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# Implementation of Hertz Contact Theory and Validation of Applicability of Hertz Contact Stiffness Model for Helical Gear using Multi Body Dynamics

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**Abstract:** The aim of this work is to evaluate the Hertz contact stiffness or Linear contact stiffness of Helical Gears in dynamic conditions. The Hertz contact occurs when two solids come in contact like sphere on sphere, cylinder on cylinder, cylinder in a groove and many other curved general contacts. In gears the contact is usually cylinder on cylinder. The Hertz model considered for the study consists of two cylinders with a spring that establishes the contact. Also the Helical Gear is modelled using Adams Gear AT. The gear is simulated for required dynamic conditions to evaluate the linear contact stiffness by establishing the relation between the Contact forces, Contact stiffness and Penetration.

**Keywords:** Hertz contact stiffness, linear contact stiffness, Helical Gear, Dynamic analysis, Contact forces and penetration

## I. INTRODUCTION

Gears are a standout amongst the most vital parts of energy transmission in different mechanical applications. As of late, there rose a need to design efficient gears because of the rising execution necessities of different power transmission applications, for example, higher load-carrying capacity, higher quality, longer working life, bring down cost, and higher speed. The most imperative contributing element of dynamics of gears is the Stiffness of the teeth, which changes always all through the operation.

Established contact mechanics is most outstandingly connected with Heinrich Hertz. Hertz tackled the contact issue of two elastic bodies with bended surfaces in 1882. This still-important established hypothesis gives a foundation to present day issues in contact mechanics. Utilizing hertz contact hypothesis to compute the contact stiffness can spare a considerable measure of time in the world of contact mechanics of gears. The evolution modern design analysis tools make it easy to apply the theory with the use of modern day contact algorithms.

The elasticity of the gear mesh plays a vital role in obtaining valuable outcomes while carrying out a noise, vibration, and harshness (NVH) analysis of a transmission system. MSC. Adams is multibody dynamics tool that helps to create exact dynamic simulation of the gear for the required operating conditions and enables to validate the gear contact mechanics. This paper will explain why it is important to account for gear contact stiffness as well as how to validate the Hertz Contact stiffness model with use of multibody dynamics tool MSC.Adams

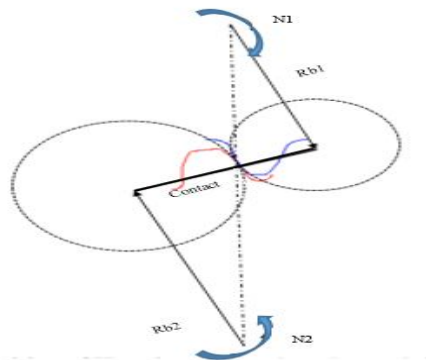


Fig. 1 Hertz Contact Model for Gear

## II. LITERATURE SURVEY

The evaluation of the gear teeth contact stiffness is very complex because of its nonlinear behavior. Contact stiffness study of gears is very important in the design of gear for its dynamic conditions. Poritsky [1] carried out the study on the stresses and deflections of the cylindrical bodies in contact. There many conditions of contact such as rolling contact and sliding contact. He applied the theory on the railway axles and gears of locomotives to study the Hertz stress behavior. The application of Hertz theory comes into place when at least one of the contacting bodies has curved surface. Ozguvan [2] carried a survey to study the mathematical models used in gear dynamics. He has classified and explained the basic principle of each model, advantages and disadvantages. Li [3] in his literature has explained several models for mesh stiffness and load distribution along the line of contact. Pedersen [4] has evaluated the tooth stiffness using the Finite element method. In unique wheel/rail models, the Hertzian contact stiffness is an essential parameter. A track structure and wheel set associated by methods for a Hertzian spring is utilized to portray high recurrence vertical vibrations. Nazmul Hasan [6] utilized Bousinesque condition for vertical twisting of rail under the focal point of a roundabout stacked surface to infer a linearized Hertzian contact firmness recipe and analyses

distinctive methodologies accessible in writing. The qualities acquired from proposed recipe are contrasted and the qualities from writing with the end goal of approval.

The contact stiffness of a mechanical joint surface impacts the get together reverberation recurrence by influencing the solidness network of the motor condition. The Hertz contact hypothesis is enhanced and encouraged to establish a mapping connection between the gathering procedure parameters and contact stiffness, which is connected in the counter kickback adapt and the point contact ball bear to build up lessened model of transmission framework. Wan Fei et al [7] in their study, the contact stiffness in anti-backlash gear is ascertained and changed over considering the driving torque and rigging preloading, while the contact firmness or stiffness in edge contact metal ball is enhanced and converged by increasing a redress factor  $\xi=1.5$  in pivotal preloading. By including adding non-massive rigid auxiliary elements, the transmission framework demonstrates made by the anti-backlash gear and the edge contact ball bear is set up to mimic the dynamic execution in ADAMS programming. The recurrence reaction estimations of the transmission framework concur well with the hypothetical esteem. Therefore, the complex hypothetical computation recipes can be supplanted by the disentangled model.

## III. METHODOLOGY

In this study, the conjugate action between the two helical gears is given by the hertz model of line contact between two cylinders. The contact conditions are similar to the Hertz model and therefore this model is widely used in *tooth contact simulation*. Though the effect of hertz contact stiffness is less on the load sharing capacity of the gear but overall mesh stiffness will be affected by the linear contact stiffness of gears in mesh.

From the survey the equation for the Hertz Contact stiffness can be defined by the following equation,

$$K_H = \frac{\pi}{4} \frac{E b}{(1-\nu^2)} \quad (1)$$

The 3-D model of the gear is generated using Adams gear AT generator. In this model, modelling option defines the type of contact such as gear fast, rigid gear and flexible tooth. For the present study flexible tooth is considered which gives accurate results because of its advanced surface to surface contact algorithm. The tooth flanks are defined by the extruded profile with the consideration of micro geometry. The contact is checked for 5 teeth on their both right and left flanks.

For the rigid body, the contact force is defined by the maximum penetration of the contact plane of one gear into the flank of the other gear. The resulting contact force is the vector sum of all the contact forces. It is given by the following equation (2)

$$F_c = K_H * p^e \quad (2)$$

Where  $F_c$  is the contact force,

$K_H$  is the contact stiffness,

$p$  is the maximum penetration and

$e$  is the exponent whose value is taken as nearly equal to 1.

For flexible teeth, the nonlinear contact algorithm which also considers the above same surface to surface contact property.

### A. Helical Gear Model

The Helical gear arrangement is modelled using Adams Gear AT. The model has approximately 14 degrees of freedom and the details are given below.

- 1) 14 Gruebler Count (approximate degrees of freedom)
- 2) 7 Moving parts (not including ground)
- 3) 3 Revolute Joints
- 4) 1 Spherical Joints
- 5) 1 Fixed Joints
- 6) 1 Inline Primitive\_Joints
- 7) 1 Inplane Primitive\_Joints
- 8) 1 Motions

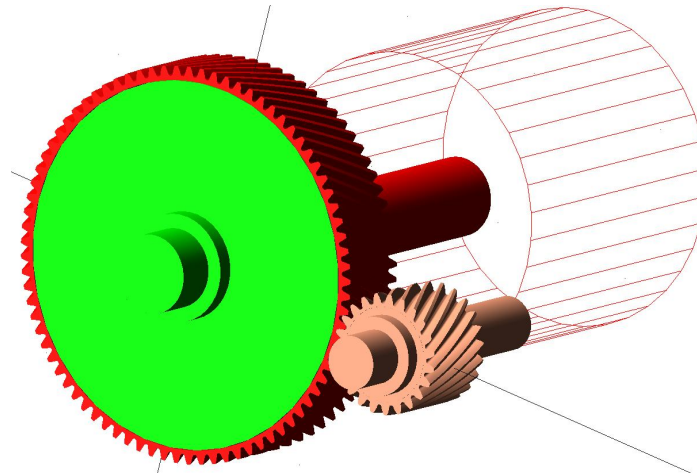


Fig. 2 Helical Gear Model in MSC. Adams

The general gear properties are as follows

	Parameter	Pinion	Gear
General Parameters			
1	Normal module ( $m_n$ )	1.395	1.395
2	Number of teeth ( $z$ )	23	81
3	Pressure angle in normal plane ( $\alpha_n$ )	20°	20°
4	Helix angle on reference circle ( $\beta$ )	24°	24°
5	Hand of Helix	LH	RH
6	Profile Shift co-efficient ( $x$ )	0.1755	-0.4611
7	Face Width ( $b$ )	30	28
8	Rim Diameter ( $d_{rim}$ )	30	116
Mass Properties			
9	Mass		
10	Density	7.83E-06	7.83E-06
11	Young's Modulus	2.05E+05	2.05E+05
12	Poisson's ratio	0.29	0.29

The output request has been made to generate the Max penetration over the time and also for the contact forces over the time. From the study, both the dynamic model and theoretical model can be compared and values of the contact stiffness can be calculated using equations (1) and (2).

### B. Hertz Contact Model

The Hertz model consists of two cylinders of infinite length. The contact is established using a spring between the two cylinders as shown in below figure. As per the calculations the theoretical contact stiffness for hertz model is obtained as 5.31E06 N/mm. The spring with the same stiffness is created between the two cylinders.



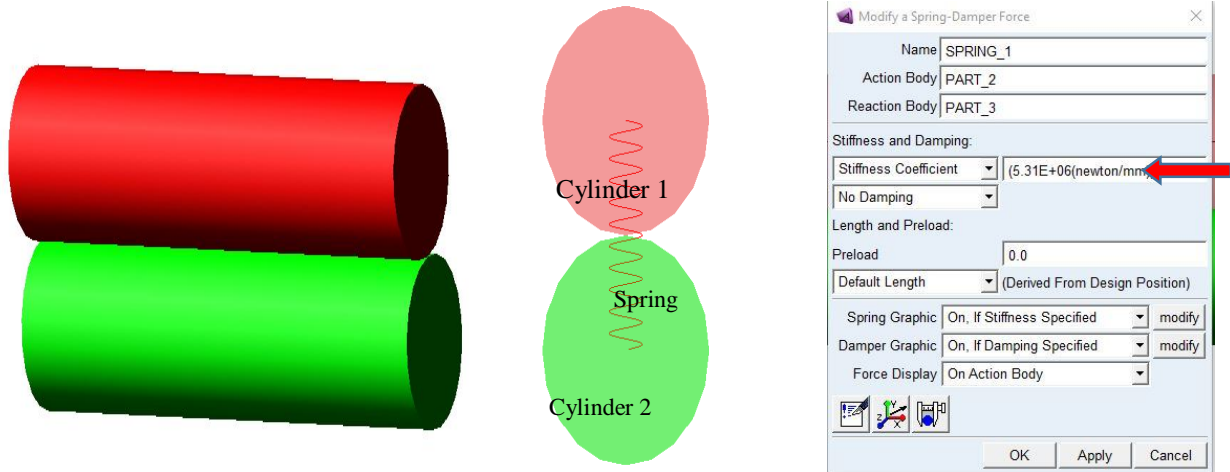


Fig. 3 Hertz Contact Model in MSC. Adams

#### IV. RESULTS AND DISCUSSION

The dynamic model of the helical gear has been run for the time period of 0.7 seconds over the time step of 5.0E-05. The results for the Dynamic simulation of contact forces and penetration are obtained. Generally, the deformation of a tooth is coming from the deformation of the wheel body, the deformation of the teeth and the 'Hertz contact'.

The forces resulting from the contact between the gears on the wheel are as follows.

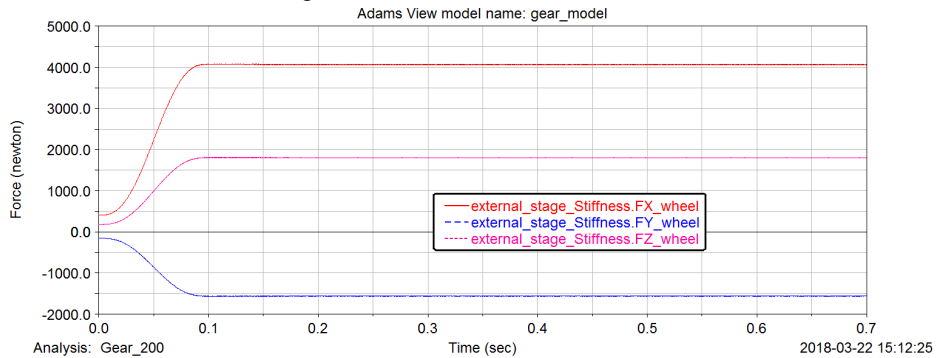


Fig. 4 Contact forces acting on wheel

The Max penetration on teeth O, M1, M2, P1 and P2 are as follows

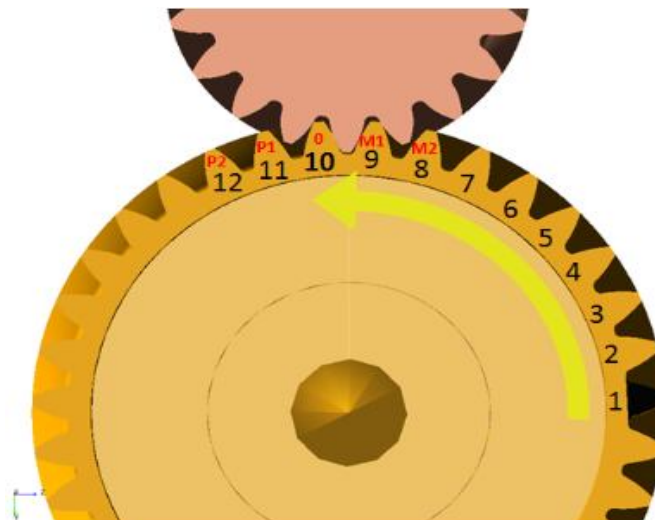


Fig. 5 Gear teeth numbering

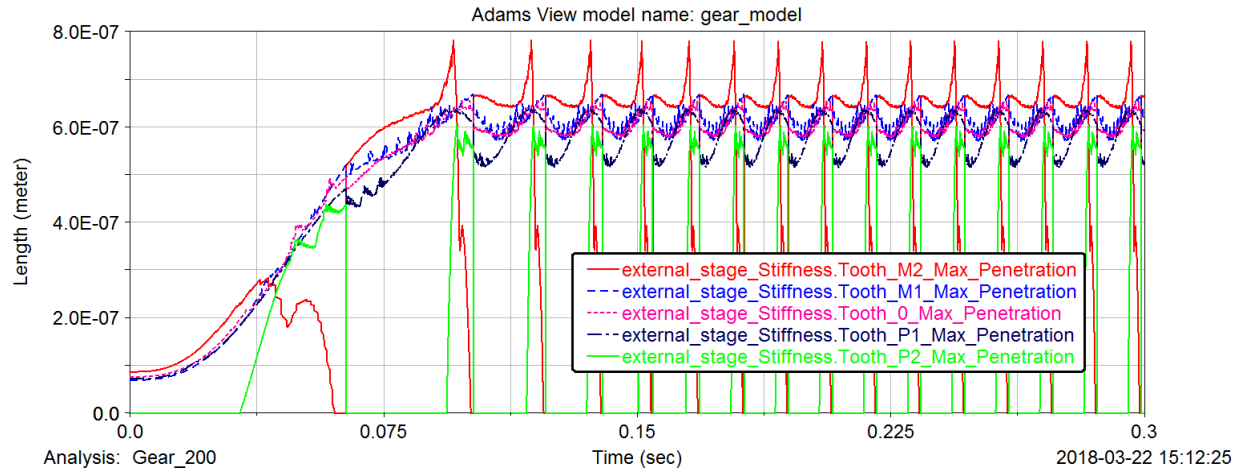


Fig. 6 Max penetration on the gear teeth

From the Analysis the absolute value of the Max Contact force is 4087 Newton and Max penetration is 7.8E-07 meter using this the linear contact stiffness for the Dynamic Helical gear model can be calculated.

$$K_H = \frac{F_c}{p^3}$$

A. Contact stiffness (Helical Gear)

$$K_H = \frac{F_c}{p^3} = \frac{4087}{7.8E-04} = 5.23E06 \frac{N}{mm}$$

B. Contact stiffness (Hertz Model)

$$K_H = \frac{\pi}{4} \frac{Eb}{(1-\gamma^2)} = \frac{\pi}{4} \frac{205000 \times 30}{(1-0.29^2)} = 5.31E06 \frac{N}{mm}$$

As per the calculations the theoretical contact stiffness for hertz model is obtained as 5.31E06 N/mm, to validate the above results the spring with the calculated stiffness value is created between the contact. After the dynamic analysis of the model, the force obtained from the spring of specified stiffness should match the contact force of the gear. When the Hertz Model is simulated the maximum force that is generated for the given spring stiffness is around 4000 N. This is the perfect evaluation of the Hertz Contact model for the gear.

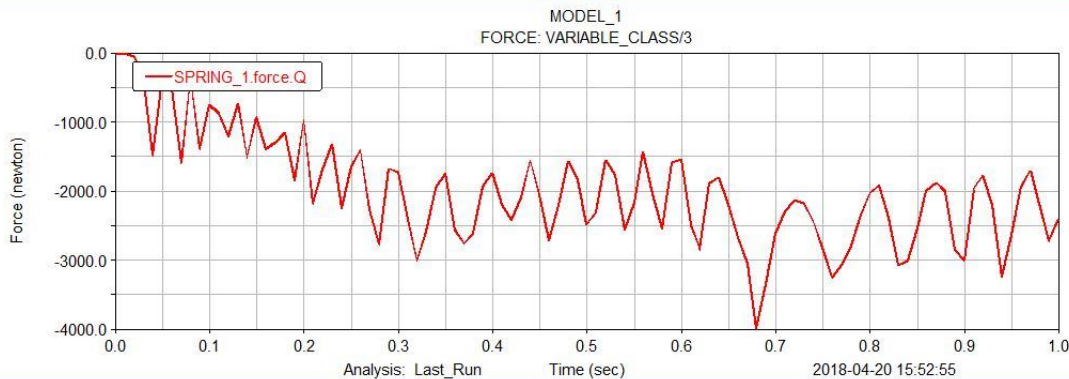


Fig. 7 Spring Force

Hertz contact stiffness value matches with the linear contact stiffness value of the helical gear. The model is verified successfully.

## V. SUMMARY

The above work represents the application of the dynamic analysis of helical gear in contact to validate the Hertz contact stiffness or linear contact stiffness. The results from the dynamic analysis of gears as well as of Hertz model are shown in figures 4,6 and 7. The equations for standard model of Hertz contact and helical gear obtained from the technical literatures are used for the work. The



main innovation of this study is in the use of simple functions for the description of the contact stiffness to extend the Hertz contact stiffness to helical gear model. The above specified method efficiently validated the stiffness value for the contact. This study summarizes an efficient and accurate method to validate the applicability of hertz contact stiffness model of gear tooth contact in dynamics using modern engineering tool like MSC. Adams

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