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Understanding the Concept of Mathematics and Mathematical Representation in Mathematics Teaching

Novi Simbolon¹, Sawaluddin Nasution², Herman Mawengkang³

^{1, 2, 3}Departemen of Mathematics, Universitas Sumatera Utara

Abstract: *The inclusion of representation as a standard component of the process in Principles and Standards for School Mathematics in addition to problem solving, reasoning, communication, and connection skills is reasonable because to think mathematics and communicate mathematical ideas one needs to represent it in various forms of mathematical representation. Besides, it can not be denied that objects in mathematics are all abstract so that to learn and understand abstract ideas that would require a representation. Representation occurs through two stages, namely internal representation and external representation. Examples of external representations include: verbal, drawing and concrete objects. Thinking of a mathematical idea that allows a person's mind to work on the basis of the idea is an internal representation. A mathematical problem posed to the student and the student can solve it, so at least the student understands the problem, so that students can plan the settlement, perform the calculations appropriately, and be able to check or review what has been processed correctly. The smoothness and flexibility of students in constructing representations is largely lacking. This is evident from at least the structured algebraic form, as well as the way in which most representations are found very little. In addition, the quantitative scores of respondents in the representation are still in the low category with a moderate tendency.*

Keywords: *Representation, Ability of student representation, understanding of mathematical concepts*

I. INTRODUCTION

The development of science and technology (Science and Technology) affects almost all human life in various fields. To be able to master the science and technology, the quality of human resources must be improved through improving the quality of lessons in school. Education not only aims to provide course material, but emphasizes how to invite students to find and build their own knowledge so that students can develop life skills (life skill) and ready to solve problems encountered in life. The objectives of mathematics subjects for all levels of primary and secondary education are so that students are able to: (1) Understand mathematical concepts, explain interconnectedness, and apply concepts or algorithms flexibly, accurately, efficiently and appropriately in problem solving, (2) Using reasoning on patterns and traits, performing mathematical manipulations in generalizing, compiling evidence, or explaining mathematical ideas and statements; (3) Solve problems that include the ability to understand problems, design mathematical models, complete models, and interpret the solutions obtained; (4) Communicating ideas with symbols, tables, diagrams, or other media to clarify circumstances or problems; and (5) Have an appreciation of the usefulness of mathematics in life, namely curiosity, attention, and interest in learning mathematics, as well as resilience and confidence in problem solving (Depdiknas, 2006).

In mathematics there are two phases of sensitivity, the name is number sense and variable sense. Number sense or often termed a sense of the number, is actually developed in such a way that the child is able to distinguish numbers that function as cardinal and ordinal. For variables, this is more difficult. Generally, mathematics learning, especially in junior high, introduces a variable by directly defining it as something symbolized by letters or alphabets, such as X, Y, and others, without going through the context that interpret it. Consequently many students are misconceptible in performing arithmetic operations on variables, example $a + b = 2ab$. Such errors often become obstacles in studying further mathematics, ie in high school or in college, even in applying mathematics in everyday life. Bruner (Ruseffendi, 1992) argues that the best way for children to learn concepts, theories and others in mathematics is to do the compilation of representations. In the initial steps of learning concepts, understanding will be more inherent when the activities that show the representation of the concept is done by the students themselves. For example in elementary school if the teacher or student wants to show a meaning 2, the students themselves present a set with 2 members. To understand the concept of addition, for example $2 + 3 = 5$, students do 2 consecutive steps, 2 boxes and 3 boxes on the number line map. The actual

representation does not indicate to the result or product embodied in a new or different configuration or construct, but a process of thinking undertaken to be able to uncover and understand the concepts, operations, and mathematical relationships of a configuration. That is, the process of mathematical representation takes place in two stages, namely internally and externally.

"For example given example a picture shows 3 groups with 5 pairs in each group. This image is then represented in the word "there are 5 pairs in each group", as the number of sentences used $5 + 5 + 5$ "symbols and hybrids used the numbers and the words" 3 baby. "This representation emphasizes the mathematical structure and its relation between various way represents multiplication That multiplication symbol is introduced several pages later as multiplication This means putting together the same group "Students are invited to see equality, say $5 + 5 + 5$ and 3×5 , with this representation placed next to each other on the page"

The above example shows how the process of internal representation that runs into the process of external representation relates to the basic concept of multiplication operations. Students must understand that multiplication operations are a form or a representation of a recurrent summation. If a teacher gives the multiplication representation directly through memorization, then the student will not mean what is meant by multiplication operation. Goldin (2002) argues that understanding the more important mathematical concepts is not the storage of past experiences, but how to recover the knowledge that has been stored in memory and relevant to the needs and can be used when necessary. The process of gaining relevant knowledge and its use is closely related to the coding of the past experience. The process is a mental activity, which is therefore called an internal representation. The internal representation of course can not be observed visually and consequently can not be judged, what is in the mind (minds on) is unknown. However, the manifestation of the minds on will be seen in words (spoken) or written in the form of statements, symbols, expressions, mathematical notations, images, graphics, and in other forms. The embodiment is called an external representation.

II. RESULTS AND DISCUSSION

Mathematic learning serves to develop the ability to calculate, measure, lower the formula, and use the mathematical formulas necessary in everyday life. So through math learning activities students can develop the ability to find, examine, use and can make generalizations. Therefore, the conceptual understanding must be really paid attention by the teacher and its usage.

Understanding The mathematical concept refers to a basic understanding. Students develop a concept when they are able to classify or classify objects or when they can associate a name with a particular group of objects. For example, the child recognizes the concept of a triangle as a plane surrounded by three straight lines. The child's understanding of the concept of a triangle can be seen when the child is able to distinguish the various other geometric shapes of the triangle. Another example is when the child calculates the multiplication of $2 \times 10 = 20$, $3 \times 10 = 30$, and $4 \times 10 = 40$, the child understands the concept of multiplication 10, the number is followed by 0. If the concept refers to a basic understanding, something someone does. For example, the process of using basic operations in addition, subtraction, multiplication and division is a type of mathematical skill. A skill can be seen from the child's performance well or poorly, quickly or slowly, easily or very difficult. Skills tend to thrive and can be improved through practice.

Troubleshooting is an application of conceptual understanding. In problem solving usually involves some combination of concepts and representations in different situations or different situations. For example, when students are asked to measure the breadth of a board, some concepts and representations are involved. Some of the concepts involved include square, parallel lines, and sides and some representations involved are representations of measuring, summing and multiplying.

A. Difficulty of Understanding Mathematical Concept of Students in Mathematics Learning

Understanding or comprehension is a level of ability that expects students to be able to understand the meaning or concepts, situations and facts that they know. In this case students not only memorized verbalistically, but understand the concept of the problem or facts asked. In order to assist the child in improving the understanding of the concept of mathematics learning, teachers need to recognize the common mistakes made by the child in completing tasks in the field of mathematics studies. In this case, there will be some errors related to the addition, subtraction, and multiplication of low grade students. As for some of these mistakes are lack of understanding of symbols, place values, calculations, the use of false processes and unreadable writing.

- 1) Lack of understanding of symbols, children generally do not have too much trouble if they are presented with questions like:

$$4 + 3 = \dots; \text{ and } 8 - 5 = \dots,$$

but will find it difficult if faced with such problems:

$$4 + \dots = 7; \quad 8 = \dots + 5; \quad \dots + 3 = 6; \quad \dots - 4 = 7; \text{ and } 8 - \dots = 5.$$

This kind of difficulty, generally because the child does not understand symbols like equal ($=$), not equal to ($\neq$), plus ($+$), less ($-$), and so on. So that children can solve mathematical problems, they must first understand the symbols.

- 2) The value of the place, there are children who do not understand the value of places such as units, tens, hundreds and so on. Uncertainty about the value of the place will make it more difficult for the child if they are faced with the base number symbol instead of ten. Therefore, many suggest that mathematics lessons in primary schools emphasize more on arithmetic or arithmetic that can be used directly in everyday life. Misconceptions about the place value are shown by children as follows:

$$\begin{array}{r} 75 \\ -27 \\ \hline 52 \end{array} \quad \begin{array}{r} 68 \\ -13 \\ \hline 71 \end{array}$$

Children who experience such mistakes can also forget to calculate the reduction or the sum is lowered so that the child is not only enough to understand the value of the place but also given enough training.

- 3) *The use of false processes.* The error in using the calculation process can be seen in the following example:

- a) Exchanging symbols

$$\begin{array}{r} 6 \\ 2 \\ -8 \\ \hline \end{array} \times \quad \begin{array}{r} 15 \\ 3 \\ -18 \\ \hline \end{array} \times$$

- b) The number of units and dozens are written regardless of place value

$$\begin{array}{r} 83 \\ 67 \\ -1410 \\ \hline \end{array} + \quad \begin{array}{r} 66 \\ 29 \\ -815 \\ \hline \end{array} +$$

- c) All the digits are added together (the algorithm is erroneous and does not pay attention to place value)

$$\begin{array}{r} 67 \\ -32 \\ 14 \\ \hline \end{array} - \quad \begin{array}{r} 58 \\ -12 \\ 16 \\ \hline \end{array} -$$

- d) Digits are added from left to right and do not pay attention to place values.

$$\begin{array}{r} 21 \\ 476 \\ 851 \\ -148 \\ \hline \end{array} + \quad \begin{array}{r} 37 \\ 753 \\ 693 \\ -1113 \\ \hline \end{array} +$$

- e) In summing dozens merged with the unit

$$\begin{array}{r} 68 \\ 8 \\ -166 \\ \hline \end{array} + \quad \begin{array}{r} 73 \\ 9 \\ -172 \\ \hline \end{array} +$$

- f) Large numbers minus small numbers regardless of place value

$$\begin{array}{r} 627 \\ 486 \\ -261 \\ \hline \end{array} - \quad \begin{array}{r} 761 \\ 489 \\ -328 \\ \hline \end{array} -$$

- g) Numbers that have been borrowed are used.

$$\begin{array}{r} 532 \\ 147 \\ -495 \\ \hline \end{array} - \quad \begin{array}{r} 423 \\ 366 \\ -167 \\ \hline \end{array} -$$

- 4) Calculations, there are children who are not familiar with the concept of multiplication, but trying to memorize the multiplication. This can lead to errors if the memorization is wrong. The error generally appears as follows

$$\begin{array}{r} 6 \\ 8 \\ -46 \\ \hline \end{array} \times \quad \begin{array}{r} 8 \\ 7 \\ -54 \\ \hline \end{array} \times$$

Multiplication lists may help to correct a child's mistake if the child has understood the concept of multiplication.

- 5) Writing unreadable, there are children who can not read their own writing, because the letters are incorrect or not straight lines. As a result, many children experience the error of not being able to read his own writing.

B. Efforts to Overcome Difficulty Learning Mathematics

Errors in the basic concepts of mathematics will cause children difficulty in learning the next concept, so it will be difficult also in learning math lesson, especially on the subject of addition, subtraction, division and multiplication. This is consistent with Gagne's theory of learning which argues that the child will only be able to complete a task, if he mastered the preceding subtasks which are prerequisites for completing the task. So the need for an effort by the teacher in overcoming these difficulties. Among the efforts made are: First, teachers should consider the principles of teaching mathematics. Secondly, Teachers need to provide various activities in learning mathematics, so the emphasis is not on memorizing.

C. Various Principles of Mathematics Teaching

There are several principles in learning mathematics that not only apply in the teaching of mathematics.

1) The Need To Prepare Children To Study Mathematic

One of the difficulties in learning mathematics is caused by the lack of readiness of children to study the field of study of mathematics. It takes a lot of time and energy to build a child's learning readiness, so that children do not experience many problems in the field of mathematics. The examples of various forms of learning activities that are the foundation for children in learning mathematics, that is:

- a) Grouping objects by their nature
- b) Know the number of group members of objects
- c) Counting objects
- d) Name the number that appears after a certain number such as "Number what happens after the number 6 "?
- e) Write numbers from 0 to 10 in the correct order.
- f) Measure and divide
- g) Sort objects from the largest to the smallest, long ones to the short ones
- h) Compile the parts into a whole.

2) *Starting From The Concrete To The Abstract*: Students can understand concepts in mathematics well if the teaching starts from the concrete to the abstract. Teachers should design three stages of learning: 1. Concrete, 2. Representation and 3. Abstract. At a concrete stage, students manipulate real objects in learning skills. For example, at a concrete stage, students must see, feel, and move 2 blocks and 3 beams to learn that they number 5 beams. In the representation stage, an image can represent a real object. As an example;

$0000 + 000 = 7$ In the abstract stage, the numbers end up replacing graphic images or symbols, for example:

$$4 + 3 = 7$$

- 3) *Providing Opportunities For Children To Practice And Repeatmengulang*: If students are required to apply concepts almost automatically, they need a lot of practice and repetition. There are many ways to provide practice and teachers should use varied methods.
- 4) *Generalize into a new situation*: Students should have sufficient opportunities to generalize their skills in many situations. For example, students can practice computing with many of the stories created by teachers or students themselves. The goal is to acquire skills in recognizing and applying computational operations to different new situations.
- 5) *Starting from the strengths and weaknesses of students*: Prior to making decisions about the techniques to be used for teaching students, teachers must understand the students' abilities and disabilities, including mathematical mastery and operations performed by students. To understand students' abilities and disabilities, there are several questions that the teacher must answer:
 - a) How does a student's inability affect mathematics learning?
 - b) How long does it take to get back to form a solid foundation for learning math?
 - c) Being aware of these abilities and inadequacies, what techniques, approaches, and learning materials will be used?
 - d) Are the students able to understand the meaning of the spoken number?
 - e) Can students read and write numbers?
 - f) Can the child perform basic operations?

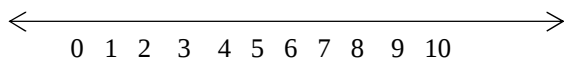
- g) Can the child determine which is greater and which is smaller?
- h) To what extent do students' language skills make learning mathematics difficult?
- i) Are there memory and attention problems that interfere with learning math. Questions can be continued as an attempt to understand students' abilities and disabilities.
- 6) The need to build a strong foundation of mathematical concepts and skills. Learning mathematics should be built on the foundations of concepts and skills. A solid foundation can be obtained if the teacher:
 - a) Emphasizing mathematics learning is more on giving answers to problems than memorizing without understanding.
 - b) Provide ample opportunity for students to generalize to various applications and experiences with various ways to solve problems from what they learn.
 - c) Teach mathematics coherently, which links between topics of one topic to another.
 - d) Present a careful learner so that students get the necessary training, and
 - e) Using a systematic program that allows concepts and skills to be taught stands on well-skilled concepts and skills.
- 7) *Provision of a balanced mathematics* : A balanced mathematical program includes a combination of three elements
 - a) Concepts,
 - b) The skills,
 - c) Troubleshooting.

All three elements must be taught in a balanced and interrelated manner.
- 8) *Use of the Calculator*: The calculator can be used after the students have the calculation skills. Thus, the use of a calculator is not to inculcate calculation skills but to instill mathematical reasoning. Many students are stuck in computing or calculation so they do not get to the reasoning aspects of a lesson. Using a child's calculator can be free from understanding the underlying mathematical concepts of the calculation. The calculator can be used for training or self-checking (self checking).

D. Various Activities For Teaching

Teaching activities should cover three categories: concepts, skills and problem solving.

- 1) *Teaching The concept of mathematics*: The concept of shape and size can be taught through game sorting. To children are given pieces of board or plastic that has different shapes and sizes. To embed the concept of shape and size, the child is asked to sort through the pieces according to their shape or size. The color concept can also be embedded through this game. Segregation should start from a simple, ie a corner only, such as shape, size, or color. If simple sorting can be done well, the game can be upgraded to complex sorting, for example breaking pieces of the same shape and size. The concept of numbers is known to children from their ability to focus attention on a single object. Therefore, to introduce the concept of numbers, the child can be invited to find objects that are similar to those shown by the teacher of a group of objects that have various properties. Members of the group of objects may differ both in terms of color, shape and size. Games using dominoes or the like can also be used to introduce the concept of numbers, groups and numbers. The concept of amounts can be taught to the child through a pair of removable boards, the left hemisphere contains a bunch of object images, and the right hemisphere contains numbers corresponding to the number of images in the left hemisphere. Playing by installing such boards children can learn about the concept of numbers. The sequence and relationship concepts can be planted through various questions asked to the child such as "what number after number 5?" Or what number is located between the numbers 5 and 7? "And so on. Before the sequence of numbers, a seating sequence may be used, such as "Who sits between Ani and Budi?", And so on. The concept of number symbols can be taught to the child through the number line, as well as the relationship between these numbers. An example of using a number line is as follows:



The concept of a pattern can be taught through a game that asks the children to find a pattern by selecting objects in a sequence that the teacher has made. Example:

Red, white, red, white, ...

0 * ^, 0 * ^, 0 * ^, ...

2 4 8, 2 4 8, 2 4 8, ...

The concept of relationships between the various sizes can be taught by giving the children the same group of objects but having

different sizes, such as the length, magnitude or severity of the groups of objects, the child is asked to sort them from the longest to the shortest. From the biggest to the most small, and so forth.

There are children who count by memorization without understanding that there is a relationship between objects and numbers. Such a child needs to get help by counting things through seeing and groping the object. Activities should be made more complex, by counting jumps or counting down.

The concept of numbers should be taught by introducing the number itself, the number of objects pointing to the number, and the word indicating the number. As an example:

0	00	000	0000	00000
1	2	3	4	5
one	two	three	four	five

The concept of size can be taught by teaching children the length of the board, weighing the weight of things, or judging the amount of money. Measurements should start from the rough to the smooth, for example from step to meter, from inch to cm, from weighing by lifting goods to the use of scales, etc.

- 2) *Teaching math skills:* The other difficulties are caused by a lack of computational skills. The shortcomings should be evaluated to determine the underlying factors, eg due to verbal, spatial, perceptual or possibly due to memory. Various mathematical skills that need attention in early childhood learning include mathematics of addition, subtraction, multiplication, division and fraction. The skills of addition are fundamental to all computational skills. The summation is a short way to calculate, and students should know that they can take the count path if they fail with the sum. The sum can be taught of "partially plus partly equal to whole". Important symbols are + and =. Just as in other fields, instruction begins with the use of concrete objects, then by using the pictures and then by using numbers. The sum should be started from a simple thing, for example: $3 + 2 = \dots$, and from here it develops into $3 + \dots = 5$, and $\dots + 2 = 5$. Skills for subtraction are taught after the child understands the sum. As well as addition, subtraction also starts from the use of concrete objects, images and then numbers. Reduction can also be taught by using a number line. The ability to perform multiplication operations is closely related to addition and subdivision. A non-aggregate child can not multiply, and a child who can not multiply is also unable to share. Multiplication is essentially a short way of addition. Therefore, if the student can not perform the multiplication operation, the child may do so by summation. Reduction is not a prerequisite of multiplication. Therefore, a child who can not perform the abatement ability may be able to solve multiplication problems if he is able to do the sum. Multiplication can be taught using a number line and can also be the following way:

$$\begin{array}{rcl}
 3 \times 6 = \dots & & 000000 \\
 & & 000000 \\
 & & 000000 \\
 & & 000 \\
 & & 000 \\
 & & 000 \\
 & & 000
 \end{array}
 \quad \text{or} \quad
 \begin{array}{r}
 000 \\
 000 \\
 000 \\
 000 \\
 000 \\
 000 \\
 000
 \end{array}$$

Distribution is a computational skill that is considered the most difficult to learn and teach. The division is the opposite of multiplication. To control it, the child must first control the multiplication. The division can also be taught by the number line and can also be taught along with the multiplication. Examples of dividing work done in conjunction with multiplication are as follows:

$$2 \times 3 = 6 \quad \longrightarrow \quad \begin{array}{l} 6 : 2 = 3 \\ 6 : 3 = 2 \end{array}$$

Fractional numbers can be taught by using geometric shapes. The first symbols taught are $\frac{1}{2}$, the next is $\frac{1}{4}$, and the symbols should be shown using a circle image that is halved, divided into four and divided into eight equal.

III. CONCLUSIONS AND SUGGESTIONS

There are several principles of teaching mathematics, namely: First, the need to prepare children to learn mathematics. Second, from the concrete to the abstract. Third, the opportunity to practice and repeat enough. Fourth, generalisasi to new situations. Fifth, depart from the strengths and weaknesses of students. Sixth, the need to build a strong foundation of mathematical concepts and skills. Seventh, the provision of a balanced mathematics program. Eighth, the use of a calculator to embed mathematical reasoning. In addition, teaching activities should include three categories: concepts, skills and problem solving.

A mathematical problem posed to the student and the student can solve it, so at least the student understands the problem, so that students can plan the settlement, perform the calculations appropriately, and be able to check or review what has been processed correctly.

The mathematical representation aspect of the student can provide illustrations, translations, disclosures, reappraisals, mines, ideas, mathematical concepts, and relationships contained in a particular configuration, construction, or problem situation presented by students in various forms in an effort to gain clarity of meaning, showing his understanding, or finding solutions to the problems he faces. Some conclusions obtained from the results of this study are as follows:

A. Some conclusions Obtained From The Results of this Study Are As Follows

- 1) The forms of representation that are widely used by students are very diverse, among others, formal forms, tables, descriptions and drawings.
- 2) Most students solve problems using tables and drawings, only a small number of students use written representation statements and symbols.
- 3) Only a small percentage of students find the general form (mathematical model) of the representation used in answering questions.
- 4) The smoothness and flexibility of students in constructing representations is largely lacking. This is evident from at least the structured algebraic form, as well as the way in which most representations are found very little.
- 5) In addition, the quantitative scores of respondents in the representation are still in the low category with a moderate tendency.

From the conclusions obtained in this study, then there are some recommendations that would need to be considered by practitioners in the field and further research, among others: It is important to give students freedom in pouring their ideas related to mathematical problems, so that they will recognize various representations of a problem. School math learning needs to pay attention to students' thinking diversity, and students understand rules, postulate, and mathematical formulas in their thinking level. This will provide a bridge for students in constructing and understanding the representation of a problem..

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