iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 Issue: IV Month of publication: April 2018

DOI: http://doi.org/10.22214/ijraset.2018.4642

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 6 Issue IV, April 2018- Available at www.ijraset.com

Strong form of Fuzzy Closed Sets in Fuzzy Topological Space

D. Ananthi[1], K. Ygapriya[2], G. Bhuvana[3], N. Sathishkumar[4], R. Ragupathiraja[5]
[1, 2, 3, 4, 5]*Asst. Prof Edayathangudy G. S., Pillay Arts and Science College Nagapattinam*

Abstract: In this paper we have introduced a new class of fuzzy sets called fuzzy strongly (gsp)*-closed sets, properties of this set are investigated and we introduce new fuzzy spaces namely, fuzzy Ts(gsp)*-space, fuzzy gT$_s$(gsp)*-space, fuzzy g*T$_s$(gsp)*- space and fuzzy g**T$_s$ (gsp)*-space. Further strongly (gsp)*-continuous mappings are also introduced and investigated.
Keywords: fuzzy Strongly (gsp)*-closed sets, fuzzy Strongly (gsp)*-continuous maps, fuzzy T$_s$(gsp)*-space, fuzzy gT$_s$(gsp)*-space, fuzzy g*T$_s$(gsp)*-space and fuzzy g**T$_s$(gsp)*-space.

I. PRELIMINARIES

Throughout this paper (X,τ) and (Y, σ) represent non-empty fuzzy topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a fuzzy subset A of a space (X,τ), cl(A) and int(A) denote the fuzzy closure and the fuzzy interior of A respectively.

A. Definition 1.1

A Subset A of fuzzy topological space (X,τ) is called;

1) Fuzzy semi open set if A ⊆ cl(int(A)) and a fuzzy semi-closed set if int(cl(A)) ⊆A.
2) Fuzzy semi pre-open set if A ⊆ cl(int(cl(A)) and a fuzzy semi-pre closed set if int(cl(int(A))) ⊂ A
3) Fuzzy regular -open set if int(cl(A))=A and a fuzzy regular -closed.

B. Definition 1.2

A Subset A of fuzzy topological space (X, τ) is called;

1) Fuzzy generalized closed set (briefly fuzzy g-closed) if cl(A) ⊆ U whenever A⊆U and U is fuzzy open in (X,τ)
2) Fuzzy g*-closed set if cl(A) ⊆U whenever A ⊆ U and U is fuzzy g open in (X,τ)
3) Fuzzy g**-closed set if cl(A) ⊆U whenever A ⊆ U and U is fuzzy g* open in (X,τ)
4) Fuzzy wg - closed set if cl(int(A)) ⊆U whenever A ⊆ U and U is fuzzy open in (X,τ)
5) Fuzzy regular generalized closed set (briefly fuzzy rg-closed) if cl(A) ⊆U whenever A ⊆ U and U is fuzzy regular open in (X,τ)
6) Fuzzy sg** - closed set if scl(A) ⊆U whenever A ⊆U and U is fuzzy g** open in (X,τ)
7) Fuzzy sg* - closed set if scl(A) ⊆ U whenever A ⊆U and U is fuzzy g* open in (X,τ)
8) Fuzzy generalized semi-closed set (briefly fuzzy gs-closed) if scl(A) ⊆U whenever A⊆U and U is fuzzy open in (X,τ)
9) Fuzzy gsp - closed set if spcl(A) ⊆ U whenever A ⊆ U and U is fuzzy open in (X,τ)
10) Fuzzy (gsp)*- closed set if cl(A) ⊆ U whenever A ⊆ U and U fuzzy gsp is open in (X,τ)

C. Definition 1.3

A function f : (X,τ) → (Y,σ) is called ;

1) fuzzy g – continuous if f^{-1}(V) is a fuzzy g-closed set of (X,τ) for every fuzzy closed set V of (Y,σ)
2) fuzzy g* – continuous if f^{-1}(V) is a fuzzy g*-closed set of (X,τ) for every fuzzy closed set V of (Y,σ)
3) fuzzy g** – continuous if f^{-1}(V) is a fuzzy g**-closed set of (X,τ) for every fuzzy closed set V of (Y,σ)
4) fuzzy rg – continuous if f^{-1} (V) is a fuzzy rg -closed set of (X,τ) for every fuzzy closed set V of (Y,σ)
5) fuzzy wg – continuous if f^{-1}(V) is a fuzzy wg -closed set of (X,τ) for every fuzzy closed set V of (Y,σ)
6) fuzzy (gsp)* – continuous if f^{-1}(V) is a fuzzy (gsp)* -closed set of (X,τ) for every fuzzy closed set V of (Y ,σ)

D. Definition 1.4

A fuzzy topological space (X ,τ) is said to be;

1) Fuzzy $T_{1/2}$* space if every fuzzy g*-closed set in it is fuzzy closed.
2) Fuzzy T_d space if every fuzzy gs -closed set in it is fuzzy g- closed.

II. FUZZY STRONGLY (GSP)* - CLOSED SETS IN FUZZY TOPOLOGICAL SPACE

We introduce the following definition

A. Definition 2.1

A subset A of a fuzzy Topological space (X,τ) is said to be a fuzzy strongly (gsp)*-closed set if cl(int(A)) ⊆U whenever A ⊆U and U is fuzzy gsp-open.

1) *Lemma 2.1:* Every fuzzy closed set is fuzzy strongly (gsp)*-closed.
2) *Proof:* Let A be a fuzzy closed. Then cl(A) = A. Let us prove that A is fuzzy strongly (gsp)* - closed. Let A⊆U and U be fuzzy gsp-open. Then cl(A) ⊆U .Since A is fuzzy closed . cl(int(A)) ⊆cl(A) ⊂U. Then cl(int(A)) ⊂ U whenever A ⊆U and U is fuzzy gsp - open.so A is fuzzy strongly (gsp)* - closed. The converse of the above proposition need not be true in general as seen in the following example.

B. Lemma 2.2

Every fuzzy g-closed set is fuzzy strongly (gsp)*-closed.

1) *Proof:* Let A be fuzzy g-closed. Then cl(A) ⊆U Whenever A ⊆U and U is fuzzy open in (X, τ).To prove A is fuzzy strongly (gsp)* - closed. Then A ⊆U and U be fuzzy (gsp) open. We have cl(A) ⊆U Whenever A ⊆U and U is fuzzy open in (X, τ).Since every fuzzy open set is (fuzzy gsp) –open. We have cl(A) ⊆U Whenever A ⊆U and U is fuzzy (gsp)-open in (X, τ).But cl(int(A)) ⊆cl(A) ⊆U whenever A ⊆U and U is (fuzzy gsp)- open in (X, τ) then cl(int(A)) ⊆U whenever A ⊆U and U is fuzzy (gsp) - open in (X, τ) so A is fuzzy strongly (gsp)*-closed .The converse of the above proposition need not be true in general as seen in the following example.

C. Lemma 2.3

Every fuzzy g*-closed set is fuzzy strongly (gsp)* - closed.

The converse of the above proposition need not be true in general as seen in the following example.

D. Lemma 2.4

Every fuzzy rg – closed set is fuzzy strongly (gsp)* - closed.

1) *Proof:* Let A be fuzzy rg-closed set. Then cl(A) ⊆U Whenever A ⊆U and U is fuzzy regular-open in (X, τ).To prove A is fuzzy strongly (gsp)* - closed. Let A ⊆U and U be fuzzy (gsp) open. Since every fuzzy regular-open set is fuzzy (gsp)-open.We have cl(A) ⊆U Whenever A ⊆U and U is fuzzy (gsp)-open in (X, τ).But cl(int(A)) ⊆ cl(A) ⊆ U whenever A ⊆U and U is fuzzy (gsp)-open in (X, τ).so cl(int(A)) ⊆U whenever A⊂U and U is fuzzy (gsp)-open in (X, τ). So A is fuzzy strongly (gsp)*-closed.
2) *Remark 2.1:* fuzzy Strongly (gsp)* - closedness is independent of fuzzy semi-closedness

E. Lemma 2.5

Every fuzzy (gsp)* - closed set is fuzzy strongly (gsp)* - closed set

1) *Proof:* Let A be fuzzy (gsp)* -closed set .Then cl(A) ⊆U Whenever A ⊆U and U is fuzzy gsp-open in (X, τ).To prove A is fuzzy strongly (gsp)* - closed. Let A ⊆U and U be fuzzy (gsp) open. Since every fuzzy (gsp)* - open set is fuzzy (gsp)-open.We have cl(A) ⊆U Whenever A ⊆U and U is fuzzy (gsp)-open in (X, τ).But cl(int(A)) ⊆cl(A) ⊆U whenever A ⊆U and U is fuzzy (gsp)-open in (X, τ)⊆cl(int(A)) ⊆U whenever A ⊆U and U is fuzzy (gsp)-open in (X, τ) so A is fuzzy strongly (gsp)* -closed.The converse of the above proposition need not be true in general as seen in the following example.

2) *Theorem 2.1:* Every fuzzy g** -closed set is fuzzy strongly (gsp)* -closed. The converse of the above proposition need not be true in general .
3) *Remark 2.2:* fuzzy Strongly (gsp)* -closed is independent of fuzzy sg**-closed.
4) *Remark 2.3:* Strongly fuzzy (gsp)* -closed is independent of fuzzy sg* -closed.

F. Lemma 2.6

Every fuzzy wg-closed set is fuzzy strongly (gsp)* - closed.

1) *Proof:* Let A be fuzzy wg-closed **.**Then cl(int(A)) ⊆U Whenever A ⊆U and U is fuzzy open in (X, τ)**.**To prove A is fuzzy strongly (gsp)* -closed .Let A ⊆U and U be fuzzy (gsp) open. Since every fuzzy wg-open set is fuzzy (gsp)- open **.**We have cl(int(A)) ⊆U whenever A ⊆U and U is fuzzy (gsp)-open in (X, τ) then cl(int(A)) ⊆U whenever A ⊆U and U is fuzzy (gsp)-open in (X, τ)**.** A is fuzzy strongly (gsp)* -closed.

III. FUZZY STRONGLY (GSP)* -CONTINUOUS MAPS

We introduce the following definitions

A. Definition 3.1

A function f:(X,τ) → (Y,σ) is called fuzzy Strongly (gsp)* -continuous if f^{-1}(V) is a fuzzy strongly (gsp)* -closed set in (X,τ) for every fuzzy closed set V of (Y,σ).

fuzzy continuous map is fuzzy strongly (gsp)* -continuous

Let f:(X,τ)→(Y,σ) be a fuzzy continuous map. Let us prove that f is fuzzy strongly (gsp)* - continuous. Let F be a fuzzy closed set in (Y,σ).Since f is fuzzy continuous f^{-1}(F) is closed in (X,τ) then f^{-1}(F) is fuzzy strongly (gsp)* -closed so f is fuzzy strongly (gsp)* -continuous the converse of the above Theorem is not true

B. Theorem 3.2

Every fuzzy g-continuous map is fuzzy strongly (gsp)* -continuous

1) *Proof:* Let f :(X,τ) → (Y,σ) be a fuzzy g-continuous**.** Let F be a fuzzy closed set in (Y,σ). Since f is fuzzy g-continuous f^{-1}(F) is fuzzy g-closed in (X,τ) .then ,f^{-1}(F) is fuzzy strongly (gsp)* -closed so f is fuzzy strongly (gsp)* - continuous fuzzy closed g**-closed fuzzy g-closed fuzzy wg-closed fuzzy sg**-closed fuzzy g*-closed fuzzy strongly (gsp)*-closed fuzzy (gsp)*-closed fuzzy Semi-closed sg*-closed sg-closed .The converse of the above Theorem is not true

C. Theorem 3.3

Every g* -continuous map is strongly (gsp)* -continuous.

1) *Proof:* Let f :(X,τ) → (Y,σ) be a g* -continuous**.** Let F be a closed set in (Y,σ)**.** Since f is g* -continuous f^{-1}(F) is g* -closed in (X,τ) **.**By Theorem (3.6),f^{-1}(F) is strongly (gsp)* -closed so f is strongly (gsp)* -continuous The converse of the above Theorem is not true

D. Theorem 3.4

Every fuzzy g** -continuous map is fuzzy strongly (gsp)* -continuous.

1) *Proof:* Let f :(X,τ) → (Y,σ) be a fuzzy g** -continuous**.** Let F be a fuzzy closed set in (Y,σ).Since f is fuzzy g** - continuous g^{-1}(F) is g** -closed in (X,τ).then f^{-1}(F) is fuzzy strongly (gsp)* -closed so f is fuzzy strongly (gsp)* -continuous The converse of the above Theorem is not true.

IV. APPLICATIONS OF FUZZY STRONGLY (GSP)* - CLOSED SETS

As application of fuzzy strongly (gsp)*-closed sets, new spaces, namely fuzzy Ts(gsp)* space , gTs(gsp)* space , fuzzy g*T s(gsp)* , fuzzy g**Ts(gsp)*space are introduced. We introduced the following definitions.

A. Definition 4.1

A fuzzy space (X, τ) is called a fuzzy Ts(gsp)* -space if every fuzzy strongly (gsp)* - closed set is closed.

B. Definition 4.2

A fuzzy space (X, τ) is called a fuzzy gTs(gsp)* - space if every fuzzy strongly (gsp)* - closed set is fuzzy g closed.

C. Definition 4.3

A fuzzy space (X, τ) is called a fuzzy g*Ts(gsp)* -space if every fuzzy strongly (gsp)* - closed set is g*-closed

D. Definition 4.4

A fuzzy space (X, τ) is called a fuzzy g**Ts(gsp)* -space if every fuzzy strongly (gsp)* - closed set is fuzzy g**-closed.

2) *Theorem 4.1:* Every fuzzy Ts(gsp)* -space is fuzzy $T_{1/2}$* -space.
3) *Proof:* Let (X, τ) be a fuzzy Ts(gsp)* - space. Let us prove that (X, τ) is a fuzzy $T_{1/2}$* - space. Let A be a fuzzy g* - closed set. Since every fuzzy g* - closed set is fuzzy strongly (gsp)* -closed, A is fuzzy strongly (gsp)* - closed. Since (X, τ) is a fuzzy Ts(gsp)* - space , A is fuzzy closed. (X,τ) is a fuzzy $T_{1/2}$* - space.
4) *Theorem 4.2:* Every fuzzy Ts(gsp)* -space is fuzzy gTs(gsp)*-space.
5) *Proof:* Let A be a fuzzy strongly (gsp)*-closed set. Then A is fuzzy closed. Since the fuzzy space is Ts(gsp)* - space. And every closed set is fuzzy g-closed .Hence A is fuzzy g- closed. (X,τ) is a fuzzy gTs(gsp)*- space. The converse is not true.
6) *Theorem 4.3:*Every fuzzy g*Ts(gsp)* -space is fuzzy gTs(gsp)* -space.
7) *Proof:* Let A be a fuzzy strongly (gsp)* -closed .Then A is fuzzy g* -closed, since the fuzzy space is a fuzzy g*Ts(gsp)* -space since. Every fuzzy g* -closed set is fuzzy g-closed .Hence A is fuzzy g-closed then (X,τ) is a fuzzy gTs(gsp)* -space. The converse is not true.
8) *Theorem 4.4:* Every g*Ts(gsp)* -space is g**Ts(gsp)* -space.
9) *Proof:* Let A be a strongly (gsp)*-closed set. Then A is g* -closed, since the space is a g*Ts(gsp)* .Since Every g**-closed set is g*-closed .Hence A is g**-closed. (X,τ) is a g**Ts(gsp)* -space.

REFERENCE

[1]. B. Ghosh, Semi-continuous and semi-closed mappings and semi-connectedness in fuzzy setting, fuzzy Sets and Systems 35(3) (1990), 345–355.
[2]. C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968), 182–190.
[3]. C.W. Baker on preserving g-super closed sets Kyungpook Math. J. 36(1996), 195-199.
[4]. G. Balasubramanian and P. Sundaram, On some generalizations of fuzzy continuous functions,Fuzzy Sets and Systems 86(1) (1997), 93–100.
[5]. G. Balasubramanian and V. Chandrasekar, Totally fuzzy semi continuous functions, Bull. CalcuttaMath. Soc. 92(4) (2000), 305–312.
[6]. G. Balasubramanian, On fuzzy pre-separation axioms, Bull. Calcutta Math. Soc. 90(6) (1998),427–434.
[7]. K. K. Azad, On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl. 82(1) (1981), 14–32.
[8]. K. M. Abd El-Hakeim, Generalized semi-continuous mappings in fuzzy topological spaces, J. Fuzzy Math. 7(3) (1999), 577–589.
[9]. L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965), 338–353.
[10]. M.K. Mishra et all on "Fuzzy super continuity" International Review in Fuzzy Mathematics July –December2012.
[11]. M.K. Mishra M. Shukla M. Fuzzy Regular Generalized Super Closed Set" International Journal of Scientfic and Research December issue July December 2012.
[12]. M.K. Mishra,et all on " Fuzzy super closed set" International Journal International Journal of Mathematics and applied Statistics.
[13]. P. M. Pu and Y. M. Liu Fuzzy topology I Neighborhood structure of a fuzzy point and More-Smith Convergence. J. Math. Anal. Appl. 76(1980) ,571-594.
[14]. P. M. Pu and Y. M. Liu Fuzzy topology II Product and quotient spaces J.Math. Anal. Appl. 77(1980) 20-37.
[15]. P. M. Pu, and Y. M. Liu, Fuzzy topology. I. Neighborhood structure of a fuzzy point and Moore-Smith convergence, J. Math. Anal. Appl. 76(2) (1980), 571–599.
[16]. R. K. Saraf and M. Khanna, On gs-closed sets in fuzzy topology, J. Indian Acad. Math. 25(1),(2003), 133–143.
[17]. R. K. Saraf, and M. Khanna, Fuzzy generalized semi pre closed sets, J. Tripura Math. Soc.3(2001) 59–68.
[18]. R. K. Saraf, and S. Mishra, Fg_-closed sets, J. Tripura Math. Soc. 2 (2000) 27–32.
[19]. R. K. Saraf, M. Caldas and S. Mishra, Results via Fg_-closed sets and Fg-closed sets, Preprint.
[20]. T. H. Yalva¸c, Semi-interior and semi closure of a fuzzy set, J. Math. Anal. Appl. 132(2) (1988),356–364.