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Regular Intuitionistic Fuzzy Graph Structure

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***Abstract:** In this paper, we discuss the notion order, degree and size of a vertex in intuitionistic fuzzy graph structure (IFGS) $\tilde{G}$ and their properties are studied. We also study the concept of regularity in intuitionistic fuzzy graph structures. Some characterization of regular IFGS on B_i -cycle are also provided and properties of regular IFGS are introduced.*
***Keywords:** Order, degree, size of vertex, regular IFGS.*
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I. INTRODUCTION

The idea of fuzzy sets was introduced by Prof. Zadeh [8] in 1965. Rosenfeld [9] in 1975 gave the concept of fuzziness in relations and graphs. Atanassov [5] introduced the idea of intuitionistic fuzzy sets. Further the notion of graph structure was discussed by Sampathkumar [1]. Dinesh and Ramakrishnan [2] gave fuzzy graph structure. The notion of intuitionistic fuzzy graph structure (IFGS) $\tilde{G}$= ($A,B_1,B_2,...B_k$) are defined and discussed by the authors in [6] ,[7] and [14]. In this paper, we discussed various properties of regularity in intuitionistic fuzzy graph structures.

II. PRELIMINARIES

In this section, we review some definitions and results that are necessary in this paper, which are mainly taken from [1], [2], [6] - [14].

1) *Definition (2.1):* An intuitionistic fuzzy graph (IFG) is of the form G = (V, E) where
a) V = {$v_1, v_2, ..., v_n$} such that $\mu_1 : V\to [0,1]$ and $\gamma_1 : V\to [0,1]$ denote the degree of membership and non membership of the element $v_i \in V$, respectively and $0 \le \mu_1(v_i) + \gamma_1 (v_i) \le 1$, for every $v_i \in V$, (i = 1,2,….,n),
b) $E \subseteq V \times V$ where $\mu_2: V\times V \to [0,1]$ and $\gamma_2 : V\times V \to [0,1]$ are such that

$$\mu_2(v_i,v_j) \le \min\{\mu_1(v_i),\mu_1(v_j)\} \text{ and } \gamma_2(v_i,v_j) \le \max\{\gamma_1(v_i),\gamma_1(v_j)\}$$

and $0 \le \mu_2 (v_i , v_j)+ \gamma_2 (v_i , v_j) \le 1$, for every $(v_i ,v_i) \in E$, (i, j = 1,2,….,n),

2) *Definition (2.2):* Let G = (V, E) be an IFG. Then the degree of a vertex v is defined by
d(v) = (dμ(v), dγ(v)) where $d\mu(v) = \sum_{u\neq v} \mu_2(v,u)$ and $d\gamma(v) = \sum_{u\neq v} \gamma_2(v,u)$.
3) *Definition (2.3):* G is said to be regular fuzzy graph if each vertex has same fuzzy degree. It is said to be k-regular fuzzy graph if (fd)(v)=k, $\forall v \in V$.
4) *Definition (2.4):* An Intuitionistic fuzzy graph G = (V, E) is said to be regular IFG if all the vertices have the same degree.
5) *Definition (2.5):* An Intuitionistic fuzzy graph is complete if
$\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$ and $\gamma_{2ij} = \max(\gamma_{2i}, \gamma_{2j})$ for all (vi ,vj)$\in$V.
6) *Definition(2.6):* The minimum degree of G is δ(G) = (δμ(G), δγ(G)) where δμ (G) = Λ {dμ (v)/v∈ V} and δγ (G) = Λ {dγ (v)/v ∈V}.
7) *Definition(2.7):* The maximum degree of G is Δ(G) = (Δμ (G), Δγ (G)) where
Δμ (G) = V{dμ (v)/v ∈V} and Δγ (G) = V {dγ (v) / v ∈V}.
8) *Definition (2.8):* Let G = (V, $R_1, R_2,..., R_k$) be a graph structure and let A be an intuitionistic fuzzy subset (IFS) on V and B_1,B_2 ,..., B_k are intuitionistic fuzzy relations (IFR) on V which are mutually disjoint, symmetric and irreflexive such that

$\mu_{B_i}(u,v) \le \mu_A(u) \wedge \mu_A(v)$ and $\nu_{B_i}(u,v) \le \nu_A(u) \vee \nu_A(v)$ $\forall u,v \in V$ and i = 1,2, ..., k. Then
$\tilde{G}$ = (A, $B_1, B_2,...,B_k$) is an intuitionistic fuzzy graph structure (IFGS) of G.

9) *Note(2.9):* Throughout this paper , unless otherwise specified $\tilde{G}$ = (A, B_1, B_2,...,B_k) will represent an intuitionistic fuzzy graph structure with respect to graph structure G = (V, R_1, R_2 ,..., R_k) and i =1, 2,..., k will refer to the number of intuitionistic fuzzy relations on V.

10) *Definition (2.10):* The μ_{B_i}-strength of connectedness between *u* and *v* is $\mu_{B_i}^{\infty}(u,v) = \bigvee_{j=1}^{\infty} \mu_{B_i}^{j}(u,v)$. and the ν_{B_i}-strength of connectedness between *u* and *v* is $\nu_{B_i}^{\infty}(u,v) = \bigwedge_{j=1}^{\infty} \nu_{B_i}^{j}(u,v)$.

III.REGULAR INTUITIONISTIC FUZZY GRAPH STRUCTURE

1) *Definition (3.1):* Let $\tilde{G}$ = (A,B_1,B_2,….,B_k) be an intuitionistic fuzzy graph structure (IFGS) of G. The μ_{B_i} - degree of vertex *u* is the sum of μ_{B_i}-edge starting from *u*. It is denoted by $d_{\mu_{B_i}}(u)$. Thus, $d_{\mu_{B_i}}(u) = \sum_{(u,v)\in B_i} \mu_{B_i}(u,v)$.

2) *Definition (3.2):* In an IFGS, the ν_{B_i} - degree of vertex *u* is the sum of ν_{B_i}-edge starting from *u*. It is denoted by $d_{\nu_{B_i}}(u)$. Thus,
$$d_{\nu_{B_i}}(u) = \sum_{(u,v)\in B_i} \nu_{B_i}(u,v).$$

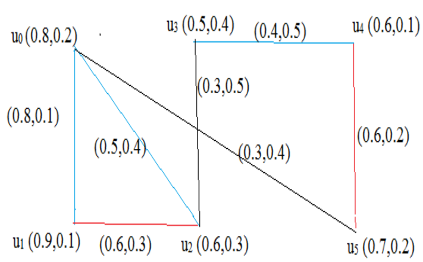
3) *Definition (3.3):* The B_i-degree of vertex *u* is $d_{B_i}(u) = \left(d_{\mu_{B_i}}(u), d_{\nu_{B_i}}(u)\right)$

Or
$$d_{B_i}(u) = \left(\sum_{(u,v)\in B_i} \mu_{B_i}(u,v), \sum_{(u,v)\in B_i} \nu_{B_i}(u,v)\right) and\ \ \mu_{B_i}(u,v) = \nu_{B_i}(u,v) = 0 \quad for\ (u,v) \notin B_i.$$

4) *Definition (3.4):* Let $\tilde{G}$ = (A,B_1,B_2,….,B_k) be an IFGS of G. The degree of vertex *u* is the sum of B_i-degrees of *u* for various i.

i.e., Degree of vertex *u* of $\tilde{G}$ = d(*u*)= $\sum_{i=1}^{k} d_{B_i}(u) = \left(\sum_{i=1}^{k} d_{\mu_{B_i}}(u), \sum_{i=1}^{k} d_{\nu_{B_i}}(u)\right)$.

5) *Example (3.5):* Consider an IFGS $\tilde{G}$ = (A, B_1, B_2, B_3) such that V = { u_0, u_1, u_2, u_3, u_4, u_5 }. Let R_1 = {(u_0,u_1),(u_0,u_2), (u_3, u_4)}, R_2 ={ {(u_1, u_2),(u_4,u_5)}, R_3 = {(u_2, u_3),(u_0, u_5)} are the relations on V. Let A = {< u_0, 0.8,0.2 >, < u_1, 0.9,0.1>, < u_2 , 0.6,0.3 >, <u_3, 0.5,0.4>, <u_4, 0.6,0.1 >, <u_5, 0.7,0.2 > }} be an IFS on V and B_1 = { (u_0, u_1), 0.8,0.1 > , < (u_0, u_2), 0.5,0.4>, < (u_3, u_4), 0.4,0.5 >}, B_2 ={< (u_1, u_2), 0.6,0.3 > ,< (u_4, u_5), 0.6,0.2 > }, B_3 = { < (u_2, u_3), 0.3,0.5 >, <(u_0, u_5), 0.3,0.4 >} are IFRs on V.



$$d_{B_1}(u_0) = \left(d_{\mu_{B_1}}(u), d_{\nu_{B_1}}(u)\right) = (1.3, 0.5), d_{B_2}(u_0) = (0,0), d_{B_3}(u_0) = (0.3, 0.4);$$
$$d_{B_1}(u_1) = (0.8, 0.1), d_{B_2}(u_1) = (0.6, 0.3), d_{B_3}(u_1) = (0,0);$$
$$d_{B_1}(u_2) = (0.5, 0.4), d_{B_2}(u_2) = (0.6, 0.3), d_{B_3}(u_2) = (0.3, 0.5);$$
$$d_{B_1}(u_3) = (0.4, 0.5), d_{B_2}(u_3) = (0,0), d_{B_3}(u_3) = (0.3, 0.5);$$
$$d_{B_1}(u_4) = (0.4, 0.5), d_{B_2}(u_4) = (0.6, 0.2), d_{B_3}(u_4) = (0,0);$$
$$d_{B_1}(u_5) = (0,0), d_{B_2}(u_5) = (0.6, 0.2), d_{B_3}(u_5) = (0.3, 0.4).$$
$$Also\ \ d(u_0) = d_{B_1}(u_0) + d_{B_2}(u_0) + d_{B_3}(u_0) = (1.6, 0.9), d(u_1) = (1.4, 0.4),$$
$$d(u_2) = (1.4, 1.2), d(u_3) = (0.7, 1.0), d(u_4) = (1.0, 0.7), d(u_5) = (0.9, 0.6).$$

6) *Definition (3.6):* Let $\tilde{G}$ be an intuitionistic fuzzy graph structure (IFGS) of G. If $d_{\mu_{B_i}}(u) = p \quad \forall u \in V$ then $\tilde{G}$ is said to be p- μ_{B_i} -regular and if $d_{\nu_{B_i}}(u) = q \quad \forall u \in V$ then $\tilde{G}$ is said to be q- ν_{B_i} -regular.

In the above example (3.5), $\tilde{G}$ is 0.3- μ_{B_3} -regular and $\tilde{G}$ is 0.6- μ_{B_2} -regular, but no ν_{B_i} -regular.

7) *Definition (3.7):* Let $\tilde{G}$ be an intuitionistic fuzzy graph structure (IFGS) of G. If $d_{B_i}(u) = (p,q) \quad \forall u \in V$ then $\tilde{G}$ is said to be (p,q)- B_i-regular and if $d(u) = (p,q) \quad \forall u \in V$ then $\tilde{G}$ is said to be (p,q)- regular IFGS.

In the above example (3.5), $\tilde{G}$ is also not B_i-regular.

8) *Theorem (3.8):* Let $\tilde{G}$ be an IFGS of G and $\tilde{G}^*$ = (V,R_1,R_2 ,...,R_k) be an odd B_i-cycle. $\tilde{G}$ is B_i-regular iff $\mu_{B_i}(u,v)$ and $\nu_{B_i}(u,v)$ are constant for all B_i-edges in R_i.

Proof: Let $\mu_{B_i}(u,v)$ and $\nu_{B_i}(u,v)$ be constant function say $\mu_{B_i}(u,v) = c_i$ and $\nu_{B_i}(u,v) = d_i$ $\forall$(u,v)$\in R_i$. i.e. $d_{\mu_{B_i}}(u)$= 2 c_i and $d_{\nu_{B_i}}(u)$.= 2 d_i $\forall\ u \in$V

$\therefore$ $\tilde{G}$ is ($2c_i$,$2d_i$) B_i-regular.

Conversely, Let $\tilde{G}$ be (r,s) B_i-regular and let B_i-edges of R_i be e_1, e_2 ,..., e_{2n+1} and let

$\mu_{B_i}(e_1) = r_1$ and let $\mu_{B_i}(e_1) = r_1$ then $\mu_{B_i}(e_2) = r - r_1$,

$\mu_{B_i}(e_3) = r-(r-r_1) = r_1$, $\mu_{B_i}(e_4) = r-r_1$, $\mu_{B_i}(e_5) = r-(r-r_1) = r_1$, ………………….....………………….

$\mu_{B_i}(e_m) = r_1$ if m is odd,

$= r-r_1$ if m is even

Similarly, let $\nu_{B_i}(e_1) = s_1$ then $\nu_{B_i}(e_2) = s-s_1$,

$\nu_{B_i}(e_3) = s-(s-s_1) = s_1$, $\nu_{B_i}(e_4) = s-s_1$, $\nu_{B_i}(e_5) = s-(s-s_1) = s_1$,

………………….....…………………..

$\nu_{B_i}(e_m) = s_1$ if m is odd,

$= s-s_1$ if m is even,

If $e_1$and e_{2n+1} are incident with u, $d_{\mu_{B_i}}(u) = r_1 + r_1 = 2r_1$ and $d_{\nu_{B_i}}(u) = s_1 + s_1 = 2s_1$

$d_{\mu_{B_i}}(u) = r$ and $d_{\nu_{B_i}}(u) = s$. $\Rightarrow 2r_1 = r$ and $2s_1 = s$

$\Rightarrow r_1 = r/2$ and $s_1 = s/2$

$\therefore d_{\mu_{B_i}}(e_m) = \frac{r}{2}$ and $d_{\nu_{B_i}}(e_m) = \frac{s}{2}$

$\Rightarrow \mu_{B_i}(u,v)$ and $\nu_{B_i}(u,v)$ are constant in R_i.

9) *Remark (3.9):* Note that theorem (3.8) also holds if $\tilde{G}^*$ is replaced by (supp (A), supp (B_1), supp (B_2),...,supp (B_k)).

10) *Theorem (3.10):* Let $\tilde{G}$ be an IFGS of G and $\tilde{G}^*$ = (V, R_1, R_2 ,..., R_k) be an even B_i-cycle. $\tilde{G}$ is B_i-regular iff $\mu_{B_i}(u,v)$ and $\nu_{B_i}(u,v)$ are constant for all B_i-edges in R_i or alternate B_i-edges in R_i have the same membership and non-membership value.

Proof: Let $\mu_{B_i}(u,v)$ and $\nu_{B_i}(u,v)$ be constant in R_i.

$\therefore$ $d_{\mu_{B_i}}(u)$ and $d_{\nu_{B_i}}(u)$ are constant $\forall\ u \in$ V.

If alternate B_i-edges in R_i have the same membership and non- membership value,

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$\Rightarrow d_{\mu_{B_i}}(u)$ and $d_{\nu_{B_i}}(u)$ are constant $\forall u \in$ V.

So $\tilde{G}$ is B_i-regular.

Conversely, Let $\tilde{G}$ be a (r,s) B_i-regular and let B_i-edges of R_i be $e_1, e_2, \ldots, e_{2n+1}$

$\therefore \mu_{B_i}(e_j) = r_1$ if j is odd,

$= r - r_1$ if j is even

and $\nu_{B_i}(e_j) = s_1$ if j is odd,

$= s - s_1$ if j is even,

If $r_1 = r - r_1$ and $s_1 = s - s_1$ $\Rightarrow 2r_1 = r$ and $2s_1 = s$

$\therefore d_{\mu_{B_i}}(e_j) = r = 2r_1 =$ constant and $d_{\nu_{B_i}}(e_j) = s = 2s_1 =$ constant.

$\Rightarrow \mu_{B_i}(u,v)$ and $\nu_{B_i}(u,v)$ are constant in R_i. If not, alternate B_i-edges in R_i have the same membership and non- membership value.

11) *Remark (3.11):* Note that theorem (3.10) is true if $\tilde{G}^*$ is replaced by (supp (A), supp (B_1), supp (B_2),…..,supp (B_k)).

12) *Definition (3.12):* The μ_{B_i}-size of an IFGS $\tilde{G}$ is $S_{\mu_{B_i}}(\tilde{G}) = \sum_{(u,v)\in B_i} \mu_{B_i}(u,v)$ and μ_{B_i}-size of $\tilde{G}$ is $S_{\nu_{B_i}}(\tilde{G}) = \sum_{(u,v)\in B_i} \nu_{B_i}(u,v)$.

13) *Definition (3.13):* The B_i -size of $\tilde{G}$ is $S_{B_i}(\tilde{G}) = \left(S_{\mu_{B_i}}(\tilde{G}), S_{\nu_{B_i}}(\tilde{G})\right) = \left(\sum_{(u,v)\in B_i} \mu_{B_i}(u,v), \sum_{(u,v)\in B_i} \nu_{B_i}(u,v)\right)$ and the size of $\tilde{G}$ is

$$S(\tilde{G}) = \sum_{i=1}^{k} S_{B_i}(\tilde{G}) = \left(\sum_{i=1}^{k} S_{\mu_{B_i}}(\tilde{G}), \sum_{i=1}^{k} S_{\nu_{B_i}}(\tilde{G})\right).$$

14) *Definition (3.14):* The μ_A-order of $\tilde{G}$ is $O_{\mu_A}(\tilde{G}) = \sum_{u\in A} \mu_A(u)$ and ν_A- order of $\tilde{G}$ is $O_{\nu_A}(\tilde{G}) = \sum_{u\in A} \nu_A(u)$.

15) *Definition (3.15):* The B_i - order of $\tilde{G}$ is $O(\tilde{G}) = \left(O_{\mu_A}(\tilde{G}), O_{\nu_A}(\tilde{G})\right) = \left(\sum_{u\in A} \mu_A(u), \sum_{u\in A} \nu_A(u)\right)$.

16) *Theorem (3.16):* The B_i -size of (r,s) B_i-regular IFGS $\tilde{G}$ of G on $\tilde{G}^*$ = (V, R_1, R_2,..., R_k) is (nr/2,ns/2) where n is the no. of vertices in V. i.e. $S_{B_i}(\tilde{G}) = \left(\frac{nr}{2}, \frac{ns}{2}\right)$.

Proof: The B_i -size of $\tilde{G}$ is $S_{B_i}(\tilde{G}) = \left(\sum_{(u,v)\in B_i} \mu_{B_i}(u,v), \sum_{(u,v)\in B_i} \nu_{B_i}(u,v)\right)$.

$$\Rightarrow S_{B_i}(\tilde{G}) = \left(S_{\mu_{B_i}}(\tilde{G}), S_{\nu_{B_i}}(\tilde{G})\right).$$

Let $d_{\mu_{B_i}}(u) = r$ and $d_{\nu_{B_i}}(u) = s$ $\forall u \in$ V

But $\sum_{u\in V} d_{\mu_{B_i}}(u) = 2\sum_{(u,v)\in B_i} \mu_{B_i}(u,v) = 2S_{\mu_{B_i}}(\tilde{G})$ *and* $\sum_{u\in V} d_{\nu_{B_i}}(u) = 2\sum_{(u,v)\in B_i} \nu_{B_i}(u,v) = 2S_{\nu_{B_i}}(\tilde{G})$.

Let no. of vertices in V be n.

$$2S_{\mu_{B_i}}(\tilde{G}) = \sum_{u\in V} d_{\mu_{B_i}}(u) = \sum_{u\in V} r = nr \quad and \quad 2S_{\nu_{B_i}}(\tilde{G}) = \sum_{u\in V} d_{\nu_{B_i}}(u) = \sum_{u\in V} s = ns.$$

$$\Rightarrow S_{\mu_{B_i}}(\tilde{G}) = \frac{nr}{2} \quad and \quad S_{\nu_{B_i}}(\tilde{G}) = \frac{ns}{2}.$$

$$\Rightarrow S_{B_i}(\tilde{G}) = \left(\frac{nr}{2}, \frac{ns}{2}\right).$$

17) *Remark (3.17):* If $\tilde{G}^*$ is replaced by (supp (A), supp (B_1), supp (B_2),... ,supp (B_k)), then theorem (3.16) also holds.

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18) *Theorem (3.18):* Let $\tilde{G}$ be a B_i-regular IFGS of G where $\widetilde{G^*}$ is a R_i-cycle then $\tilde{G}$ is B_i- cycle and it cannot be an intuitionistic fuzzy B_i –tree.

Proof: Let $\tilde{G}$ be a B_i-regular on R_i-cycle $\widetilde{G^*}$.

Then $\mu_{B_i}(u,v)$ and $\nu_{B_i}(u,v)$ are either constant for all B_i-edges in R_i or alternate B_i-edges in R_i have the same membership and non- membership value.

$\therefore$ there does not exist unique B_i-edge (u,v) in R_i s,t, $\mu_{B_i}(u,v) = \wedge \mu_{B_i}(x,y)$ *and* $\nu_{B_i}(u,v) = \vee \nu_{B_i}(x,y)$

$\Rightarrow \tilde{G}$ is an intuitionistic fuzzy B_i-cycle.

$\Rightarrow \tilde{G}$ cannot be an intuitionistic fuzzy B_i-tree.

19) *Remark (3.19):* Note that theorem (3.18) is true if $\widetilde{G^*}$ is replaced by (supp (A), supp (B_1), supp (B_2),….,supp (B_k)).

20) *Theorem (3.20):* Let $\tilde{G}$ be a B_i-regular IFGS of G on an odd B_i-cycle then $\tilde{G}$ does not have an intuitionistic fuzzy B_i-bridge. Hence $\tilde{G}$ does not have an intuitionistic fuzzy B_i-cut vertex.

Proof: Let $\tilde{G}$ be a B_i-regular on an odd B_i-cycle $\widetilde{G^*}$

Then $\mu_{B_i}(u,v)$ and $\nu_{B_i}(u,v)$ are constant for all B_i-edges in R_i by theorem (3.8).

$\Rightarrow$ If we remove an intuitionistic fuzzy B_i-bridge, it will not reduce the strength of connectedness between any pair of vertices.

$\Rightarrow \tilde{G}$ will not have an intuitionistic fuzzy B_i-bridge.

Also a vertex is an intuitionistic fuzzy B_i–cut vertex iff it is a common vertex of two B_i-bridges.

$\therefore$ $\tilde{G}$ does not have an intuitionistic fuzzy B_i–cut vertex.

21) *Theorem (3.21):* Let $\tilde{G}$ be a B_i-regular IFGS of G on an even B_i-cycle then $\tilde{G}$ either does not have an intuitionistic fuzzy B_i-bridge or it has $r_i/2$ intuitionistic fuzzy B_i-bridges where $r_i = |B_i|$ i.e. r_i = no. of edges. Also $\tilde{G}$ does not have a B_i –cut vertex.

Proof: Let $\tilde{G}$ be a B_i-regular on an even B_i-cycle $\widetilde{G^*}$.

Then by theorem (3.10), $\mu_{B_i}(u,v)$ and $\nu_{B_i}(u,v)$ are either constant for all B_i-edges in R_i or alternate B_i-edges in R_i have the same membership and non- membership value.

a) *Case I:* Let $\mu_{B_i}(u,v)$ and $\nu_{B_i}(u,v)$ be a constant in R_i.

$\Rightarrow$ If we remove an intuitionistic fuzzy B_i-edge, it will not reduce the strength of connectedness between any pair of vertices.

$\Rightarrow \tilde{G}$ will not have an intuitionistic fuzzy B_i-bridge.

$\Rightarrow \tilde{G}$ will not have a B_i-cut vertex.

b) *Case II:* Alternate B_i-edges in R_i have the same membership and non- membership value.

$\widetilde{G^*}$ is a B_i-cycle.

$\therefore$ B_i-edges with greater membership and lesser non- membership value are the B_i-bridges of $\tilde{G}$.

$\Rightarrow$ there are $r_i/2$ such B_i-edges where r_i = no. of edges.

Two B_i-bridges will not have a common vertex.

Hence $\tilde{G}$ does not have a B_i-cut vertex.

22) *Remark (3.22):* If $\widetilde{G^*}$ is replaced by (supp (A), supp (B_1), supp (B_2),...,supp (B_k)), then theorem (3.21) is true.

23) *Theorem (3.23):* A B_i-connected (r,s) B_i-regular IFGS $\tilde{G}$ of G where r > 0, s > 0 with no. of vertices greater than or equal to 3, cannot have an end vertex of B_i–paths.

Proof: Let $d_{\mu_{B_i}}(u) > 0$ and $d_{\nu_{B_i}}(u) > 0$ $\forall u \in$ V.

$\Rightarrow$ each vertex is adjacent to atleast one vertex by a B_i-edge.

If possible, let v be an end vertex of a B_i–path.

Let (u,v) be in R_i , $d_{\mu_{B_i}}(u)$ = r = $\mu_{B_i}(u,v)$ and $d_{\nu_{B_i}}(u)$ = s = $\nu_{B_i}(u,v)$

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$\Rightarrow \tilde{G}$ is B_i-connected and no. of vertices are greater than or equal to 3.

$\Rightarrow u$ is adjacent to some vertex $z \neq v$ by a B_i –edge.

$\Rightarrow d_{\mu_{B_i}}(u) \geq \mu_{B_i}(u,v)+\mu_{B_i}(v,z) > \mu_{B_i}(u,v) = r$ and $d_{\nu_{B_i}}(u) \geq \nu_{B_i}(u,v)+\nu_{B_i}(v,z) > \nu_{B_i}(u,v) = s.$

$\Rightarrow$ $d_{\mu_{B_i}}(u) > r$ and $d_{\nu_{B_i}}(u) > s$ which is a contradiction.

Hence $\tilde{G}$ cannot have an end vertex of B_i–paths.

24) *Definition (3.24):* The minimum μ_{B_i}-degree of $\tilde{G}$ is the minimum of $d_{\mu_{B_i}}(u)$. Thus, $\delta_{\mu_{B_i}}(u) = \wedge\{d_{\mu_{B_i}}(u) : u \in A\}$ and the minimum ν_{B_i}- degree of $\tilde{G}$ is the minimum of $d_{\nu_{B_i}}(u)$.

i.e. $\delta_{\nu_{B_i}}(u) = \wedge\{d_{\nu_{B_i}}(u) : u \in A\}$.

25) *Definition (3.25):* The minimum B_i-degree of $\tilde{G}$ is $\delta_{B_i}(\tilde{G}) = \left(\delta_{\mu_{B_i}}(\tilde{G}),\ \delta_{\nu_{B_i}}(\tilde{G})\right)$ *or* $\delta_{B_i}(\tilde{G}) = \wedge\left\{\left(d_{\mu_{B_i}}(u), d_{\nu_{B_i}}(u)\right) : u \in A\right\}$ and the minimum degree of $\tilde{G}$ is $\delta(\tilde{G}) = \wedge\left\{\delta_{B_i}(\tilde{G}) : i = 1,2,...,k\right\}$. *i.e..* $\delta(\tilde{G}) = \left(\wedge\delta_{\mu_{B_i}}(\tilde{G}),\ \wedge\delta_{\nu_{B_i}}(\tilde{G})\right)$ where $i = 1,2,...,k$

26) *Definition (3.26):* The maximum μ_{B_i}-degree of IFGS $\tilde{G}$ is the maximum of $d_{\mu_{B_i}}(u)$. Thus, $\Delta_{\mu_{B_i}}(u) = \vee\{d_{\mu_{B_i}}(u) : u \in A\}$ and the maximum ν_{B_i}- degree of $\tilde{G}$ is the maximum of $d_{\nu_{B_i}}(u)$.

i.e., $\Delta_{\nu_{B_i}}(u) = \vee\{d_{\nu_{B_i}}(u) : u \in A\}$.

27) *Definition (3.27):* The maximum B_i-degree of $\tilde{G}$ is $\Delta_{B_i}(\tilde{G}) = \left(\Delta_{\mu_{B_i}}(\tilde{G}), \Delta_{\nu_{B_i}}(\tilde{G})\right)$ *or* $\Delta_{B_i}(\tilde{G}) = \vee\left\{\left(d_{\mu_{B_i}}(u), d_{\nu_{B_i}}(u)\right) : u \in A\right\}$ and the maximum degree of $\tilde{G}$ is $\Delta(\tilde{G}) = \vee\left\{\Delta_{B_i}(\tilde{G}) : i = 1,2,...,k\right\}$. *i.e..* $\Delta(\tilde{G}) = \left(\vee\Delta_{\mu_{B_i}}(\tilde{G}),\ \vee\Delta_{\nu_{B_i}}(\tilde{G})\right)$ where $i = 1,2,...,k.$

28) *Definition (3.28):* The minimum degree of IFGS $\tilde{G}$ is the minimum of $\delta_{B_i}(\tilde{G})$ $\therefore \delta(\tilde{G}) = \wedge\left\{\delta_{B_i}(\tilde{G}) : i = 1,2,3,....,k\right\}$ and the maximum degree of IFGS $\tilde{G}$ is the maximum of $\Delta_{B_i}(\tilde{G})$ $\therefore \Delta(\tilde{G}) = \vee\left\{\Delta_{B_i}(\tilde{G}) : i = 1,2,3,....,k\right\}$.

29) *Proposition (3.29):* In an IFGS, if we take sum i.e. addition of membership degree and non membership degree of all the nodes (or vertices) of IFGS, it is always equal to double the sum of membership value and non membership value of all the edges of IFGS.

i.e. $\sum d_{B_i}(u) = \left(\sum d_{\mu_{B_i}}(u), \sum d_{\nu_{B_i}}(u)\right)$

Or
$$= \left(2\sum_{u \neq v} \mu_{B_i}(u,v),\ 2\sum_{u \neq v_i} \nu_{B_i}(u,v)\right)$$

Proof: Let $\tilde{G}$ be an IFGS and let V = { $u_1, u_2, u_3, \ldots\ldots, u_n$ }

$$\sum d_{B_i}(u_j) = \left(\sum d_{\mu_{B_i}}(u_j), \sum d_{\nu_{B_i}}(u_j)\right) \qquad \forall u_j \in V \quad for \quad j = 1,2,....,n.$$

$$\sum d_{B_i}(u_j) = \left(d_{\mu_{B_i}}(u_1), d_{\nu_{B_i}}(u_1)\right) + \left(d_{\mu_{B_i}}(u_2), d_{\nu_{B_i}}(u_2)\right) + \ldots\ldots \left(d_{\mu_{B_i}}(u_n), d_{\nu_{B_i}}(u_n)\right)$$

$$= \left(\mu_{B_i}(u_1,u_2), \nu_{B_i}(u_1,u_2)\right) + \left(\mu_{B_i}(u_1,u_3), \nu_{B_i}(u_1,u_3)\right) + \ldots\ldots\ldots\ldots + \left(\mu_{B_i}(u_1,u_n), \nu_{B_i}(u_1,u_n)\right) +$$
$$\left(\mu_{B_i}(u_2,u_1), \nu_{B_i}(u_2,u_1)\right) + \left(\mu_{B_i}(u_2,u_3), \nu_{B_i}(u_2,u_3)\right) + \ldots\ldots\ldots\ldots + \left(\mu_{B_i}(u_2,u_n), \nu_{B_i}(u_2,u_n)\right) +$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots +$$
$$+ \left(\mu_{B_i}(u_n,u_1), \nu_{B_i}(u_n,u_1)\right) + \left(\mu_{B_i}(u_n,u_2), \nu_{B_i}(u_n,u_2)\right) + \ldots\ldots\ldots\ldots + \left(\mu_{B_i}(u_n,u_{n-1}), \nu_{B_i}(u_n,u_{n-1})\right)$$
$$= 2\left(\left(\mu_{B_i}(u_1,u_2), \nu_{B_i}(u_1,u_2)\right) + \left(\mu_{B_i}(u_1,u_3), \nu_{B_i}(u_1,u_3)\right) + \ldots\ldots\ldots\ldots + \left(\mu_{B_i}(u_1,u_n), \nu_{B_i}(u_1,u_n)\right)\right)$$
$$= \left(2\sum_{u \neq v} \mu_{B_i}(u,v),\ 2\sum_{u \neq v_i} \nu_{B_i}(u,v)\right)$$

Hence the result is proved.

30) *Example (3.30):* Consider IFGS $\tilde{G}$ and V as shown in Example (3.5), clearly

Here $\sum_{j=0}^{5} d(u_j) = (7, 4.8)$ *and* $\left(2\sum_{u\neq v}\mu_{B_i}(u_j,u_s), 2\sum_{u\neq v_i}\nu_{B_i}(u_j,u_s)\right) = (2\times 3.5, 2\times 2.4) = (7, 4.8).$

31) *Proposition (3.31):* In an IFGS having n vertices, the maximum membership degree and maximum non membership degree of any vertex is n-1.

Proof: Let $\tilde{G}$ be an IFGS and no. of vertices = n.

The maximum membership degree of any edge is 1and the no. of edges incident on a vertex can be atmost n-1.
Thus, the maximum degree of membership of any vertex in an IFGS having n vertices is n-1.
Similarly, the maximum degree of non membership of any edge is 1and the no. of edges incident on a vertex can be atmost n-1.
Thus, the maximum degree of non membership of any vertex in an IFGS having n vertices is n-1.

32) *Proposition (3.32):* In an IFGS, the number of vertices with odd degree of membership and that of odd degree of non membership is even.

Proof: Let $\tilde{G}$ be an IFGS and V be the set of all vertices.

Let the vertices have both odd and even degree of membership.

$$\sum d_{\mu_{B_i}}(u) = \sum_{odd} d_{\mu_{B_i}}(u) + \sum_{even} d_{\mu_{B_i}}(u), \quad \forall\, u \in V \qquad \text{-- (1)}$$

where $\sum d_{\mu_{B_i}}(u)$ = Sum of the degree of membership of all the vertices in $\tilde{G}$

= sum of 2 sums each of taken over vertices with degree of membership as even and odd respectively.

Also by above Proposition (3.29),

$$\sum d_{\mu_{B_i}}(u) = 2\sum_{u\neq v}\mu_{B_i}(u,v) \qquad \text{-- (2)}$$

$\Rightarrow \sum d_{\mu_{B_i}}(u)$ *is even by above equation* (2)

$\sum_{even} d_{\mu_{B_i}}(u)$ is even since it is the sum of even numbers,

$\sum_{odd} d_{\mu_{B_i}}(u)$ is also even (by equation (1))

$\therefore \sum_{odd} d_{\mu_{B_i}}(u)$ = even no.

$\because$ *each* $\sum d_{\mu_{B_i}}(u)$ *is odd*

$\Rightarrow$ *the* total no. of terms in the sum = even

($\because$ their sum is an even no.)

$\therefore$ The number of vertices with odd degree of membership is even.

Similarly the number of vertices with odd degree of non membership is also even.

33) *Example (3.33):* Consider IFGS $\tilde{G}$ as given in example (3.5),

$d(u_0) = (1.6, 0.9),\ d(u_1) = (1.4, 0.4), d(u_2) = (1.4, 1.2),$
$d(u_3) = (0.7, 1.0), d(u_4) = (1.0, 0.7), d(u_5) = (0.9, 0.6).$

In this example, vertices with odd degree of membership are u_3 and u_5 and their no. = 2 (even).
Similarly the number of vertices with odd degree of non membership (i.e. u_0 and u_4) = 2(even).
Degree of membership of u_3 = 0.7, Degree of membership of u_5 = 0.9,
Degree of membership of u_0 = 0.9, Degree of membership of u_4 = 0.7
These are all odd.

34) *Proposition (3.34):* Let $\tilde{G}$ be a complete IFGS, then it will have at least one pair of vertices with same degree of membership and at least one pair of vertices with same degree of non membership.

Proof: Let $\tilde{G}$ be a complete IFGS and V = {$u_1, u_2, u_3, \ldots, u_n$} be the set of all vertices.

Case I: Let $\mu_A(u_j)$ and $\nu_A(u_j)$ are equal $\forall u_j \in$ V,

Then clearly $\mu_{B_i}(u_j, u_k)$ and $\nu_{B_i}(u_j, u_k)$ are all equal.

$\Rightarrow$ The B_i-degree of all vertices are all equal.

$\Rightarrow$ The degree of all vertices are all equal.

Therefore, the result is proved.

Case II: Let $\mu_A(u_j)$ and $\nu_A(u_j)$ are different $\forall$ $u_j \in$ V,

$$\Rightarrow d_{\mu_{B_i}}(u_j) = \sum_{u_j \neq u_k} \mu_{B_i}(u_j, u_k) \text{ and } d_{\nu_{B_i}}(u_j) = \sum_{u_j \neq u_k} \nu_{B_i}(u_j, u_k) \qquad (\because u_j \text{ are adjacent to } u_k)$$

Hence the required result is proved.

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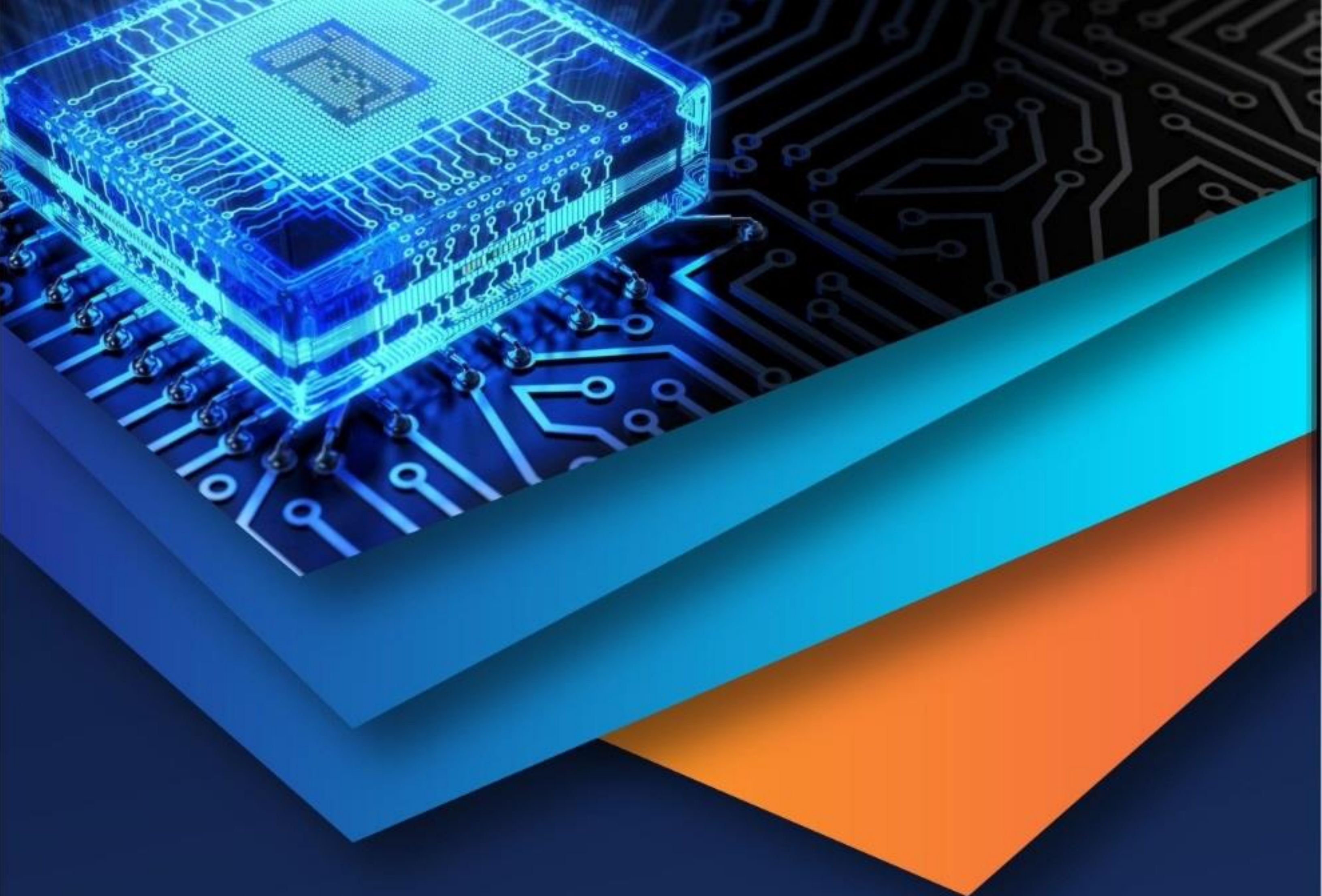

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