Wind Speed Augmentation Model for Empty Conical Diffusers for Use in Diffuser Augmented Wind Turbines

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Abstract: Diffusers have been used to augment the wind speed in diffuser augmented wind turbines. However, there is no known method to estimate the wind speed augmentation by these diffusers. This study presents a mathematical model that estimates the wind speed augmentation by empty conical diffusers for use in diffuser augmented wind turbines (DAWT). The model is used by DAWT wind energy systems engineers in optimizing the power output of the DAWT. The model is based on the diffuser length \((L)\), diffuser expansion angle \((\theta)\) and the diffuser inlet diameter \((D)\). The model equation and the experimental data are correlated with \(R^2 = 0.9751\) and \(RMSE = 0.034\). It was shown that the diffuser expansion angle \((\theta)\), a predictor contributes more to the desired output as compared to the non-dimensional length \((L/D)\).

Keywords: Wind speed augmentation, Non-dimensional length \((L/D)\), Diffuser augmented wind turbine (DAWT), Diffuser expansion angle \((\theta)\)

I. INTRODUCTION

The use of ducts around horizontal axis wind turbines to enhance wind energy extraction has been under study for several decades. The most common duct used is the diffuser. A wind turbine is placed at the inlet (narrower side) section of the diffuser. A sub-atmospheric pressure region is created at the diffuser exit plane. This low pressure region generates an increased air mass flow through the diffuser inlet in order to equalize the pressure imbalance [1], [2], [3]. The air mass is concentrated and accelerated past the wind turbine. Therefore, the wind speed at the wind turbine is greater than that of free air.

Air flow behaviour and performance of a diffuser depends on its geometrical shape and flow parameters [4]. Geometrical shape parameters comprise the non-dimensional length \((L/D)\), the ratio (Ar) of the inlet and outlet cross-sectional areas of the diffuser and the diffuser expansion angle \((\theta)\). Flow parameters determine flow conditions such as turbulence intensity, inlet swirl, boundary layer thickness, Reynolds’ number and inlet velocity profile.

The impact of the geometrical parameters on diffuser performance has led many researchers to investigate the effect of these parameters on diffuser performance [5], [6]. Reference [7] found out that, the pressure recovery coefficient \((C_{pr})\), depends on \(L/D\) and the expansion angle \((\theta)\).

Reference [8] in their work, “Characteristics of a highly efficient propeller type small wind turbine with diffuser”, investigated the effect the diffuser shape had on the wind speed and concluded that the wind speed inside the diffuser is greatly influenced by the length \((L)\) and expansion angle \((\theta)\) of the diffuser, and maximum speed increased 1.76 times with the selection of the appropriate diffuser shape. Reference [9] found out that the ratio of the free stream velocity and wind velocity recorded in the inlet section of an empty diffuser \((V/V_0)\) increases linearly with the expansion angle \((\theta)\) and reaches a maximum at 10°. Reference [10] experimentally found out that the wind speed in the diffuser was greatly influenced by the expansion angle \((\theta)\), flange height, hub ratio, centre body length and inlet shroud length.

Geometrical shape parameters are key in the performance and behaviour of diffusers. However, there is no known model basing on diffuser geometrical shape parameters that estimate wind speed augmentation by conical diffusers. The present study presents the development of a mathematical model which estimates the wind speed augmentation by empty conical diffusers. This is a follow up paper based on optimum geometrical shape parameters of conical diffusers which were experimentally determined in [11].
II. MATERIALS AND METHODS

Optimum geometrical shape parameters for conical diffusers used in DAWT were experimentally determined. The optimum parameters were used to develop the wind speed augmentation model for conical diffusers.

A. Determination Of Optimum Geometrical Shape Parameters For Conical Diffusers

In [11], experimental work with empty conical diffusers was presented. The thrust of the experiments was to determine the relationship between the wind speed augmentation \( \frac{V_s}{V_0} \) and the geometrical shape parameters of the diffusers. The geometrical parameters under study were the diffuser expansion angle (\( \theta \)) and the non-dimensional length \( \frac{L}{D} \). Fig. 1 shows the experimental set up that was used.

Fig.1. Experimental set up for the wind speed augmentation measurement [11]

From the experiments, optimum geometrical parameters that gave maximum wind speed augmentation \( \left( \frac{V_s}{V_0} \right)_{\text{max}} \) at the throat of the diffuser were determined. Columns 1-4 of Table I summarizes the obtained results.

<table>
<thead>
<tr>
<th>( \frac{L}{D} )</th>
<th>( \left( \frac{V_s}{V_0} \right)_{\text{max}} )</th>
<th>Optimum diffuser expansion angle (( \theta ))</th>
<th>Validity limits of the model (( \theta ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.48</td>
<td>0.252944</td>
<td>14.5</td>
</tr>
<tr>
<td>1.0</td>
<td>1.49</td>
<td>0.191889</td>
<td>11</td>
</tr>
<tr>
<td>1.5</td>
<td>1.50</td>
<td>0.130833</td>
<td>7.5</td>
</tr>
<tr>
<td>2.0</td>
<td>1.52</td>
<td>0.095944</td>
<td>5.5</td>
</tr>
<tr>
<td>2.5</td>
<td>1.53</td>
<td>0.078500</td>
<td>4.5</td>
</tr>
<tr>
<td>3.0</td>
<td>1.55</td>
<td>0.061056</td>
<td>3.5</td>
</tr>
</tbody>
</table>

With reference to Table I, from \( \frac{L}{D} = 0.5 \) to \( \frac{L}{D} = 3 \), \( \left( \frac{V_s}{V_0} \right)_{\text{max}} \) increased from 1.48 to 1.55. An increment of 4.7% was achieved in this regard. This means that larger diffusers have greater wind speed augmentation. It is also observed that each \( \frac{L}{D} \) ratio has its own optimum diffuser expansion angle and these expansion angles decrease with increase in \( \frac{L}{D} \).

B. Development of the Wind Speed Augmentation \( \left( \frac{V_s}{V_0} \right) \) Model

The experimental results presented and discussed in [11] were used to develop a mathematical model which estimates the wind speed augmentation by an empty conical diffuser. A mathematical model is the use of a computational or mathematical language to describe a dynamic behaviour of a physical system [12]. Any given mathematical model can be described using three schematic block diagrams, namely the input block, mathematical block and the output block as shown in Fig. 2.
The MATLAB software was used in developing the model. To build and develop the model, the diffuser expansion angle (θ) and $L/D$ were the predictors and $V_x/V_0$ was the desired response. The experimental data had two variables $\theta$ and $L/D$. This dictated that our model be a bivariate polynomial model of the form given by equation (1):

$$f(xy) = p_{00} + p_{01}x + p_{10}x^2 + p_{11}xy + p_{21}x^2y + p_{22}x^2y^2 + \ldots + p_{nn}x^n y^m \quad (1)$$

where $x$ and $y$ are variables and $p$ is a constant.

### III. RESULTS AND DISCUSSION

In this section, the developed wind speed augmentation model for conical diffusers and the comparison of the experimental data and the model is presented. A detailed ranking of predictors by importance of weight contribution to output is also presented.

#### A. Wind Speed Augmentation ($V_x/V_0$) Model

The developed model is given in equation (2). It has 19 different predictor combinations. The modelled wind speed augmentation is strongly correlated to empirical data with $R^2 = 0.9751$ and a root mean square error (RMSE) = 0.

$$\frac{V_x}{V_0} = p_0 + p_1 \theta + p_2 \theta^2 + p_3 \theta^3 + p_4 \theta^4 + \left[p_5 + p_6 \theta + p_7 \theta^2 + p_8 \theta^3 + p_9 \theta^4 \right] \left[\frac{L}{D}\right] + \left[p_5 + p_6 \theta + p_7 \theta^2 + p_8 \theta^3 + p_9 \theta^4 \right] \left[\frac{L}{D}\right]^2 + \left[p_5 + p_6 \theta + p_7 \theta^2 + p_8 \theta^3 + p_9 \theta^4 \right] \left[\frac{L}{D}\right]^3 + \left[p_{14} + p_{15} \theta \left[\frac{L}{D}\right]^4 + p_{16} \left[\frac{L}{D}\right]^5 \right]$$

(2)

The validity of the model is given in the last column of Table I. Table II shows the values of the scaling constants ($p_0 - p_{19}$) of the predictors of the polynomial regression model equation (2).

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Scaling Symbol</th>
<th>Scaling constant</th>
<th>Desired Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>$p_0$</td>
<td>1.0362</td>
<td>$V_x/V_0$</td>
</tr>
<tr>
<td>0</td>
<td>$p_1$</td>
<td>3.2365</td>
<td></td>
</tr>
<tr>
<td>$L/D$</td>
<td>$p_2$</td>
<td>-0.2100</td>
<td></td>
</tr>
<tr>
<td>$\theta^2$</td>
<td>$p_3$</td>
<td>-1.6655</td>
<td></td>
</tr>
<tr>
<td>$0(L/D)$</td>
<td>$p_4$</td>
<td>-1.9960</td>
<td></td>
</tr>
<tr>
<td>$(L/D)^2$</td>
<td>$p_5$</td>
<td>0.3973</td>
<td></td>
</tr>
<tr>
<td>$\theta^3$</td>
<td>$p_6$</td>
<td>-35.0900</td>
<td></td>
</tr>
<tr>
<td>$0(L/D)$</td>
<td>$p_7$</td>
<td>7.3220</td>
<td></td>
</tr>
<tr>
<td>$(L/D)^2$</td>
<td>$p_8$</td>
<td>4.7842</td>
<td></td>
</tr>
<tr>
<td>$\theta^4$</td>
<td>$p_9$</td>
<td>-0.2706</td>
<td></td>
</tr>
<tr>
<td>$0(L/D)$</td>
<td>$p_{10}$</td>
<td>28.6781</td>
<td></td>
</tr>
<tr>
<td>$(L/D)^3$</td>
<td>$p_{11}$</td>
<td>52.2431</td>
<td></td>
</tr>
<tr>
<td>$\theta^5$</td>
<td>$p_{12}$</td>
<td>-20.6286</td>
<td></td>
</tr>
<tr>
<td>$(L/D)^2$</td>
<td>$p_{13}$</td>
<td>-1.2033</td>
<td></td>
</tr>
<tr>
<td>$(L/D)^4$</td>
<td>$p_{14}$</td>
<td>0.0781</td>
<td></td>
</tr>
<tr>
<td>$\theta^6$</td>
<td>$p_{15}$</td>
<td>-38.1745</td>
<td></td>
</tr>
<tr>
<td>$(L/D)^3$</td>
<td>$p_{16}$</td>
<td>15.2757</td>
<td></td>
</tr>
<tr>
<td>$0(L/D)^2$</td>
<td>$p_{17}$</td>
<td>2.1870</td>
<td></td>
</tr>
<tr>
<td>$(L/D)^4$</td>
<td>$p_{18}$</td>
<td>0.0743</td>
<td></td>
</tr>
<tr>
<td>$(L/D)^5$</td>
<td>$p_{19}$</td>
<td>-0.0078</td>
<td></td>
</tr>
</tbody>
</table>
Equation (2) gives the general wind speed augmentation model by empty conical diffusers of $0.5 \leq L/D \leq 3$. This model can be used by DAWT designers to determine the optimum wind speed augmentation by an empty conical diffuser before the turbine rotor is inserted. This enables the designers to optimize the power output of the DAWT system. However, for design purposes, only one $L/D$ is needed. Respective model equations can be deduced by substituting the desired $L/D$ and the predictor coefficients as given in Table II in the model equation (2). As an example, a design equation for $L/D = 1$ is given by substituting for $L/D$ in equation (2) and one gets:

$$V_i / V_0 = 1.0232 + 4.8957\theta - 12.7851\theta^2 + 1.8774\theta^3 - 9.4964\theta^4$$  \hspace{1cm} (3)

Substituting $\theta$ values in equation (3), one gets the optimum angle which correspond to $L/D = 1$, that is, 11°. Therefore, for $L/D = 1$, to get optimum wind speed augmentation which would give optimum wind power output by the DAWT, the conical diffuser should have an expansion angle of 11°. It should however be emphasized that the $\theta$ values should be within the limits of the model as given in the last column of Table I.

B. Ranking of Predictors by Importance of Weight Contribution to Output

To determine the contribution of the predictors to the model, the Relief algorithm was used. It ranks the predictors according to their weight contribution to the desired output. The Relief algorithm was applied to the 19 terms expressed as predictors and given in Table II. The benefit of ranking predictors using Relief algorithm is to be able to differentiate between primary and secondary contributors. In the Relief Algorithm when the weight of contribution to the output is positive the predictor is said to be a primary contributor. In addition, if the weight of the contribution of the predictor to the output is negative the predictor is said to be a secondary contributor. The weight of contribution by the Relief Algorithm for the predictors ranges between -1 and 1 with large positive weights assigned to important predictors. Table III shows predictors and their assigned numbers according to weight of contribution.

<table>
<thead>
<tr>
<th>Assigned Number</th>
<th>Predictor</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>$\theta^2(L/D)^2$</td>
<td>0.0922</td>
</tr>
<tr>
<td>16</td>
<td>$\theta^3(L/D)^3$</td>
<td>0.0798</td>
</tr>
<tr>
<td>17</td>
<td>$\theta^2(L/D)^2$</td>
<td>0.0771</td>
</tr>
<tr>
<td>4</td>
<td>$\theta(L/D)$</td>
<td>0.0556</td>
</tr>
<tr>
<td>7</td>
<td>$\theta^2(L/D)$</td>
<td>0.0544</td>
</tr>
<tr>
<td>8</td>
<td>$\theta^3(L/D)^3$</td>
<td>0.0518</td>
</tr>
<tr>
<td>13</td>
<td>$\theta(L/D)$</td>
<td>0.0454</td>
</tr>
<tr>
<td>11</td>
<td>$\theta^2(L/D)$</td>
<td>0.0406</td>
</tr>
<tr>
<td>18</td>
<td>$\theta(L/D)^2$</td>
<td>0.0400</td>
</tr>
<tr>
<td>15</td>
<td>$\theta^3(L/D)$</td>
<td>0.0313</td>
</tr>
<tr>
<td>1</td>
<td>$\theta$</td>
<td>0.0197</td>
</tr>
<tr>
<td>3</td>
<td>$\theta^2$</td>
<td>0.0196</td>
</tr>
<tr>
<td>6</td>
<td>$\theta^3$</td>
<td>0.0180</td>
</tr>
<tr>
<td>10</td>
<td>$\theta^4$</td>
<td>0.0165</td>
</tr>
<tr>
<td>2</td>
<td>$(L/D)$</td>
<td>0.0042</td>
</tr>
<tr>
<td>5</td>
<td>$(L/D)^2$</td>
<td>0.0035</td>
</tr>
<tr>
<td>9</td>
<td>$(L/D)^3$</td>
<td>0.0024</td>
</tr>
<tr>
<td>14</td>
<td>$(L/D)^4$</td>
<td>0.0014</td>
</tr>
<tr>
<td>19</td>
<td>$(L/D)^5$</td>
<td>5.8935e-04</td>
</tr>
</tbody>
</table>

With reference to Table III, it can be seen that predictors that are purely $L/D$ dependent contributed the least with their weight of contribution decreasing as the power of $L/D$ increases. Basing on the Relief weight ranking, Fig. 3 was generated with the Y axis representing the predictor weight and the X axis represents predictor rank. Each predictor rank number given in Fig. 3, relates to the
predictor number as given in equation (2) and the vertical axis gives the corresponding weight of each predictor in the development of the model.

![Graph of model predictor ranking](image)

Fig. 3. A graph of model predictor ranking

It is observed that all the 19 predictors are primary contributors. The predictors contributing the most are predictors assigned by the numbers 12, 16 and 17 and this corresponds to \( \theta^2 (L/D)^2 \), \( \theta^3 (L/D)^2 \) and \( \theta^2 (L/D)^3 \) (with weight contributions of 0.0922, 0.0798 and 0.0771 respectively). The predictors contributing the least by weight are those assigned by the numbers 19, 14 and 9 and this corresponds to \( (L/D)^5 \), \( (L/D)^4 \) and \( (L/D)^3 \) (with weight contributions of 5.8935e-04, 0.0014 and 0.0024 respectively). It can be seen that \( \theta \) as a predictor contributes more to the desired output as compared \( L/D \). This means that the diffuser expansion angle is more important than \( L/D \) in wind speed augmentation in a conical diffuser.

C. Comparison of Experimental Data and the Developed Polynomial Regression Model

The polynomial regression model was compared with the experimental data. Fig. 4 and 5 show graphs of the comparison of the model and the experimental data. Only graphs for \( L/D = 0.5 \) and \( L/D = 3 \) are shown. For each \( L/D \), graphs of the experimental data and the model, the model equation and the corresponding RMSE are given.

Model equation for \( L/D = 0.5 \):

\[
\frac{V_x}{V_0} = 1.0013 + 3.2887\theta - 2.8883\theta^2 - 12.7874\theta^3 + 9.5909\theta^4 \pm 0.039
\]

(4)

![Graphs for the comparison of the model and experimental data for \( L/D = 0.5 \)](image)

Fig. 4 Graphs for the comparison of the model and experimental data for \( L/D = 0.5 \)
Model equation for $L/D = 3$:

$$
\frac{V_x}{V_0} = 1.1064 + 13.8355\theta - 106.3079\theta^2 - 15.8420\theta^3 - 85.8454\theta^4 \pm 0.028
$$

Equations (4) and (5) give model equations for $L/D = 0.5$ and $L/D = 3$ respectively. In Fig. 4, $(V_x/V_0)_{max} = 1.48$ was achieved at $14.5^\circ$ (0.252944 rad.) and in Fig. 5, $(V_x/V_0)_{max} = 1.55$ was achieved at $3.5^\circ$ (0.061056 rad). Therefore, $14.5^\circ$ and $3.5^\circ$ are the optimum design angles for $L/D = 0.5$ and $L/D = 3$ respectively. To achieve optimum wind power output, DAWT of $L/D = 0.5$ and $L/D = 3$ should have diffuser expansion angles of $14.5^\circ$ and $3.5^\circ$ respectively. The RMSE was used as the goodness of fit of the model. It gives the average deviation of the model from the experimental data. For $L/D = 0.5$ a RMSE of 0.039 was obtained and 0.028 for $L/D = 3$. The deviations are quite minimal. This indicates that the developed model agrees with the experimental data. The model can reliably be used for conical diffuser design.

Fig. 5 Graphs for the comparison of the model and experimental data for $L/D = 3$

Equations (4) and (5) give model equations for $L/D = 0.5$ and $L/D = 3$ respectively. In Fig. 4, $(V_x/V_0)_{max} = 1.48$ was achieved at $14.5^\circ$ (0.252944 rad.) and in Fig. 5, $(V_x/V_0)_{max} = 1.55$ was achieved at $3.5^\circ$ (0.061056 rad). Therefore, $14.5^\circ$ and $3.5^\circ$ are the optimum design angles for $L/D = 0.5$ and $L/D = 3$ respectively. To achieve optimum wind power output, DAWT of $L/D = 0.5$ and $L/D = 3$ should have diffuser expansion angles of $14.5^\circ$ and $3.5^\circ$ respectively. The RMSE was used as the goodness of fit of the model. It gives the average deviation of the model from the experimental data. For $L/D = 0.5$ a RMSE of 0.039 was obtained and 0.028 for $L/D = 3$. The deviations are quite minimal. This indicates that the developed model agrees with the experimental data. The model can reliably be used for conical diffuser design.

**IV. CONCLUSION**

A polynomial regression model which estimates wind speed augmentation by an empty conical diffuser was developed. The model is useful to DAWT engineers in the design stage of the optimization of the power output of DAWT wind energy systems. Each $L/D$ has its own particular optimum angle which give maximum wind speed augmentation. This is obtained from the corresponding model equation of that $L/D$. The model and the experimental data were correlated with $R^2 = 0.9751$ and RMSE = 0.034. It was illustrated that the expansion angle ($\theta$) as a predictor contributes more to the desired output as compared to $L/D$. This means that during the design and construction stages, more attention should be given to the diffuser expansion angle since wind speed augmentation is heavily depended on it.

**REFERENCES**

