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Optimal Replenishment Policies and Profits for Two-Echelon Inventory Problems with Stochastic Demand

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Abstract - In this paper, a new mathematical model is developed to optimize replenishment policies and profits of a two-echelon distribution inventory system under demand uncertainty. The system consists of a factory warehouse at the upper echelon and a number of supermarkets at the lower echelon. The supermarkets face stochastic stationary demand where sales price and inventory replenishment periods are uniformly fixed over the echelons. Adopting a Markov decision process approach, the states of a Markov chain represent possible states of demand for the inventory item. The replenishment cost, holding cost, shortage cost and sales price are combined with demand and inventory positions in order to generate the profit matrix corresponding to a given echelon. The matrix represents the long run measure of performance for the decision problem. The objective is to determine in each echelon of the planning horizon an optimal replenishment policy so that the long run profits are maximized for a given state of demand. Using weekly equal intervals, the decisions of when to replenish additional units are made using dynamic programming over a finite period planning horizon. A numerical example demonstrates the existence of an optimal state-dependent replenishment policy and profits over the echelons.
Keywords: Echelon, inventory, profits, replenishment, stochastic demand

I. INTRODUCTION

The goal of a supply chain network is to procure raw materials, transform them into intermediate goods and then final products. Finally, delivery of products to customers is required through a distribution system that includes an echelon inventory system. The system spans procurement, manufacturing and distribution with inventory management as one key element. To cope with current turbulent market demands, there is still need to adopt coordinated inventory control across supply chain facilities by establishing optimal replenishment policies in a stochastic demand environment. In practice, large industries continually strive to optimize replenishment policies of products in multi-echelon inventory systems. This is a considerable challenge when the demand for manufactured items follows a stochastic trend. One major challenge is usually encountered: determining the most desirable period during which to replenish additional units of the item in question given a periodic review production-inventory system when demand is uncertain.

Rodney and Roman [1] examined the optimal policies study in the context of a capacitated two-echelon inventory system. This model includes installations with production capacity limits, and demonstrates that a modified base stock policy is optimal in a two-stage system when there is a smaller capacity at the downstream facility. This is shown by decomposing the dynamic programming value function into value functions dependent upon individual echelon stock variables. The optimal structure holds for both stationary and non stationary customer demand.

Axsater S [2] similarly examined a simple decision rule for decentralized two-echelon inventory control. A two-echelon distribution inventory system with a central warehouse and a number of retailers is considered. The retailers face stochastic demand and the system is controlled by continuous review installation stock policies with given batch quantities. A back order cost is provided to the warehouse and the warehouse chooses the reorder point so that the sum of the expected holding and backorder costs are minimized. Given the resulting warehouse policy, the retailers similarly optimize their costs with respect to the reorder points. The study provides a simple technique for determining the backorder cost to be used by the warehouse.

In related work by Haji R [3], a two-echelon inventory system is considered consisting of one central warehouse and a number of non-identical retailers. The warehouse uses a one-for-one policy to replenish its inventory, but the retailers apply a new policy that is each retailer orders one unit to central warehouse in a predetermined time interval; thus retailer orders are deterministic not

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random.

Abhijeet S and Saroj K [4] considered vendor managed Two-Echelon inventory system for an integrated production procurement case. Joint economic lot size models are presented for the two supply situations, namely staggered supply and uniform supply. Cases are employed that describe the inventory situation of a single vendor supplying an item to a manufacturer that is further processed before it is supplied to the end user. Using illustrative examples, the comparative advantages of a uniform sub batch supply over a staggered alternative are investigated and uniform supply models are found to be comparatively more beneficial and robust than the staggered sub batch supply.

The literature cited provide profound insights by authors that are crucial in analyzing two-echelon inventory systems in a stochastic demand setting. However, a new dynamic approach is sought in order to relate state-transitions with customers, demand and price of item at the respective echelons in an effort to optimize replenishment policies and profits in a multistage decision setting.

In this paper, a two-echelon production- inventory system is considered whose goal is to optimize replenishment policies and the sales revenue associated with item sales. At the beginning of each period, a major decision has to be made, namely whether to replenish additional units of the item or not to replenish and keep the item at prevailing inventory position in order to sustain demand at a given echelon. The paper is organized as follows. After describing the mathematical model in §2, consideration is given to the process of estimating the model parameters. The model is solved in §3 and applied to a special case study in §4.Some final remarks lastly follow in §5.

II. MODEL FORMULATION

A. Notation and assumptions

i,j	=	States of demand
F	=	Favorable state
U	=	Unfavorable state
h	=	Inventory echelon
n,N	=	Stages
Z	=	Replenishment policy
N^Z	=	Customer matrix
N^Z_{ij}	=	Number of customers
D^Z	=	Demand matrix
D^Z_{ij}	=	Quantity demanded
Q^Z	=	Demand transition matrix
Q^Z_{ij}	=	Demand transition probability
P^Z	=	Profit matrix
P^Z_{ij}	=	Profits
e^Z_i	=	Expected profits
a^Z_i	=	Accumulated profits
c_r	=	Unit replenishment costs
c_h	=	Unit holding costs
c_s	=	Unit shortage costs
p	=	Unit sales price

$i,j \varepsilon \{F,U\}$ $h \varepsilon \{1,2\}$ $Z \varepsilon \{0,1\}$ n=1,2,…………………….N

We consider a two-echelon inventory system consisting of a manufacturing plant producing a single product in batches for a designated number of supermarkets at echelon 1.At echelon 2; customers demand the product at supermarkets. The demand during each time period over a fixed planning horizon for a given echelon (h) is classified as either *favorable* (denoted by state F) or *unfavorable* (denoted by state U) and the demand of any such period is assumed to depend on the demand of the preceding period. The transition probabilities over the planning horizon from one demand state to another may be described by means of a Markov chain. Suppose one is interested in determining an optimal course of action, namely to replenish additional units of the item (a decision denoted by Z=1) or not to replenish additional units of the item (a decision denoted by Z=0) during each time period over

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the planning horizon, where Z is a binary decision variable. Optimality is defined such that the maximum expected profits are accumulated at the end of N consecutive time periods spanning the planning horizon under consideration. In this paper, a two-echelon (h =2) and two-period (N=2) planning horizon is considered.

B. Finite - period dynamic programming problem formulation

Recalling that the demand can either be in state F or in state U, the problem of finding an optimal replenishment policy may be expressed as a finite period dynamic programming model.

Let $P_n(i,h)$ denote the optimal expected profits accumulated during the periods *n,n+1,…...,**N*** given that the state of the system at the beginning of period *n* is $i\epsilon\{ F, U \}$.The recursive equation relating P_n and P_{n+1} is

$$P_n(i,h) = max_Z[Q^Z_{iF}(h)\ P^Z_{iF}(h) + P_{n+1}(F,h), Q^Z_{iU}(h)\ P^Z_{iU}(h) + P_{n+1}(U,h)] \quad (1)$$

$i\epsilon\{F , U \}$, $h=\{1,2\}$, $n= 1,2,\ldots\ldots N$

together with the final conditions

$P_{N+1}(F , h) = P_{N+1}(U , h) = 0$

This recursive relationship may be justified by noting that the cumulative profits $P^Z_{ij}(h)+ P_{N+1}(j)$ resulting from reaching state $j \epsilon\{F, U\}$ at the start of period n+1 from state $i \epsilon\{ F, U \}$ at the start of period *n* occurs with probability $Q^Z_{ij}(h)$.

Clearly, $e^Z(h) = [Q^Z_{ij}(h)] [R^Z(h)]^T$, $Z \epsilon\{0,1\}$, $h \epsilon\{1,2\}$ (2)

where 'T' denoted matrix transposition, and hence the dynamic programming recursive equations

$$P_N(i) = max_Z [e^Z_i(h) + Q^Z_{iF}(h)P_{N+1}(F) + Q^Z_{iU}(h)P_{N+1}(U)] \quad (3)$$

$$P_N(i,h) = max_Z[e^Z_i(h)] \quad (4)$$

result where (4) represents the Markov chain stable state.

1) *Computing $Q^Z(h)$ and $P^Z(h)$:* The demand transition probability from state $i\epsilon\{ F, U \}$ to state $j \epsilon\{ F, U \}$,given replenishment policy $Z \epsilon\{ 0,1 \}$ may be taken as the number of customers observed over echelon h with demand initially in state i and later with demand changing to state j, divided by the sum of customers over all states. That is,

$$Q^Z_{ij}(h) = N^Z_{ij}(h)/((N^Z_{iF}(h) + (N^Z_{iU}(h)) \quad i\epsilon\{F , U \}, Z \epsilon\{0,1\} , h= \{1, 2\} \quad (5)$$

When demand outweighs on-hand inventory, the profit matrix $P^Z(h)$ may be computed by means of the relation

$$P^Z(h) = p[D^Z(h)] - (c_r + c_h + c_s)[D^Z(h) - I^Z(h)]$$

Therefore,

$$P^Z_{ij}(h) = \begin{cases} p\ D^Z_{ij}(h) - (c_r + c_h + c_s)[D^Z_{ij}(h) - I^Z_{ij}(h)] & if\ \ D^Z_{ij}(h) > I^Z_{ij}(h) \\ p\ D^Z_{ij}(h) \quad c_h I^Z_{ij}(h) & if\ \ D^Z_{ij}(h) \le I^Z_{ij}(h) \end{cases} \quad (6)$$

for all $i,j\epsilon\{ F, U \}$, $h \epsilon\{1,2\}$ and $Z\epsilon\{0,1\}$.

The justification for expression (6) is that $D^Z_{ij}(h)$ – $I^Z_{ij}(h)$ units must be replenished to meet excess demand. Otherwise replenishment is cancelled when demand is less than or equal to on-hand inventory.

The following conditions must, however hold:

1. Z=1 when $c_r > 0$ and Z=0 when $c_r = 0$
2. $c_s > 0$ when shortages are allowed and $c_s= 0$ when shortages are not allowed.

III. OPTIMIZATION

The optimal replenishment policy and profits are found in this section for each period over echelon h separately.

A. Optimization during period 1

When demand is favorable (ie. in state F), the optimal replenishment policy during period 1 is

$$Z = \begin{cases} 1 & if \;\; e^Z_F(h) > e^Z_U(h) \\ 0 & if \;\; e^Z_F(h) \le e^Z_U(h) \end{cases}$$

The associated profits are then

$$P_1(F,h) = \begin{cases} e^1_F(h) & if \;\; Z = 1 \\ e^0_F(h) & if \;\; Z = 0 \end{cases}$$

Similarly, when demand is unfavorable (ie. in state U), the optimal replenishment policy during period 1 is

$$Z = \begin{cases} 1 & if \;\; e^1_U(h) > e^0_U(h) \\ 0 & if \;\; e^1_U(h) \le e^0_U(h) \end{cases}$$

In this case, the associated profits are

$$P_1(U,h) = \begin{cases} e^1_U(h) & if \;\; Z = 1 \\ e^0_U(h) & if \;\; Z = 0 \end{cases}$$

B. Optimization during period 2

Using (2),(3) and recalling that $a^Z_i(h)$denotes the already accumulated profits at the end of period 1 as a result of decisions made during that period, it follows that

$$a^Z_i(h) = e^Z_i(h) + Q^Z_{iF}(h) max[e^1_F(h), e^0_F(h)] + Q^Z_{iU}(h) max[e^1_U(h), e^0_U(h)]$$

$$a^Z_i(h) = e^Z_i(h) + Q^Z_{iF}(h) P_2(F,h) + Q^Z_{iU}(h) P_2(U,h)$$

Therefore when demand is favorable(ie.in state F),the optimal replenishment policy during period 2 is

$$Z = \begin{cases} 1 & if \;\; a^1_F(h) > a^0_F(h) \\ 0 & if \;\; a^1_F(h) \le a^0_F(h) \end{cases}$$

while the associated profits are

$$P_2(F,h) = \begin{cases} a^1_F(h) & if \;\; Z = 1 \\ a^0_F(h) & if \;\; Z = 0 \end{cases}$$

Similarly, when the demand is unfavorable (ie. in state U), the optimal replenishment policy during period 2 is

$$Z - \begin{cases} 1 & if \quad a_U^1(h) > a_U^0(h) \\ 0 & if \quad a_U^1(h) \leq a_U^0(h) \end{cases}$$

In this case the associated profits are

$$P_2(U,h) = \begin{cases} a_U^1(h) & if \quad Z = 1 \\ a_U^0(h) & if \quad Z = 0 \end{cases}$$

IV. CASE STUDY

In order to demonstrate use of the model in §2-3, real case applications from *Sameer Agriculture and LivestockLtd,* a production plant for milk powder and three supermarkets: *Capital supermarket, Nakumatt supermarket ,Tuskys supermarket* and *MIllenium supermarket* in Uganda are presented in this section. The production plant supplies milk powder at supermarkets (echelon 1), while end customers come to supermarkets for milk powder product (echelon 2).The demand for milk powder fluctuates every week at both echelons. The production plant and supermarkets want to avoid excess inventory when demand is Unfavorable (state U) or running out of stock when demand is Favorable (state F) and hence seek decision support in terms of an optimal replenishment policy and the associated profits of milk powder product in a two-week planning period. The network topology of a two-echelon inventory system for milk powder product is illustrated in Figure 1 below:

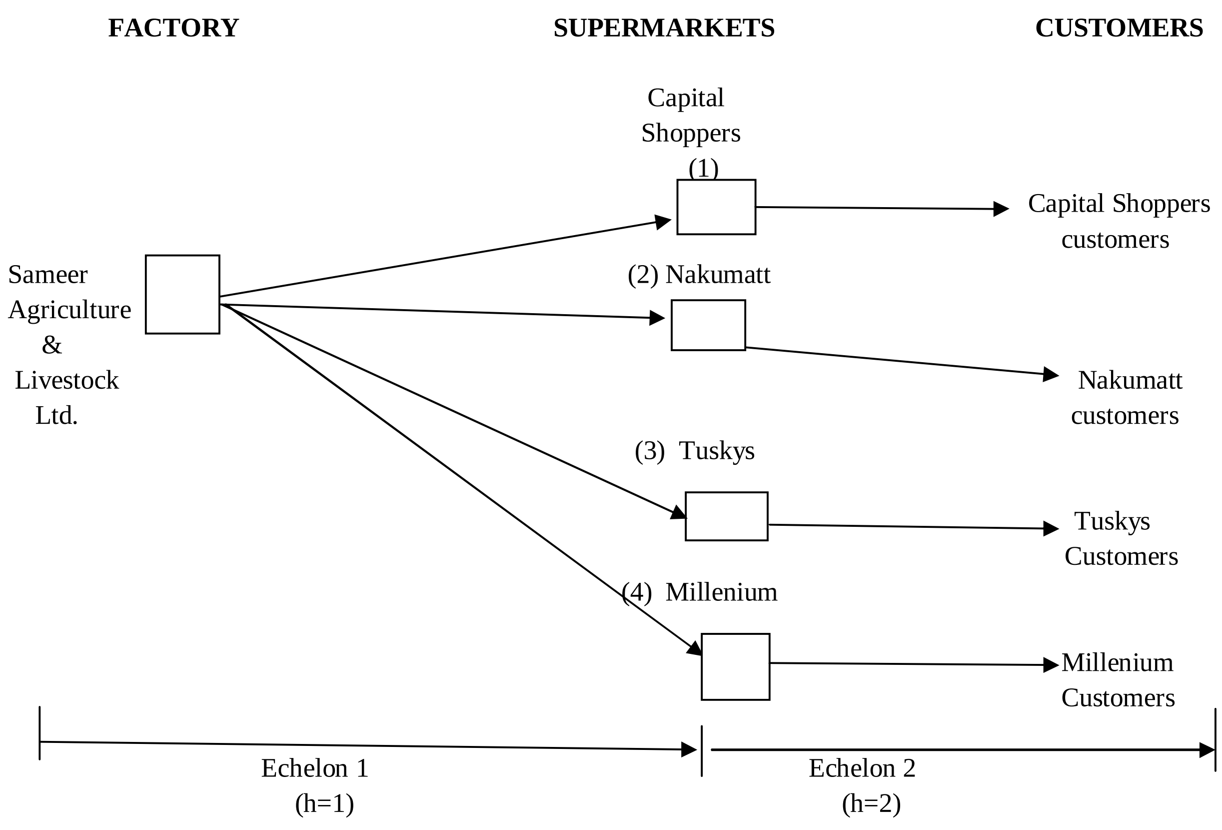

Figure 1: A two-echelon inventory system of milk powder product

A. Data collection

Samples of customers, demand and inventory levels were taken for 400 gms packets of milk powder at echelons 1 and 2 over the state-transitions and the respective replenishment policies for twelve weeks.

TABLE I
CUSTOMERS, DEMAND, REPLENISHMENT POLICIES AND SALES PRICE (IN US$) GIVEN STATE-TRANSITIONS, AND ECHELONS OVER TWELVE WEEKS

STATE TRANSITION (i,j)	ECHELON (h)	REPLENISHMENT POLICY (Z)	CUSTOMERS $N^Z_{ij}(h)$	DEMAND $D^Z_{ij}(h)$	INVENTORY $I^Z_{ij}(h)$	SALES PRICE (p)
FF	1	1	91	156	95	2.5
FU	1	1	71	15	93	2.5
UF	1	1	64	107	93	2.5
UU	1	1	13	11	94	2.5
FF	1	0	82	123	43.5	2.5
FU	1	0	30	78	45	2.5
UF	1	0	55	78	46.5	2.5
UU	1	0	25	15	45.5	2.5
FF	2	1	45	93	145	2.5
FU	2	1	59	60	40	2.5
UF	2	1	59	59	35.5	2.5
UU	2	1	13	11	79.5	2.5
FF	2	0	54	72	81	2.5
FU	2	0	40	77	78.5	2.5
UF	2	0	45	75	79.5	2.5
UU	2	0	11	11	78.5	2.5

In either case, the unit replenishment cost (c_r) is \$1.50, the unit holding cost per week (c_h) is \$0.50 and the unit shortage cost per week (c_s) is \$0.75

B. Computation of Model Parameters

Using (5) and (6), the state transition matrices and profit matrices (in million UGX) at each respective echelon for week1 are

$$Q^1(1) = \begin{bmatrix} 0.5617 & 0.4383 \\ 0.8312 & 0.1688 \end{bmatrix} \qquad P^1(1) = \begin{bmatrix} 222.25 & -32.25 \\ 229 & -43 \end{bmatrix}$$

$$Q^1(2) = \begin{bmatrix} 0.4327 & 0.5673 \\ 0.8194 & 0.1806 \end{bmatrix} \qquad P^1(2) = \begin{bmatrix} 123.8 & 95 \\ 82.9 & 32 \end{bmatrix}$$

for the case when additional units were replenished (Z=1) during week 1,

while these matrices are given by

$$Q^0(1) = \begin{bmatrix} 0.7322 & 0.2678 \\ 0.6875 & 0.3125 \end{bmatrix} \qquad P^0(1) = \begin{bmatrix} 218.63 & 90.75 \\ 86.63 & 113.75 \end{bmatrix}$$

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$$Q^0(2) = \begin{bmatrix} 0.5745 & 0.4255 \\ 0.8036 & 0.1964 \end{bmatrix} \qquad P^0(2) = \begin{bmatrix} 202.5 & 196.25 \\ 198.75 & 196.25 \end{bmatrix}$$

For the case when additional units were not replenished (Z=0) during week 1.

When additional units are replenished (Z = 1), the matrices $Q^1(1)$, $P^1(1)$, $Q^1(2)$ and $P^1(2)$ yield the profits (in million UGX)

$$e_F^1(1) = (0.5617)(222.25) + (0.4383)(-32.25) = 110.70$$
$$e_U^1(1) = (0.8312)(229) + (0.1688)(-43) = 183.09$$
$$e_F^1(2) = (0.4327)(123.8) + (0.5673)(95) = 107.46$$
$$e_U^1(2) = (0.8194)(82.9) + (0.1806)(-32) = 30.09$$

However, When additional units are *not* replenished (Z=0), the matrices $Q^0(1)$, $P^0(1)$, $Q^0(2)$ and $P^0(2)$ yield the profits (in million UGX)

$$e_F^0(1) = (0.7322)(218.63) + (0.2678)(90.75) = 184.38$$
$$e_U^0(1) = (0.6875)(86.63) + (0.3125)(113.75) = 95.105$$
$$e_F^0(2) = (0.5745)(202.5) + (0.4255)(196.25) = 199.84$$
$$e_U^0(2) = (0.8036)(198.75) + (0.1964)(196.25) = 198.26$$

When additional units are replenished (Z=1), the accumulated profits at the end of week 2 follows:

Echelon 1:

$$a_F^1(1) = 110.70 + (0.5617)(184.38) + (0.4383)(183.09) = 294.51$$
$$a_U^1(1) = 183.09 + (0.8312)(184.38) + (0.1688)(183.09) = 367.25$$

Echelon 2:

$$a_F^1(2) = 107.46 + (0.4327)(184.38) + (0.5723)(198.26) = 301.7$$
$$a_U^1(2) = 30.09 + (0.8194)(184.38) + (0.1806)(198.26) = 216.98$$

When additional units are *not* replenished (Z=0), the accumulated profits at the end of week 2 follows:

Echelon 1:

$$a_F^0(1) = 184.38 + (0.7322)(184.38) + (0.2678)(183.09) = 368.4$$
$$a_U^0(1) = 95.105 + (0.6875)(184.38) + (0.3125)(183.09) = 279.08$$

Echelon 2:

$$a_F^0(2) = 199.84 + (0.5745)(184.38) + (0.4255)(198.26) = 390.13$$
$$a_U^0(2) = 198.26 + (0.8036)(184.38) + (0.1964)(198.26) = 385.37$$

C. The Optimal Replenishment Policy

Week1: Echelon 1

Since184.38 > 110.70, it follows that Z=0 is an optimal replenishment policy for week 1 with associated profits of $184.38 for the case of favorable demand. Since 183.09 > 95.105, it follows that Z=1 is an optimal replenishment policy for week 1 with associated profits of $183.09 for the case when demand is unfavorable.

Week1: Echelon 2

Since 199.84 > 107.46, it follows that Z=0 is an optimal replenishment policy for week 1 with associated profits of $199.84 when demand is favorable. Since 198.26 > 30.09, it follows that Z=0 is an optimal replenishment policy for week 1 with associated profits of $198.26 when demand is unfavorable.

Week 2: Echelon 1

Since 368.4 > 294.51, it follows that Z=0 is an optimal replenishment policy for week 2 with associated accumulated profits of $368.4 when demand is favorable. Since 367.25 > 279.08, it follows that Z=1 is an optimal replenishment policy for week 2 with associated accumulated profits of $367.25 when demand is unfavorable.

Week 2: Echelon 2

Since 390.13 > 301.7, it follows that Z=0 is an optimal replenishment policy for week 2 with associated accumulated profits of $390.13 for the case of favorable demand. Since 385.37 > 216.98, it follows that Z=0 is an optimal replenishment policy for week 2 with associated accumulated profits of $385.37 for the case of unfavorable demand.

V. CONCLUSION

A two-echelon inventory model with stochastic demand was presented in this paper. The model determines an optimal replenishment policy and profits of an item with stochastic demand. The decision of whether or not to replenish additional units is modeled as a multi-period decision problem using dynamic programming over a finite planning horizon. Results from the model indicate optimal replenishment policies and profits over the echelons for the given problem. As a profit maximization strategy in echelon-based inventory systems, computational efforts of using Markov decision process approach provide promising results. However, further extensions of the research are sought in order to analyze replenishment policies for maximizing profits under non stationary demand conditions over the echelons. In the same spirit, our model raises a number of salient issues to consider: Lead time of milk powder during the replenishment cycle and customer response to abrupt changes in price of the product. Finally, special interest is thought in further extending our model by considering replenishment policies in the context of Continuous Time Markov Chains (CTMC).

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