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Study of Heat Transfer in the MHD Flow of a Second-Order Fluid through a Porous Channel

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Abstract: The problem of study of heat transfer in the MHD flow of an incompressible second-order fluid through a porous channel has been discussed. Behaviour of the temperature profile has been studied for the different sets of values of Reynolds number (R), second-order parameter (τ_2) and Hartmann number (S).
Keywords: Heat Transfer; Second-Order Fluid; Porous Channel; transverse Magnetic Field.

I. INTRODUCTION

The heat transfer in the flow of an electrically conducting fluid between porous boundaries is of practical interest in problems of gaseous diffusion etc. Terrill and Shrestha[1] have discussed the problem of steady laminar flow of an incompressible viscous fluid in a two dimensional channel when the walls are of different permeability and studied the effects of magnetic field when the fluid is electrically conducting[2]. The problem of flow of a second-order fluid with heat transfer in a channel with porous walls has been considered by Agarwal[3]. Sharma & Singh[4] have studied the numerical solution of the flow of second-order fluid through a channel with porous walls under a transverse magnetic field.

The purpose of the present paper is an attempt to study the heat transfer in the flow of a second-order fluid through a channel with porous walls under a transverse magnetic field by regular perturbation technique. The second-order effects on the temperature profile are illustrated graphically for different values of the Hartmann and Reynolds number. The results are also obtained for the Newtonian fluid by taking the second-order parameter to be zero.

II. FORMULATION OF THE PROBLEM

The constitutive equation of an incompressible second-order fluid as suggested by Colemann and Noll[5] can be written as:

$$\tau_{ij} = -p\delta_{ij} + 2\mu_1 d_{ij} + 2\mu_2 e_{ij} + 4\mu_3 c_{ij} \quad \text{--} \ (1)$$

where

$$d_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}),$$
$$e_{ij} = \tfrac{1}{2}(a_{i,j} + a_{j,i}) + u^{m}{}_{,i}u_{m,j},$$
$$c_{ij} = d_{im}\, d^{m}{}_{j}. \quad \text{---} \ (2)$$

p is the hydro-static pressure; τ_{ij} is the stress-tensor; u_i and a_i are the velocity and acceleration vector and μ_1, μ_2, μ_3 represent material constants whose values are given by $\mu_1 = 18.5$, $\mu_2 = -0.2$ and $\mu_3 = 1.0$ (all expressed C.G.S. units) for a 5.46 percent solution of poly-iso-butylene in cetane at 30^0C as suggested by Markovitz[6].

The heat transfer in the steady two dimensional flow of an incompressible second-order fluid in a channel, of width 2h consisting of two porous walls (coinciding with the plane $y = \pm h$) of equal permeability is considered. The whole system of the channel is constructed in such a manner that its bottom and top becomes perfectly insulated and does not transmit the heat. A constant magnetic field H_0 is applied normal to the axis of the channel. The induced magnetic field has been neglected in the flow since the magnetic Reynolds number is small. A uniform suction V is applied to the both the walls of the channel. Let us choose the origin of a rectangular co-ordinate system in the middle of the channel with x and y axes respectively in a plane parallel and perpendicular to the channel walls. Let u and v be the components of the velocity in x and y directions respectively.

Following Terrill and Shrestha[1] a stream function ψ can be defined as

$$\psi(x,\xi) = (hU - Vx)\, f(\xi) \quad \text{--------------------------------------} \ (3)$$

where U is the entrance velocity and ξ (= y/h) is the dimensionless distance while 2h is the distance between the channel walls. In non-dimensional form the velocity field by Terrill and Shrestha[1] is taken as:

$$u(x,\xi) = (U - Vx/h)\, f'(\xi)$$

$v(\xi) = V f(\xi)$ -- (4)

where dash denotes differentiation with respect to ξ. The expression (4) suggests that u is a function of x and ξ, while v is a function of ξ only. Using this fact, the constitutive equation (1) the equation of continuity and momentum equations can be written as:

$\partial u / \partial x + (1/h)\, \partial v / \partial \xi = 0$ --- (5)

$u\, \partial u / \partial x + (v/h)\, \partial u / \partial \xi = -(1/\rho)\, \partial p / \partial x + (\nu_1/h^2)\, \partial^2 u / \partial \xi^2 + \nu_2 [\, (1/h^2)\, \partial^2 / \partial \xi^2 \{ u\, \partial u / \partial x + (v/h)\, \partial v / \partial \xi \} + (2/h^2)\, \partial / \partial \xi (\partial u / \partial x . \partial v / \partial \xi)] + (\nu_3 / h^2)\, \partial / \partial x\, (\partial u / \partial \xi)^2 - \mu_e^2 H_0^2\, \sigma\, u / \rho$ ------------------------ (6)

$v \partial v / \partial \xi = -(1/\rho)\, \partial p / \partial \xi + (\nu_1 / h)\, \partial^2 v / \partial \xi^2 + \nu_2 [(2/h)\, \partial^2 / \partial \xi^2 \{(v/h) \partial v / \partial \xi\} + 2 \partial / \partial x\, (\partial u / \partial x . \partial u / \partial \xi) + (4/h^2) \{ \partial u / \partial \xi . \partial^2 u / \partial \xi^2 + \partial v / \partial \xi . \partial^2 v / \partial \xi^2 \} - \partial^2 / \partial x \partial \xi \{ u\, \partial u / \partial x + (v/h)\, \partial u / \partial \xi \}] + (\nu_3 / h^2) [4\, \partial / \partial \xi (\partial v / \partial \xi)^2 + \partial / \partial \xi (\partial u / \partial \xi)^2]$ ---------------- (7)

$\rho\, c_v (u\, \partial T / \partial x + v\, \partial T / \partial y) = k (\partial^2 T / \partial x^2 + \partial^2 T / \partial y^2) + \Phi$ --------------------- (8)

where ρ is the density, μ_e is the magnetic permeability, σ is the electric conductivity, ν_1 (= μ_1/ρ) is the kinematic viscosity, ν_2 (= μ_2/ρ) is the kinematic elastico-viscosity, ν_3 (= μ_3/ρ) is the kinematic coefficient of cross-viscosity, c_v is the specific heat at constant volume, k is the thermal conductivity and $\xi = y/h$ is the dimensionless distance.

The viscous dissipation function (Φ) is given by $\Phi = \tau^i_j\, d^j_i$ --- (9)

where τ^i_j is the mixed deviatoric stress tensor

The boundary conditions are,

$u(x, \pm 1) = 0,\ (\partial u / \partial \xi)_{\xi=0} = 0,$

$v(x, 0) = 0,\ v(x, 1) = V,\ v(x, -1) = -V,$

$T(x, 1) = T_1,\ T(x, -1) = T_{-1}.$ --------------------------------------- (10)

Substituting (4) in equation (6) and (7) and eliminating p from the obtained equation, we get

$f^{iv} + R (f' f'' - f f''') + \tau_1 (f f^{v} - f' f^{iv}) - S^2 f'' = 0,$ ---------------------------(11)

where R (= Vh/ν_1) is the suction Reynolds number, τ_1(= $\nu_2 V / h\nu_1$) is an elastico-viscous parameter governing the effects of elastico-viscosity of the fluid and S [= $\mu_e H_0\, h\, (\sigma / \mu_1)^{1/2}$] is the Hartmann number.

Equation (8) together with equation (4) suggests the form of the temperature distribution as follows:

$T = T_{-1} + (\nu_1 V) [\, \phi(\xi) + (U/V - x/h)^2 \psi(\xi)] / (h\, C_v)$.----------------------------- (12)

Using equation (12) in equation (8) and equating the coefficient of $(U/V - x/h)^2$ and terms independent of $(U/V - x/h)^2$ on both sides of the resulting equation, we obtain

$\phi'' - 2RPf\phi' + 2\psi + 8RPf'^2 + 8R^2 P\, \tau_2\, f f' f'' = 0,$ ----------------------------- (13)

$\psi'' - 2RPf\psi' + 4RP\psi f' + 2RPf''^2 + 2R^2 P\, \tau_2 (f f'' f''' - f' f''^2) = 0.$ ------- (14)

where P = $\mu_1 c_v / k$ is the Prandtl number, $\tau_2 = 2\mu^2 / (h^2 \rho)$ is the second-order parameter.

The expression of the temperature distribution in the dimensionless form can be expressed as:

$T^* = (T - T_{-1}) / (T_1 - T_{-1}) = E (\phi + \zeta^2 \psi),$ ------------------------------------- (15)

where $\zeta = (U/V - x/h)$ is the dimensionless distance.

III. SOLUTION OF THE PROBLEM

Assuming the relationships $\tau_1 = -R\tau_1$ ($\tau_1 \geq 0$) and $S^2 = RS_1^2$ eqn. (11) becomes

$f^{iv} + R (f' f'' - f f''') - R\tau_1 (f f^{v} - f' f^{iv}) - RS_1^2 f'' = 0$ ---------------------- (16)

For small values of the suction Reynolds number R, we can develop a regular perturbation scheme for solving eqns. (13), (14) & (16) by expanding f, ϕ and ψ in powers of R. Substituting

$f(\xi) = \sum R^n f_n(\xi)$ -- (17)

$\phi(\xi) = \sum R^n \phi_n(\xi)$ -- (18)

$\psi(\xi) = \sum R^n \psi_n(\xi)$ --- (19)

In eqns. (13),(14) &(16) and equating the like powers of R on the two sides of the resulting equations, we obtain the following sets of equations:

$f_0^{iv} = 0,$

$f_1^{iv} + f_0' f_0'' - f_0 f_0''' - \tau_1(f_0 f_0^{v} - f_0' f_0^{iv}) - S_1^2 f_0'' = 0,$

$f_2^{iv} + f_1' f_0'' + f_0' f_1'' - f_1 f_0''' - f_0 f_1''' - \tau_1(f_1 f_0^{v} + f_0 f_1^{v} - f_1' f_0^{iv} - f_0' f_1^{iv}) - S_1^2 f_1'' = 0.$ --------------------------------- (20)

$\psi_0'' = 0,$

$\psi_1'' - 2 P f_0 \psi_0' + 4 P \psi_0 f_0' + 2 P f_0''^2 = 0,$

$\psi_2'' - 2P(f_1 \psi_0' + f_0 \psi_1') + 4P(\psi_1 f_0' + f_1' \psi_0 + f_0'' f_1'') + 2P\tau_2 (f_0 f_0'' f_0''' - f_0' f_0''^2) = 0$ ------------------------- (21)

$\phi_0'' + 2 \psi_0 = 0,$

$\phi_1'' - 2 P f_0 \phi_0' + 2 \psi_1 + 8 P f_0'^2 = 0,$

$\phi_2'' - 2P (f_1 \phi_0' + f_0 \phi_1') + 2 \psi_2 + 16 P f_0' f_1' + 8P\tau_2 f_0 f_0' f_0'' = 0.$ --------(22)

Boundary condition (10) can be rewritten as:

$f_n(0) = f_n'(1) = f_n''(0) = 0 \quad \forall n$

$f_0(1) = 1, \quad f_n(1) = 0 \quad n \geq 1$

$\phi_n(-1) = 0 \quad \forall n \quad \phi_0(1) = 1/E = w$ (say),

$\phi_n(1) = 0, \quad n \geq 1 \quad \psi_n(\pm 1) = 0 \quad \forall n$ ------------------------------- (23)

The solution of the equation (20), (21), (22) subjected to the boundary condition (23) is given as follows:

$f_0(\xi) = (1/2)(3\xi - \xi^3),$

$f_1(\xi) = -(1/280)(\xi^7 - 3\xi^3 + 2\xi) - (S_1^2/40)(\xi^5 - 2\xi^3 + \xi),$

$f_2(\xi) = (1/1293600)(14\xi^{11} - 385\xi^9 + 198\xi^7 + 876\xi^3 - 703\xi) - (\tau_1/280)\{(3\xi^7 - 9\xi^3 + 6\xi) + S_1^2(\xi^7 - 3\xi^3 + 2\xi)\} - S_1^2\{(1/100800)(15\xi^9 + 108\xi^7 - 54\xi^5 - 276\xi^3 + 207\xi) + (S_1^2/8400)(5\xi^7 - 21\xi^5 + 27\xi^3 - 11\xi)\}.$ --------------- (24)

$\psi_0(\xi) = 0,$

$\psi_1(\xi) = (3/2)P(1 - \xi^4),$

$\psi_2(\xi) = 3P^2\{383/280 - \xi^8/56 - \xi^6/10 + \xi^4/2 - (3/2)\xi^2\} - P\{(9/280)(1 - \xi^4)^2 + (S_1^2/10)(1 + 2\xi^6 - 3\xi^4)\} - (3/5)P\tau_2(1 - \xi^6).$ ----------------------------- (25)

$\phi_0(\xi) = (w/2)(\xi + 1),$

$\phi_1(\xi) = (wP/40)(10\xi^3 - \xi^5 - 9\xi) - (P/2)(21\xi^2 + \xi^6 - 6\xi^4 - 16),$

$\phi_2(\xi) = P^2[(29\xi^{10}/840 - 51\xi^8/140 + 37\xi^6/20 - 9\xi^4/2 - 1149\xi^2/280 + 595/84) + (w/40)(1391\xi/2520 - 9\xi^3/2 + 99\xi^5/20 - 15\xi^7/14 + 5\xi^9/72)] - P[11/168 - 33\xi^2/280 + 11\xi^4/140 - 3\xi^6/140 - 3\xi^8/280 + \xi^{10}/168 - S_1^2(2\xi^2/5 - 13\xi^8/280 + \xi^6/5 - 7\xi^4/20 - 57/280) + \tau_2(3 - 3\xi^2/5 - 3\xi^8/10 + 12\xi^6/5 - 9\xi^4/2) - w\{(71\xi/100800 - \xi^3/840 + 3\xi^5/5600 - \xi^9/20160) + S_1^2(19\xi/8400 - \xi^7/1680 + \xi^5/400 - \xi^3/240)\}].$ ------ (26)

IV. RESULTS AND DISCUSSIONS

A. The values of the function f_0, f_1 and f_2 are identical to those obtained by Sharma and

B. For $\tau_2 = 0$ the results are in good agreement with those obtained by Terrill and Shrestha

C. For S = 0 the results are matching with those obtained by Agarwal[3].

The variation of the temperature profile at P=0.4, $\zeta = 0.4$, E = 1, $S_1 = 1$, $\tau_2 = -1$ for R = 0.01, 0.1, 1.0 is represented in fig (1). It is evident that for R = 0.01 temperature increases linearly with ξ throughout the channel, for R = 0.1 temperature slightly increases with ξ throughout the channel and for R = 1.0 temperature increases very rapidly first and start decreasing rapidly thereafter.

The variation of the temperature profile at P=0.4, $\zeta = 0.4$, E = 1, $S_1 = 1$, R = 1 for $\tau_2 = 0, 0.1, 1.0$ is represented in fig (2). It is evident that temperature increases very rapidly first and start decreasing rapidly thereafter.

The variation of the temperature profile at P=0.4, $\zeta = 0.4$, E = 1, R = 1, $\tau_2 = -1$ for $S_1 = 0, 1, 2$ is represented in fig (3). It is evident that temperature increases very rapidly first and start decreasing rapidly thereafter.

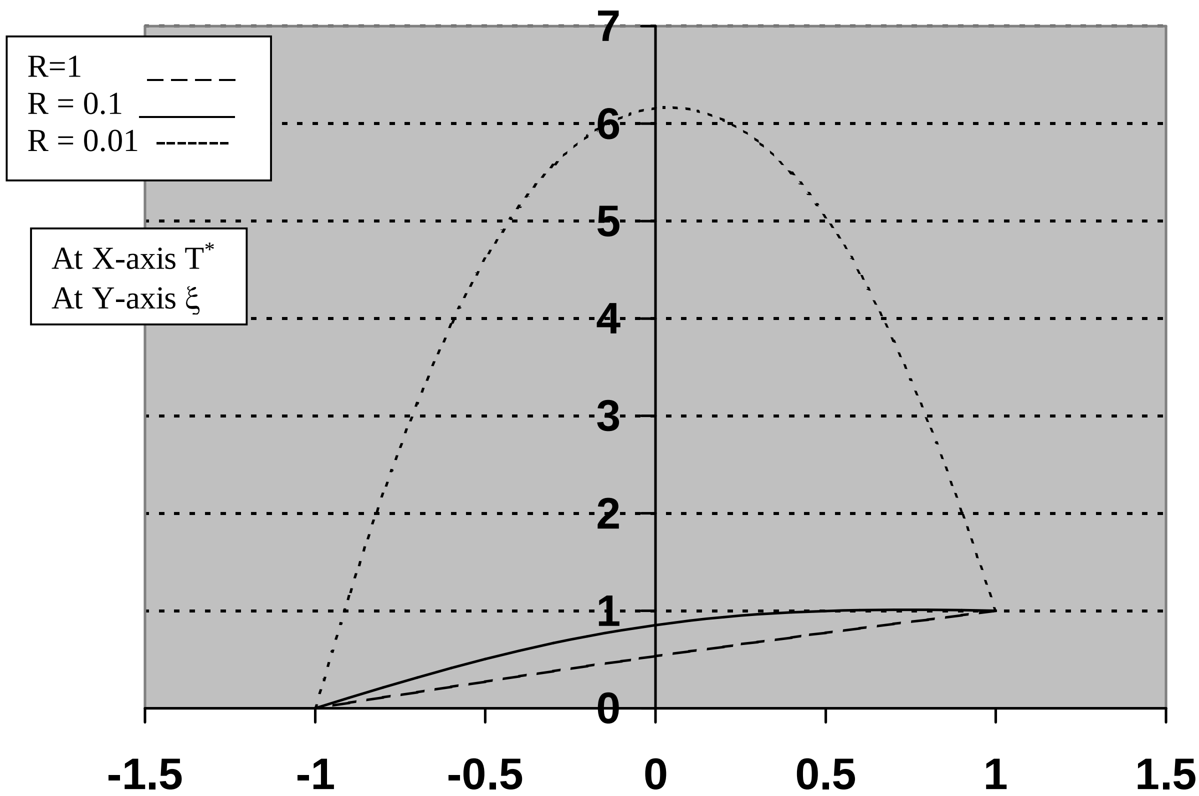

Fig (1) Variation of the temperature T^* With ξ for different values of Reynolds Number (R)

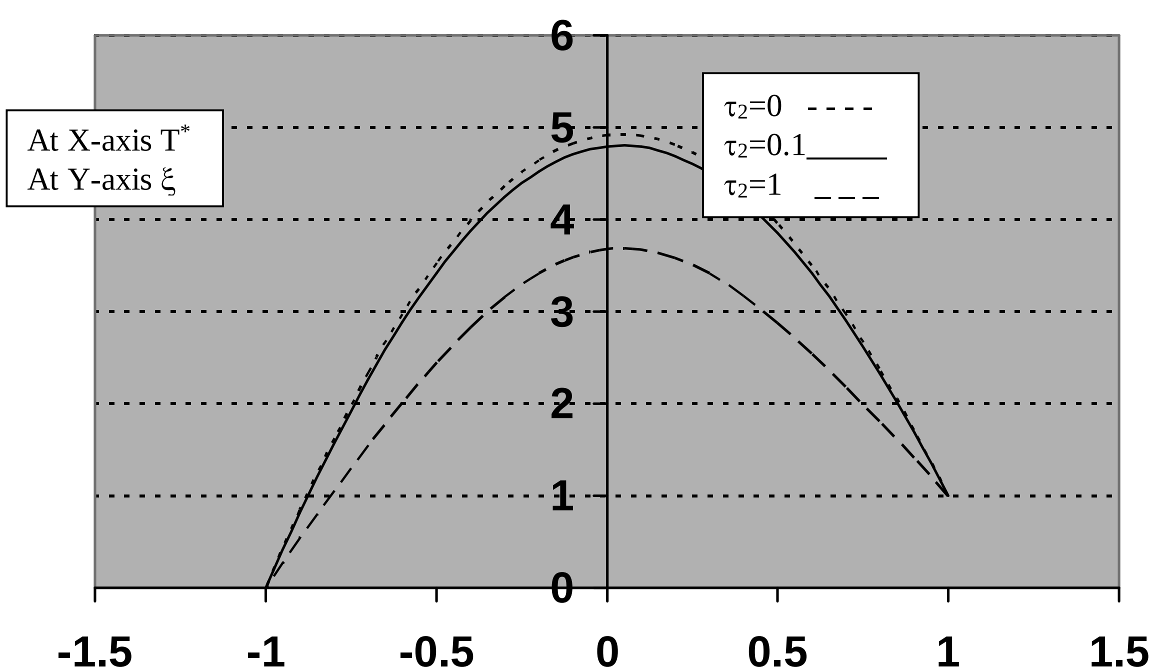

Fig (2) Variation of the temperature T^* with ξ for different values of τ_2

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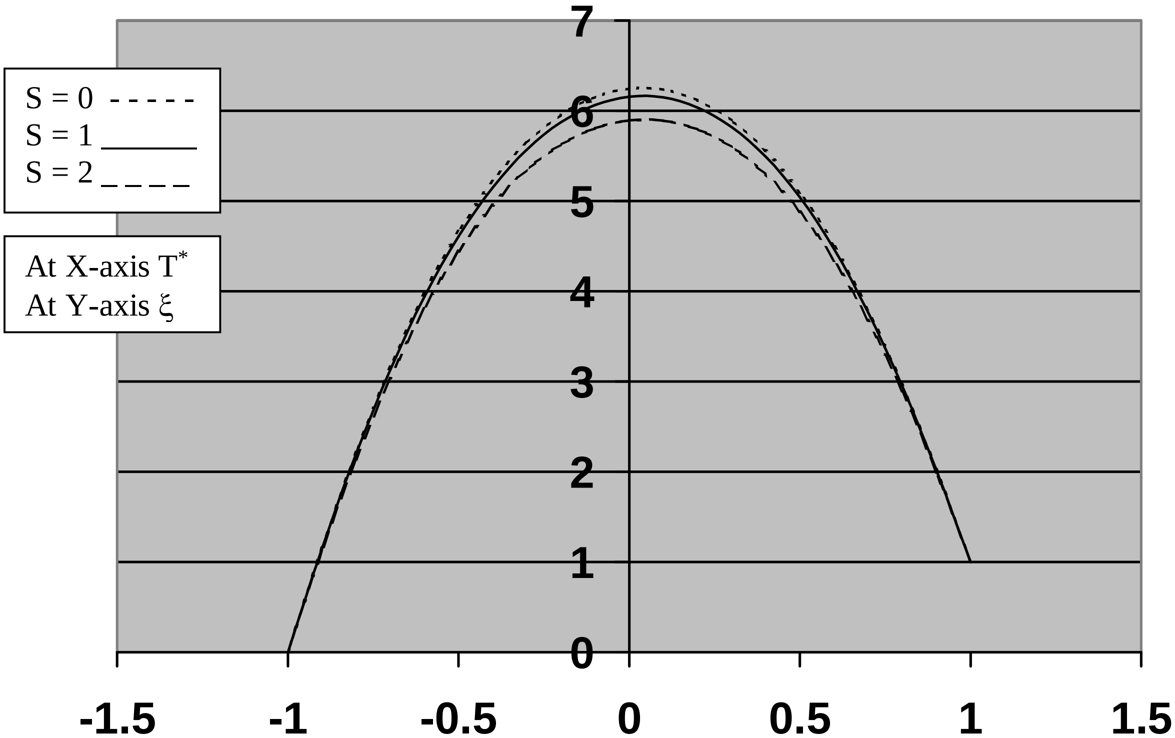

Fig (3) Variation of the temperature T* With ξ for different values of Hartman Number (S)

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