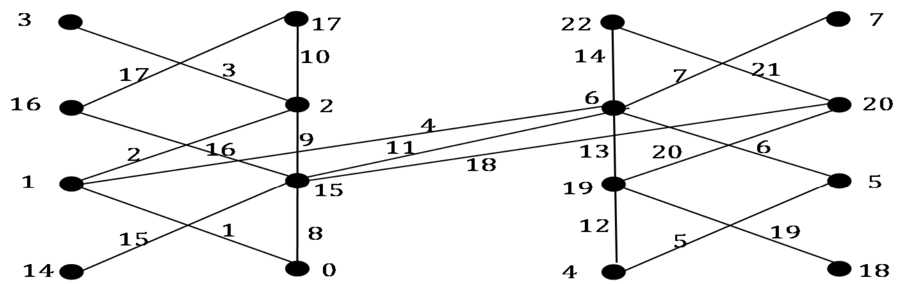

Figure 3.5 $S(H_4) : n \equiv 0 \pmod 4$

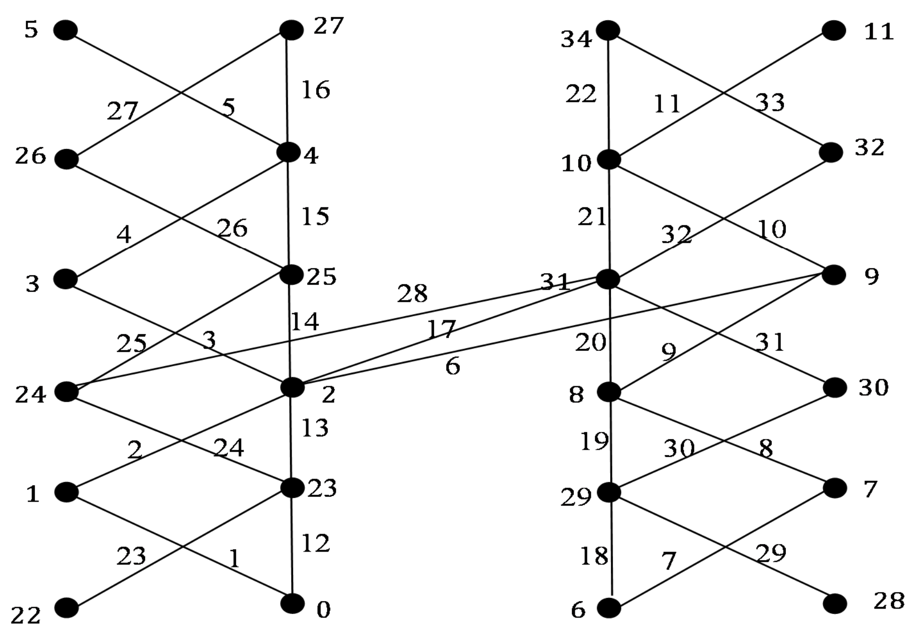

Figure 3.6 $S(H_6) : n \equiv 2 \pmod 4$

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Theorem 3.7: $S(P_n \Theta\ 2k_1)$ is Mean Graph.

Proof: Let $V[S(P_n\Theta\ 2k_1)] = \{ [(u_i,v_i) : 1 \leq i \leq n] \cup [(u_{ij},v_{ij}) : 1 \leq i \leq n,\ 1 \leq j \leq 2] \}$

$E[S(P_n\Theta\ 2k_1)] = [(u_iu_{i+1}) : 1 \leq i \leq n-1] \cup [(u_iu_{ij}) \cup (v_iu_{ij}) \cup (u_iv_{ij}) : 1 \leq i \leq n,\ 1 \leq j \leq 2] \cup [(v_iu_{i+1}) : 1 \leq i \leq n-1] \cup [(u_iv_{i+1}) : 1 \leq i \leq n-1$

Let $f : V[S(P_n\Theta\ 2k_1)] \rightarrow \{0,1,2,\ldots\ldots\ldots,q\}$

Case: (i) when $n \equiv 0 \pmod 2$

$f(u_{2i-1}) = 6i-5 \quad 1 \leq i \leq n/2$

$f(u_{2i}) = 6n+2+6(i-1) \quad 1 \leq i \leq n/2$

$f(v_{2i-1,1}) = 6(i-1) \quad 1 \leq i \leq n/2$

$f(v_{2i-1,2}) = 6i-4 \quad 1 \leq i \leq n/2$

$f(u_{2i,1}) = 6i-3 \quad 1 \leq i \leq n/2$

$f(u_{2i,2}) = 6i-1 \quad 1 \leq i \leq n/2$

$f(v_{2i}) = 6i-2 \quad 1 \leq i \leq n/2$

$f(v_{2i-1}) = 6(n+i)-7 \quad 1 \leq i \leq n/2$

$f(v_{2i,1}) = 6n-5+6i \quad 1 \leq i \leq n/2$

$f(v_{2i,2}) = 6n-3+6i \quad 1 \leq i \leq [n/2]-1$

$f(v_{n,2}) = 9n-2$

$f(u_{2i-1,1}) = 6n-8+6i \quad 1 \leq i \leq n/2$

$f(u_{2i-1,2}) = 6n-6+6i \quad 1 \leq i \leq n/2$

The induced edge labeling are

$f^*(u_{2i-1}v_{2i-1,1}) = 6i-5 \quad 1 \leq i \leq n/2$

$f^*(u_{2i-1}v_{2i-1,2}) = 6i-4 \quad 1 \leq i \leq n/2$

$f^*(u_{2i}v_{2i,1}) = 6n-4+6i \quad 1 \leq i \leq n/2$

$f^*(u_{2i}v_{2i,2}) = 6n-3+6i \quad 1 \leq i \leq n/2$

$f^*(u_iu_{i+1}) = 3n-1+3i \quad 1 \leq i \leq n-1$

$f^*(u_iu_{i,1}) = 3n-3+3i \quad 1 \leq i \leq n$

$f^*(u_iu_{i,2}) = 3n-2+3i \quad 1 \leq i \leq n$

$f^*(v_{2i}u_{2i,1}) = 6i-2 \quad 1 \leq i \leq n/2$

$f^*(v_{2i}u_{2i,2}) = 6i-1 \quad 1 \leq i \leq n/2$

$f^*(v_{2i-1}u_{2i-1,1}) = 6n-7+6i \quad 1 \leq i \leq n/2$

$f^*(v_{2i-1}u_{2i-1,2}) = 6(n-1)+6i \quad 1 \leq i \leq n/2$

$f^*(v_{2i-1}u_{2i}) = 6n-5+6i \quad 1 \leq i \leq n/2$

$f^*(v_{2i}u_{2i+1}) = 6i \quad 1 \leq i \leq [n/2]-1$

$f^*(v_{2i}u_{2i-1}) = 6i-3 \quad 1 \leq i \leq n/2$

$f^*(v_{2i+1}u_{2i}) = 6n-2+6i \quad 1 \leq i \leq [n/2]-1$

Hence, distinct induced edge labels shows that $S(P_n\Theta\ 2k_1)$ (n : even)is a Mean graph. For example, $S(P_4\Theta\ 2k_1)$ is mean Graphs an shown in the figure 3.5.

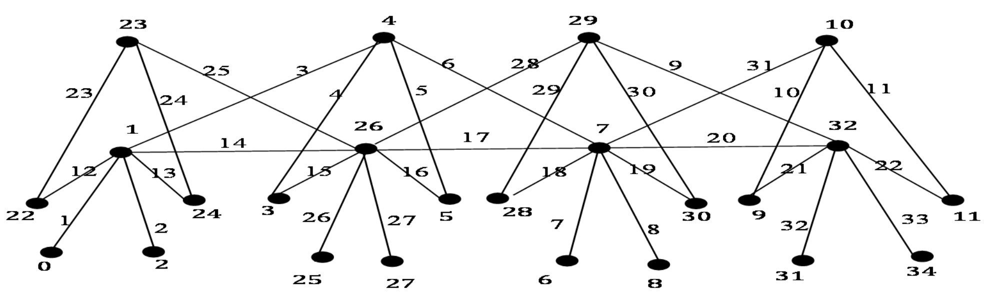

Figure 3.8 $S(P_4\Theta\ 2k_1)$: $n \equiv 0 \pmod 2$

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Case: (ii) When $n \equiv 1 \pmod 2$

$f(u_{2i-1}) = 6i-5 \qquad 1 \le i \le [n+1]/2$

$f(u_{2i}) = 6n-4+6i \qquad 1 \le i \le [n-1]/2$

$f(v_{2i-1,1}) = 6(i-1) \qquad 1 \le i \le [n+1]/2$

$f(v_{2i-1,2}) = 6i-4 \qquad 1 \le i \le [n+1]/2$

$f(v_{2i,1}) = 6(n+i)-5 \qquad 1 \le i \le [n-1]/2$

$f(v_{2i,2}) = 6(n+i)-3 \qquad 1 \le i \le [n-1]/2$

$f(u_{2i,1}) = 6i-3 \qquad 1 \le i \le [n-1]/2$

$f(u_{2i,2}) = 6i-1 \qquad 1 \le i \le [n-1]/2$

$f(u_{2i-1,1}) = 6(n+i)-8 \qquad 1 \le i \le [n+1]/2$

$f(u_{2i-1,2}) = 6(n+i-1) \qquad 1 \le i \le [n-1]/2$

$f(u_{n,2}) = 9n-2$

$f(v_{2i}) = 6i-2 \qquad 1 \le i \le [n-1]/2$

$f(v_{2i-1}) = 6(n+i)-7 \qquad 1 \le i \le [n+1]/2$

The induced edge labeling are

$f^*(u_{2i-1}v_{2i-1,1}) = 6i-5 \qquad 1 \le i \le [n+1]/2$

$f^*(u_{2i-1}v_{2i-1,2}) = 6i-4 \qquad 1 \le i \le [n+1]/2$

$f^*(u_{2i}v_{2i,1}) = 6(n+i)-4 \qquad 1 \le i \le [n-1]/2$

$f^*(u_{2i}v_{2i,2}) = 6(n+i)-3 \qquad 1 \le i \le [n-1]/2$

$f^*(u_iu_{i+1}) = 3(n+i)-1 \qquad 1 \le i \le n-1$

$f^*(u_iu_{i,1}) = 3(n+i-1) \qquad 1 \le i \le n$

$f^*(u_iu_{i,2}) = 3(n+i)-2 \qquad 1 \le i \le n$

$f^*(v_{2i}u_{2i,1}) = 6i-2 \qquad 1 \le i \le [n-1]/2$

$f^*(v_{2i}u_{2i,2}) = 6i-1 \qquad 1 \le i \le [n-1]/2$

$f^*(v_{2i-1}u_{2i-1,1}) = 6(n+i)-7 \qquad 1 \le i \le [n+1]/2$

$f^*(v_{2i-1}u_{2i-1,2}) = 6(n+i-1) \qquad 1 \le i \le [n+1]/2$

$f^*(v_{2i-1}u_{2i}) = 6(n+i)-5 \qquad 1 \le i \le [n-1]/2$

$f^*(v_{2i}u_{2i+1}) = 6i \qquad 1 \le i \le [n-1]/2$

$f^*(v_{2i}u_{2i-1}) = 6i-3 \qquad 1 \le i \le [n-1]/2$

$f^*(v_{2i+1}u_{2i}) = 6(n+i)-2 \qquad 1 \le i \le [n-1]/2$

Hence, distinct induced edge labels shows that $S(P_n\Theta\, 2k_1)$ $(n : odd)$is a Mean graph.

For example, $S(P_3\Theta\, 2k_1)$ is Mean Graphs an shown in the figure 3.6.

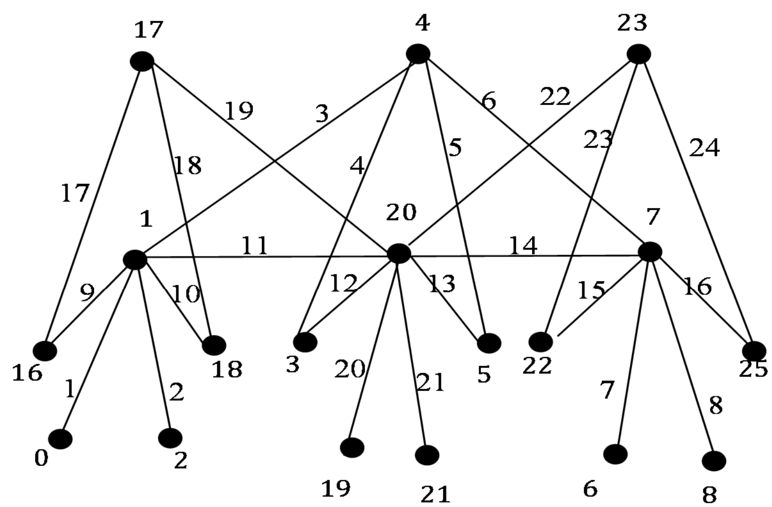

Figure 3.9 $S(P_3\Theta\, 2k_1)$: $n \equiv 1 \pmod 2$

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