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Some New Graphs on k-Super Mean Labeling

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Abstract: ***Let G be a*** (p,q) ***graph and*** $f : V(G) \to \{1,2,3 \ldots . p+q+k-1\}$ ***be an injection. For each edge*** $e = uv$, ***let*** $f^*(e) = \frac{f(u)+f(v)}{2}$ ***if*** $f(u)+f(v)$ ***is even and*** $f^*(e) = \frac{f(u)+f(v)+1}{2}$ ***if*** $f(u)+f(v)$ ***is odd ,then*** f ***is called k- Super mean labeling if*** $f(V) \cup \{f^*(e) : e \in E(G)\} = \{1,2,3, \ldots \ldots . p+q+k-1\}$. ***A graph that admits a k-Super mean labeling is called k-Super mean graph.***

In this paper we investigate k – super mean labeling of $< C_m , K_{1,n} >$ ***and*** $< C_m * K_{1,n} >$

Keywords: **k–Super mean labeling, k–Super mean graph,** $Q_n \odot K_1$, $[P_n : D(T_2)], T(C_n)$ ***AMS Subject Classification--- 05C78***

I. INTRODUCTION

All graphs in this thesis are finite, simple and undirected. Terms not defined here are used in the sense of Harary [7]. The symbols V(G) and E(G) will denote the vertex set and edge set of a graph G. Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967. Labeled graphs serve as useful models for a broad range of applications such as X-ray, crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. Particularly interesting applications of graph theory can be found in [1-4]. The concept of mean labeling was introduced and studied by S. Somasundaram and R. Ponraj [12]. The concept of super mean labeling was introduced and

studied by D. Ramya et al [11]. Further some results on super mean graphs are discussed in [8,9,10,13,15]. B. Gayathri and M. Tamilselvi [5-6, 14] extended super mean labeling to k-super mean labeling. In this paper we investigate k–super mean labeling of $S(Q_n \odot K_1)$, $[P_n : D(T_2)]$, $T(C_n)$ and $(C_3 \times P_n) \cup D(T_m)$. Here k denoted as any positive integer greater than or equal to 1.

II. MAIN RESULTS

A. Definition 2.1:

Let G be a (p, q) graph and $f: V(G) \to \{1,2,3, \ldots, p+q\}$ be an injection. For each edge e = uv, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd, then f is called super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2,3, \ldots, p+q\}$. A graph that admits a super mean labeling is called super mean graph.

B. Definition 2.2

Let G be a (p, q) graph and $f: V(G) \to \{k, k+1, k+2, \ldots, p+q+k-1\}$ be an injection. For each edge e = uv, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v))$ is odd, then f is called k - super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, k+2, \ldots, p+q+k-1\}$. A graph that admits a k - Super mean labeling is called k-Super mean graph.

C. Definition 2.3

A subdivision of a graph G is a graph resulting from the subdivision of each edge by a new vertex.

D. Definition 2.4

A quadrilateral snake (Q_n) is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to a new vertices v_i and w_i respectively and then joining v_i and w_i for $1 \le i \le n$.

E. Definition 2.5

A triangular snake (T_n) is obtained from a path by identifying each edge of the path with an edge of the cycle C_3.

F. Definition 2.6

A double triangular snake D(T_n) consists of two triangular snake that have a common path. That is, a double triangular snake is obtained from a path v_1, v_2, … , v_n by joining v_i and v_{i+1} to a new vertices w_i for i = 1,2,…,n-1and to a new vertices u_i for i =1,2,…,n-1.

G. Definition 2.7

The corona of Q_n with K_1 , $Q_n \odot K_1$ is the graph obtained by taking one copy of Q_n and n copies of K_1 and joining the i th vertex of Q_n with an edge to every vertex in the i th copy of K_1.

H. Theorem 2.8

The graph $S(Q_n \odot K_1)$ is a k-Super mean graph for all $n \geq 2$.

Proof:

Let $V\big(S(Q_n \odot K_1)\big) = \{u_i, v_i, v_i'; 1 \leq i \leq n\} \cup \{u_i', w_i, w_i', s_i, s_i', x_i, x_i', y_i, y_i', z_i; 1 \leq i \leq n-1\}$ and $E\big(S(Q_n \odot K_1)\big) = \{e_i'' = (u_i, v_i'), e_i''' = (v_i', v_i); 1 \leq i \leq n\} \cup \{e_i = (u_i, u_i'), e_i' = (u_i' u_{i+1}), e_i^{iv} = (u_i, w_i'), e_i^{v} = (w_i', w_i), e_i^{vi} = (w_i, z_i), e_i^{vii} = (z_i, s_i), e_i^{viii} = (w_i, x_i'), e_i^{ix} = (x_i', x_i), e_i^{x} = (y_i', y_i), e_i^{xi} = (s_i, y_i'), e_i^{xii} = (s_i', s_i), e_i^{xiii} = (u_{i+1}, s_i'); 1 \leq i \leq n\}$be the vertices and edges of $S(Q_n \odot K_1)$ respectively.

Define $f: V\big(S(Q_n \odot K_1)\big) \to \{k, k+1, k+2, \dots, 27n+k-23\}$ by

$f(u_i) = k + 27i - 23;\ 1 \leq i \leq n$
$f(v_i) = k + 27i - 27;\ 1 \leq i \leq n$
$f(v_1') = k + 2$
$f(v_i') = k + 27i - 29;\ 2 \leq i \leq n$
$f(w_i) = k + 27i - 18;\ 1 \leq i \leq n-1$
$f(w_i') = k + 27i - 21;\ 1 \leq i \leq n-1$
$f(x_i) = k + 27i - 13;\ 1 \leq i \leq n-1$
$f(x_i') = k + 27i - 15;\ 1 \leq i \leq n-1$
$f(z_i) = k + 27i - 5;\ 1 \leq i \leq n-1$
$f(y_i) = k + 27i - 4;\ 1 \leq i \leq n-1$
$f(y_i') = k + 27i - 12;\ 1 \leq i \leq n-1$
$f(s_i) = k + 27i - 9;\ 1 \leq i \leq n-1$
$f(s_i') = k + 27i + 2;\ 1 \leq i \leq n-1$
$f(u_i') = k + 27i - 17;\ 1 \leq i \leq n-1$

Now the induced edge labels are

$f^*(e_i) = k + 27i - 20;\ 1 \leq i \leq n-1$
$f^*(e_i') = k + 27i - 6;\ 1 \leq i \leq n-1$
$f^*(e_1'') = k + 3$
$f^*(e_i'') = k + 27i - 26;\ 2 \leq i \leq n$
$f^*(e_1''') = k + 1$
$f^*(e_i''') = k + 27i - 28;\ 2 \leq i \leq n$
$f^*\big(e_i^{iv}\big) = k + 27i - 22;\ 1 \leq i \leq n-1$
$f^*(e_i^{v}) = k + 27i - 19;\ 1 \leq i \leq n-1$
$f^*\big(e_i^{vi}\big) = k + 27i - 11;\ 1 \leq i \leq n-1$
$f^*\big(e_i^{vii}\big) = k + 27i - 7;\ 1 \leq i \leq n-1$
$f^*\big(e_i^{viii}\big) = k + 27i - 16;\ 1 \leq i \leq n-1$
$f^*\big(e_i^{ix}\big) = k + 27i - 14;\ 1 \leq i \leq n-1$
$f^*(e_i^{x}) = k + 27i - 8;\ 1 \leq i \leq n-1$
$f^*\big(e_i^{xi}\big) = k + 27i - 10;\ 1 \leq i \leq n-1$
$f^*\big(e_i^{xii}\big) = k + 27i - 3;\ 1 \leq i \leq n-1$

$f^*(e_i^{xiii}) = k + 27i + 3;\ 1 \le i \le n-1$

Here p = 13n-10, q = 14n-12 and p + q =27n-22.

Clearly, $f(V) \cup \{f^*(e): e \in E(S(Q_n \odot K_1))\} = \{k, k+1, \dots, k+27n-23\}$.

So f is a k – Super mean labeling.

Hence $S(Q_n \odot K_1)$ is a k – Super mean graph for all $n \ge 2$.

I. Example 2.9

100 – Super mean labeling of $S(Q_4 \odot K_1)$ is given in figure 1:

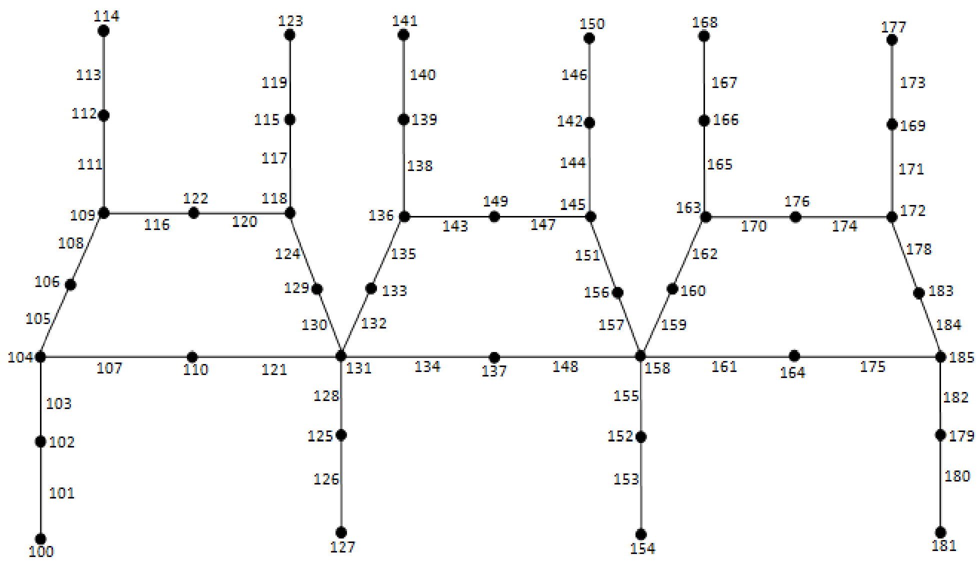

Figure 1: 100 –Super mean labeling of $S(Q_4 \odot K_1)$

J. Definition 2.10

Let G be a graph with fixed vertex v, and let $[P_m;G]$ be the graph obtained from m copies of G by joining v_i and v_{i+1} by means of an edge for some j and $1 \le i \le m-1$.

K. Theorem 2.11

The graph $[P_n: D(T_2)]$ is a k-Super mean graph for all $n \ge 2$.

Proof:

Let $V([P_n: D(T_2)]) = \{u_i, v_i, w_i, z_i; 1 \le i \le n\}$ and $E([P_n: D(T_2)]) = \{e_i = (u_i, u_{i+1}), e_i' = (u_i, v_i), e_i^{ii} = (u_i, w_i), e_i''' = (v_i, z_i), e_i^{iv} = (w_i, z_i), e_i^{v} = (v_i, w_i); 1 \le i \le n\}$ be the vertices and edges of $[P_n: D(T_2)]$respectively.

Define $f: V([P_n: D(T_2)]) \to \{k, k+1, k+2, \dots, k+10n-2\}$ by

$f(u_1) = k + 2$

$f(u_i) = k + 10i - 4;\ 2 \le i \le n$

$f(v_i) = k + 10i - 10;\ 1 \le i \le n$

$f(w_i) = k + 10i - 2;\ 1 \le i \le n$

$f(z_1) = k + 6$

$f(z_i) = k + 10i - 8\ ; 2 \le i \le n$

Now the induced edge labels are

$f^*(e_i) = k + 10i - 1;\ 1 \le i \le n-1$

$f^*(e_1') = k + 1$

$f^*(e_i') = k + 10i - 7;\ 2 \le i \le n$

$f^*(e_1'') = k + 5$

$f^*(e_i'') = k + 10i - 3;\ 2 \leq i \leq n$
$f^*(e_1''') = k + 3$
$f^*(e_i''') = k + 10i - 9;\ 2 \leq i \leq n$
$f^*(e_1^{iv}) = k + 7$
$f^*(e_i^{iv}) = k + 10i - 5;\ 2 \leq i \leq n$
$f^*(e_i^{v}) = k + 10i - 6;\ 1 \leq i \leq n$
Here p = 4n, q = 6n-1 and p + q =10n-1.
Clearly, $f(V) \cup \{f^*(e): e \in E([P_n: D(T_2)])\} = \{k, k+1, \dots, k+10n-2\}$.
So f is a k – Super mean labeling.
Hence $[P_n: D(T_2)]$ is a k – Super mean graph for all $n \geq 2$.

L. Example 2.12
425 – Super mean labeing of $[P_4: D(T_2)]$ is given in figure 2:

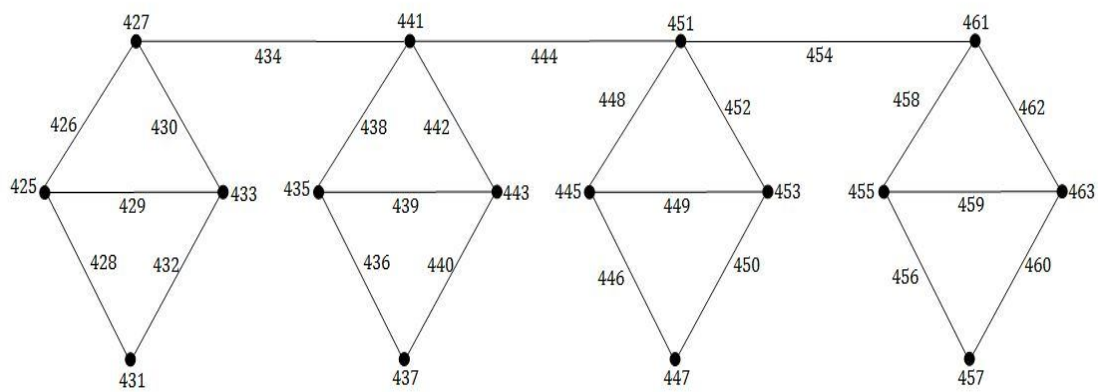

Figure 2: 425 –Super mean labeling of $[P_4: D(T_2)]$.

M. Definition 2.13
A total graph T(G) of a graph G is a graph whose vertex set is V(G) ∪E(G) and two vertices are adjacent in T(G) if they are adjacent or incident in G.

N. Theorem 2.14
The graph $T(C_n)$ is a k-Super mean graph, if n is odd and for all $n \geq 3$.
Proof:
Let $V(T(C_n)) = \{v_i, v_i'; 1 \leq i \leq n\}$ and $E(T(C_n)) = \{e_i = (v_i, v_{i+1}), e_i' = (v_i, v_i'), e_i^{ii} = (v_{i+1}, v_i'), e_i''' = (v_i', v_{i+1}'); 1 \leq i \leq n\}$be the vertices and edges of $T(C_n)$ respectively.
Let $n = 2l + 1$.
Define $f: V(T(C_n)) \to \{k, k+1, k+2, \dots, k+6n-1\}$ by
$f(v_i) = k + 2i - 2;\ 1 \leq i \leq l + 1$
$f(v_{l+1+i}) = k + 2(l+1) + 2i - 1;\ 1 \leq i \leq l$
$f(v_i') = k + 4n + 2i - 2; 1 \leq i \leq l + 1$
$f(v_{l+i+1}') = k + 4n + 2(l+1) + 2i - 1;\ 1 \leq i \leq l$
Now the induced edge labels are
$f^*(e_i) = k + 2i - 1;\ 1 \leq i \leq l$
$f^*(e_{l+i}) = k + 2l + 2i; 1 \leq i \leq l$
$f^*(e_n) = k + n$
$f^*(e_i') = k + 2n + 2i - 2;\ 1 \leq i \leq l + 1$

$f^*(e'_{l+i+1}) = k + 2n + 2(l+1) + 2i - 1;\ 1 \le i \le l$
$f^*(e''_i) = k + 2n + 2i - 1;\ 1 \le i \le l$
$f^*(e''_{l+1+i}) = k + 2n + 2(l+1) + 2i - 2;\ 1 \le i \le l$
$f^*(e^{ii}_n) = k + 3n$
$f^*(e^{iii}_i) = k + 4n + 2i - 1;\ 1 \le i \le l$
$f^*(e^{iii}_{l+i+1}) = k + 4n + 2l + 2i;\ 1 \le i \le l$
$f^*(e^{iii}_n) = k + 5n$
Here p = 2n, q = 4n and p + q =6n.
Clearly, $f(V) \cup \{f^*(e): e \in E(T(C_n))\} = \{k, k+1, \dots, k+6n-1\}$.
So f is a k – Super mean labeling.
Hence $T(C_n)$is a k – Super mean graph if n is odd and for all $n \ge 3$.

O. Example 2.15
99 – Super mean labeling of $T(C_7)$ is given in figure 3:

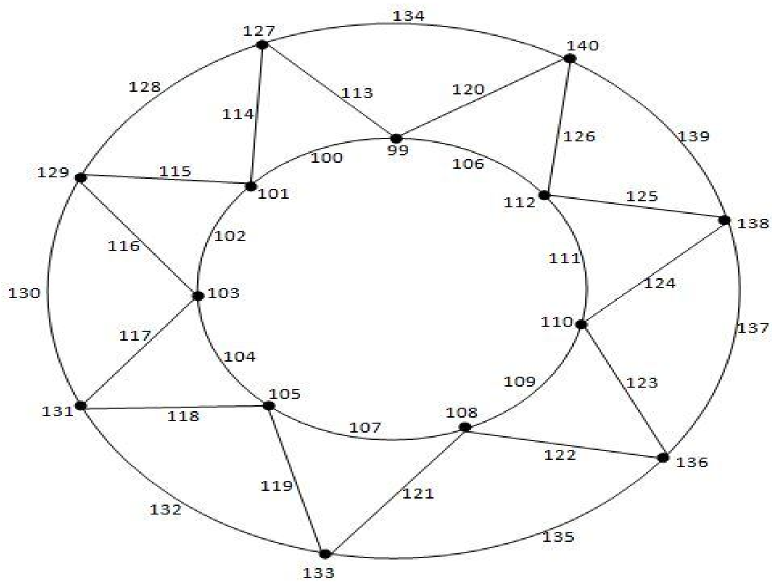

Figure 3: 149 – Super mean labeling of $T(C_7)$

P. Theorem 2.16
The graph $(C_3 \times P_n) \cup D(T_m)$ is a k-Super mean graph for all $m, n \ge 2$.
Proof:
Let us denote $(C_3 \times P_n) \cup D(T_m)$ by G.
Let $V(G) = \{u_i, v_i, w_i; 1 \le i \le n\} \cup \{x_i; 1 \le i \le m\} \cup$ and $E(G) = \{e_i = (u_i, w_i), e'_i = (u_i, v_i), e''_i = (v_i, w_i), ; 1 \le i \le n\} \cup \{e'''_i = (u_i, u_{i+1}), e^{iv}_i = (, v_i, , v_{i+1}), e^{v}_i = (w_i, w_{i+1}); 1 \le i \le n-1\} \cup \{e^{vi}_i = (x_i, x_{i+1}), e^{vii}_i = (x_i, y_i), e^{viii}_i = (x_{i+1}, y_i), e^{ix}_i = ((x_i, z_i), e^{x}_i = (x_{i+1}, z_i); 1 \le i \le m-1\}$be the vertices and edges of G respectively.
Define $f: V(G) \to \{k, k+1, k+2, \dots, k+9n+7m-10\}$ by

$$f(u_i) = \begin{cases} k + 9i - 7; & if\ i\ is\ odd, 1 \le i \le n \\ k + 9i - 4; & if\ i\ is\ even, 1 \le i \le n \end{cases}$$

$$f(v_i) = \begin{cases} k + 9i - 9; & if\ i\ is\ odd, 1 \le i \le n \\ k + 9i - 7; & if\ i\ is\ even, 1 \le i \le n \end{cases}$$

$$f(w_i) = \begin{cases} k + 9i - 4; & if\ i\ is\ odd, 1 \le i \le n \\ k + 9i - 9; & if\ i\ is\ even, 1 \le i \le n \end{cases}$$

$f(x_1) = \begin{cases} f(w_n) + 1; \text{ if } i \text{ is odd} \\ f(u_n) + 1; \text{ if } i \text{ is even} \end{cases}$

$f(x_i) = \begin{cases} f(w_n) + 8i - 7; \text{ if } i \text{ is odd}, 2 \le i \le m \\ f(u_n) + 8i - 7; \text{ if } i \text{ is even}, 2 \le i \le m \end{cases}$

$f(y_i) = \begin{cases} f(w_n) + 8i - 5; \text{ if } i \text{ is odd}, 1 \le i \le m-1 \\ f(u_n) + 8i - 5; \text{ if } i \text{ is even}, 1 \le i \le m-1 \end{cases}$

$f(z_i) = \begin{cases} f(w_n) + 8i - 1; \text{ if } i \text{ is odd}, 1 \le i \le m-1 \\ f(u_n) + 8i - 1; \text{ if } i \text{ is even}, 1 \le i \le m-1 \end{cases}$

Now the induced edge labels are

$f^*(e_i) = \begin{cases} k + 9i - 5; \text{ if } i \text{ is odd}, 1 \le i \le n \\ k + 9i - 6; \text{ if } i \text{ is even}, 1 \le i \le n \end{cases}$

$f^*(e_i') = \begin{cases} k + 9i - 8; \text{ if } i \text{ is odd}, 1 \le i \le n \\ k + 9i - 5; \text{ if } i \text{ is even}, 1 \le i \le n \end{cases}$

$f^*(e_i'') = \begin{cases} k + 9i - 6; \text{ if } i \text{ is odd}, 1 \le i \le n \\ k + 9i - 8; \text{ if } i \text{ is even}, 1 \le i \le n \end{cases}$

$f^*(e_i''') = k + 9i - 1;\ 1 \le i \le n-1$

$f^*(e_i^{iv}) = k + 9i - 3;\ 1 \le i \le n-1$

$f^*(e_i^{v}) = k + 9i - 2;\ 1 \le i \le n-1$

$f^*(e_i^{vi}) = \begin{cases} f(w_n) + 8i - 3; \text{ if } i \text{ is odd}, 1 \le i \le m-1 \\ f(u_n) + 8i - 3; \text{ if } i \text{ is even}, 1 \le i \le m-1 \end{cases}$

$f^*(e_i^{vii}) = \begin{cases} f(w_n) + 8i - 6; \text{ if } i \text{ is odd}, 1 \le i \le m-1 \\ f(u_n) + 8i - 6; \text{ if } i \text{ is even}, 1 \le i \le m-1 \end{cases}$

$f^*(e_i^{viii}) = \begin{cases} f(w_n) + 8i - 2; \text{ if } i \text{ is odd}, 1 \le i \le m-1 \\ f(u_n) + 8i - 2; \text{ if } i \text{ is even}, 1 \le i \le m-1 \end{cases}$

$f^*(e_i^{ix}) = \begin{cases} f(w_n) + 8i - 4; \text{ if } i \text{ is odd}, 1 \le i \le m-1 \\ f(u_n) + 8i - 4; \text{ if } i \text{ is even}, 1 \le i \le m-1 \end{cases}$

$f^*(e_i^{x}) = \begin{cases} f(w_n) + 8i; \text{ if } i \text{ is odd}, 1 \le i \le m-1 \\ f(u_n) + 8i; \text{ if } i \text{ is even}, 1 \le i \le m-1 \end{cases}$

Here p =3n+3m-2, q = 6n+4m-7and p + q =9n+7m-9.

Clearly, $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, \dots, k + 9n + 7m - 10\}$.

So f is a k – Super mean labeling.

Hence $(C_3 \times P_n) \cup D(T_m)$ is a k-Super mean graph for all $m, n \ge 2$.

Q. Example 2.17

20 – Super mean labeling of $(C_3 \times P_2) \cup D(T_3)$ is given in figure 4:

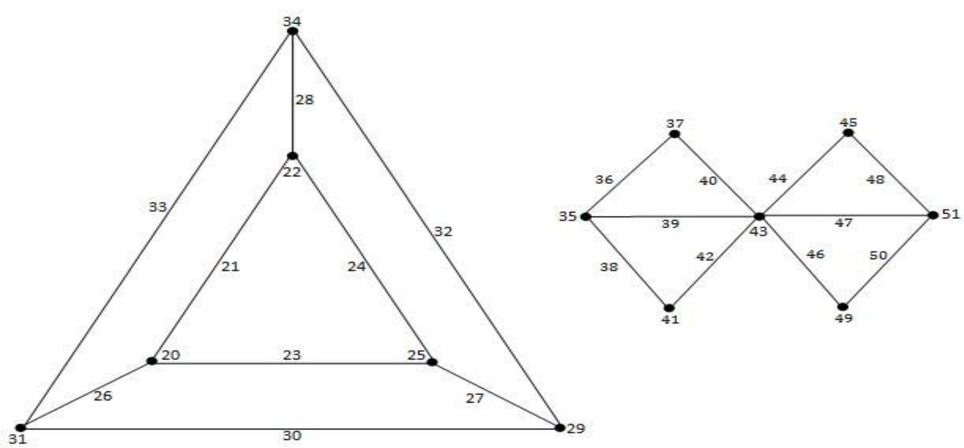

Figure 4: 20 – Super mean labeling of $(C_3 \times P_2) \cup D(T_3)$

III. CONCLUSIONS

Graph labeling has its own applications in communication networks and astronomy. so, enormous types of labeling have grown. Towards this, k-super mean labeling is also a kind of labeling which is an extension of super mean labeling. we discussed k-super mean labeling of the graphs $S(Q_n \odot K_1)$, $[P_n: D(T_2)]$, $T(C_n)$, $(C_3 \times P_n) \cup D(T_m)$.

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REFERENCES

[1] G.S. Bloom, S.W. Golomb, Applications of numbered undirected graphs, Proc. IEEE, 65 (1977), 562-570.
[2] G.S. Bloom, S.W. Golomb, Numbered complete graphsunusual rulers and assorted applications, Theory andApplications of Graphs-Lecture notes in Math., Springer Verlag, New York, 642 (1978), 53-65.
[3] G.S. Bloom, D.F. Hsu, On graceful digraphs and a problem in network addressing, CongressusNumerantium, 35 (1982) 91-103.
[4] J.A. Gallian, A dynamic survey of graph labeling, Electronic Journal of Combinatorics, 18 (2015) # DS6.
[5] B. Gayathri, M. Tamilselvi, M. Duraisamy, k-super mean labeling of graphs, In: Proceedings of the International Conference on Mathematicsand Computer Sciences, Loyola College, Chennai (2008), 107-111.
[6] B. Gayathri and M. Tamilselvi, k-super mean labeling of some trees and cycle related graphs, Bulletin of Pure and Applied Sciences, Volume 26E(2) (2007) 303-311.
[7] F. Harary, Graph Theory, Addison Wesley, Massachusetts (1972).
[8] P. Jeyanthi and D. Ramya, Super mean labeling of some classes of graphs, International J. Math. Combin., 1 (2012) 83-91.
[9] P. Jeyanthi, D. Ramya and P. Thangavelu, On super mean graphs, AKCE J. Graphs Combin., 6 No. 1 (2009) 103-112.
[10] D. Ramya, R. Ponraj and P. Jeyanthi, Super mean labeling of graphs, ArsCombin., 112 (2013) 65-72.
[11] Rosa, On certain valuations of the vertices of a graph Theory of Graphs (Internet Symposium, Rome, July (1966)), Gordon and Breach, N.Y. and Duhod, Paris (1967) 349-355.
[12] S. Somasundaram and R. Ponraj, Mean labeling of graphs, National Academy Science Letter, 26 (2003), 210-213.
[13] P.Sugirtha, R. Vasuki and J. Venkateswari, Some new super mean graphs, International Journal of Mathematics Trends and Technology, Vol. 19 No. 1 March 2015.
[14] M. Tamilselvi, A study in Graph Theory- Generalization of super mean labeling, Ph.D. Thesis, Vinayaka Mission University, Salem, August (2011).
[15] M. Tamilselvi and Akilandeswari K and N. Revathi, Some Results on k-Super Mean Labeling, International Journal of Scientific Research, Volume 5 Issue 6, June 2016, P. No. 2149-2153.