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Oscillation Theorems for a Second Order Delay Difference Equations

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Abstract: The Oscillation Theorems for a Second Order Delay Difference Equations of the form

$$\Delta^2 u_n + p_n u_{(n-\sigma(n))} = 0, n \in \mathbb{N}(a) \dots\dots\dots(1)$$

The method uses techniques based on signum functions. Example is inserted to illustrate the result.

Keywords: Delay Difference equation, signum functions, Boundedness

2010MSC: 39A10

1. INTRODUCTION

The purpose of this paper is to present conditions for all solutions of the linear delay difference equations of the form

$$\Delta^2 u_n + p_n u_{(n-\sigma(n))} = 0, n \in \mathbb{N}(a) \dots\dots\dots(1)$$

Where Δ is the forward difference operator i.e.

$$\Delta u_n = u_{n+1} - u_n [\Delta^2 u_n = \Delta(\Delta u_n)]$$
 to be oscillatory, and to

present an oscillation theorem for the more general equation

$$\Delta(r_n \Delta u_n) + p_n f(u_n, u_{g(n)}) = 0 \dots\dots\dots(2)$$

assume the condition without further mentions $\{r_n\}, \{p_n\}$

positive real sequence on interval $n \in \mathbb{N}(a)$

$\{r_n\} > 0, \{p_n\} \geq 0$ and $0 \leq \sigma(n) \leq m$. The assumptions

on f and g are stated proceeding Lemma 2 in section.

Results on the growth and boundness of non-oscillatory solutions are presented in section 3. By a solution of (1) and (2) we mean real sequence $\{u_n\}$ satisfying equation (1) for $n \in \mathbb{N}(a)$. We consider only such solutions which are non-trivial for all large n . A solution $\{u_n\}$ of (1) and (2) is called non-oscillatory if it is eventually positive or negative. Otherwise it is called Oscillatory.

The problem of determining oscillation theorem for second order nonlinear difference equation has been the subject of investigation in [1-3]. Among the papers dealing with this subject we refer to [4] in which oscillatory theorem of linear difference equations of second order have been established.

The purpose of this note is to give some new criteria (sufficient conditions for oscillation of all solutions of (1). The

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results we obtain the discrete analogues of some theorem for a non-linear differential equation of second order with delay due to katasatos [5] and staikes &petsoulat[6].

2. MAIN RESULTS

The following Lemma allows the use of technique introduced by Coles [1] to give a short proof of a classical oscillation Theorem for equation (3).

Lemma 1

If $p_n \geq 0, (p_n \neq 0), 0 \leq \sigma(n) \leq m$ for all $n \geq a$ and $\{u_n\}$ is a solutions of equation (1) that is positive, then $\Delta u_n > 0$ for all n sufficiently large and there is a constant $\kappa > 0$ such that

$$\frac{u_{(n-\sigma(n))}}{u_n} \geq \kappa$$

Proof.

If $\{u_n\}$ is positive, then so is $u_{(n-\sigma(n))}$ and there for $\Delta^2 u_n \leq 0$. This means the $\{u_n\}$ is bounded non-oscillatory so that if $\Delta u_n \leq 0$, $\{u_n\}$ become zero again. Therefore which is contrary to the hypothesis. Hence $\Delta u_n > 0$ for all n sufficiently large. Now suppose that n_0 is larger than the last zero of u_{n-m} . Then $n \geq n_0, u_{n-m} \leq u_{(n-\sigma(n))}$ and

$$\frac{u_{(n-\sigma(n))}}{u_n} \geq \frac{u_{n-m}}{u_n}$$

Let g be a function whose graph is the line tangent to the graph of $\{u_n\}$ at $(n-m, u_{n-m})$ for some $n \geq n_0$; that is

$$g_s = \Delta u_{n-m}(s-t+m) + u_{n-m}$$

Since $\{u_n\}$ is bounded,

$$\frac{u_{n-m}}{u_n} \geq \frac{u_{n-m}}{g_n} = \frac{g_{n-m}}{g_n}$$

Let

$$x = \frac{-u_{n-m}}{\Delta u_{n-m} + n - m}$$

Then $g_x = 0$ and because of similar triangles we have

$$\frac{u_{n-m}}{u_n} \geq \frac{g_{n-m}}{g_n} = \frac{(x-n+m)}{(x-n)} = \frac{u_{n-m}}{[u_{n-m} + m\Delta u_{n-m}]} \dots\dots\dots(4)$$

But Δu_{n-m} is decreasing so that the last member of (4) increases to a positive limit κ as $n \rightarrow \infty$. Hence

$$\frac{u_{(n-\sigma(n))}}{u_n} \geq \frac{u_{n-m}}{u_n} \geq \kappa$$

for all n sufficiently large.

Theorem 1

If $\{p_n\} \geq 0, 0 \leq \sigma(n) \leq m$ for all $n > a$ and $\sum_{s=a}^{\infty} p_s = \infty$, then a

proof:

If $\{u_n\}$ be a non oscillatory solution of the equation (1)

and there $\Delta^2 u_n \leq 0, \Delta u_n \leq 0$

for sufficiently large n Let $\omega_n = \frac{\Delta u_n}{u_n}$. Then (1) becomes

$$\Delta \omega_n + \omega_n^2 + \frac{p_n u_{(n-\sigma(n))}}{u_n} = 0$$

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And since the divergence of the summation $\sum_{s=a}^{\infty} p_s$ implies that

$p_s \neq 0$ it follows from Lemma 1 that

$$\Delta\omega_n + \omega_n^2 + \kappa p_n \leq 0$$

Thus for n_0 sufficiently large and $n \geq n_0$

$$\omega_n + \sum_{s=n_0}^n \omega_s^2 \leq \omega_{n_0} - \kappa \sum_{s=n_0}^n p_s < 0 \dots \dots \dots (5)$$

Let $h_n = \sum_{s=n_0}^n \omega_s^2$. Then it follows from (5) that $\Delta h_n \geq h_n^2$

from which it follows that

$$n - n_0 \leq \frac{1}{h_{n_0}} - \frac{1}{h_n} < \frac{1}{h_{n_0}}$$

For large n . Which contradiction to fact that

$\omega_n \geq 0$ for $n \geq n_0$.

If $r_n = 1, \{p_n\} \geq 0, g_n \rightarrow \infty$ as $n \rightarrow \infty$ the all solutions equation (2) is oscillatory.

The following lemma is helpful in proving an oscillation theorem for (2) with non-constant r

If $f(y, w)$ has the sign of y and w when they have the same sign, and $f(y, w)$ is non-decreasing in y and w . For

the oscillation Theorem presented here we make the following assumptions on f and g :

- (i) $g_n \rightarrow \infty$ as $n \rightarrow \infty$
- (ii) If y and w are one sign, the $f(y, w)$ has that sign
- (iii) $f(y, w)$ is bounded and zero.

Note that condition (ii) and (iii) is satisfied and f is non decreasing in y and w .

Lemma 2

If $\{p_n\} \geq 0, (p_n \neq 0), r_n \geq 0, \sum_{s=a}^{\infty} \frac{1}{r_s} = \infty$ Conditions (i)-

(iii) hold and $\{u_n\}$ is a solutions of equation (1) that is positive, then $\Delta u_n \geq 0$ for all n sufficiently large

Proof

If $\{u_n\} > 0$ for large n and lemma is false. Then there is a point n_0 larger than the last zero u_n and larger than the last zero

$u_{g(n)}$ such that $\Delta u_{n_0} < 0$ for $n \geq n_0$

$\Delta(r_n \Delta u_n) = -p_n f(u_n, u_{g(n)}) \leq 0$ and therefore

$r_n \Delta u_n \leq r_{n_0} \Delta u_{n_0} < 0$ it follows dividing by r_n and summing

from n_0 to n and $n \rightarrow \infty$ then $\{u_n\}$ is negative. This is contrary to the hypothesis. A similar argument treats the case of

eventually positive or eventually negative If $\{u_n\} < 0$ for large n

Theorem 2

If $p_n \geq 0, r_n \geq 0, \sum_{s=a}^{\infty} \frac{1}{r_s} = \infty, \sum_{s=a}^{\infty} p_s = \infty$ Conditions (i)-(iii)

hold and $\{u_n\}$ is a solutions of equation (2) is oscillatory if interval (a, ∞)

Proof:

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Suppose there is a solutions $\{u_n\}$ on the interval (a, ∞) and that If $\{u_n\} > 0$ for large n .

By lemma 2 $\Delta u_n \geq 0$ for all n sufficiently large so that $\{u_n\}$ is non-decreasing; it then follows from conditions (i)-(iii) that there is positive constant ℓ such that $\ell \leq f(g(n), u_{g(n)})$ for all n sufficiently large. Thus $\Delta(r_n \Delta(u_n) + p_n \ell \leq 0$ and for n_0 sufficiently and $n > n_0$ and Taking summation

$$r_n \Delta u_n - r_{n_0} \Delta u_{n_0} + \ell \sum_{s=n_0}^n p_s \leq 0 \dots \dots \dots (6)$$

But $\ell \sum_{s=n_0}^n p_s \rightarrow \infty$ as $n \rightarrow \infty$ this contrary that to lemma. A

similar argument left to reader if $u_n < 0$ for large n

3. Boundedness and Non-Oscillation

The proof of Lemma 2 suggests the following theorem

Therom 3

If (i)-(iii) hold $p_n \geq 0$ and $\{u_n\}$ is a non-oscillatory solution of (2) on an interval (a, ∞) , then there are non-

negative constants ℓ_1, ℓ_2 such that $|u_n| \leq \ell_1 + \ell_2 \sum_{s=n_0}^n \frac{1}{r_s}$. In

particular, if $\sum_{s=n_0}^n p_s < \infty$ then all solutions existing on (a, ∞)

are oscillatory or bounded.

Proof:

If $\{u_n\}$ is non-oscillatory solution of (2) that is positive then $\Delta(r_n \Delta u_n) \leq 0$, for all n sufficiently large.

Summing the n_0 to n

$$0 < u_n \leq u_{n_0} + r_{n_0} \Delta u_{n_0} \sum_{s=n_0}^n \frac{1}{r_s}$$

We take $\ell_1 = |u_{n_0}|, \ell_2 = r_{n_0} |\Delta u_{n_0}|$ if $\{u_n\}$ is negative for all large n , the process used above leads to the inequality

$$u_n \geq u_{n_0} + r_{n_0} \Delta u_{n_0} \sum_{s=n_0}^n \frac{1}{r_s}$$

In this case

$$0 < |u_n| = -u_n \leq -u_{n_0} - r_{n_0} \Delta u_{n_0} \sum_{s=n_0}^n \frac{1}{r_s}$$

This complete the proof

Theorem 4

Let $p_n \geq 0$, suppose that $\{p_n\}$ is real sequence with $0 \leq \sigma(n) \leq m$ suppose that $\gamma \geq 1, \gamma$ is ratio of odd integers the signum fuctions :

$\text{sgn } u_n = 1$ if $\{u_n\} > 0, \text{sgn } u_n = -1$ if $\{u_n\} < 0, \text{sgn } 0 = 0$

Then the condition

$$\sum_{s=a}^{\infty} s p_s = \infty \dots \dots \dots (7)$$

Is necessary condition that all solutions of the equation

$$\Delta^2 u_n + p_n |u_{(n-\sigma(n))}^{1-\gamma}| \text{sgn } u_n = 0 \dots \dots \dots (8)$$

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Are oscillatory. For $\gamma = 1$, (7) is a sufficient condition that all bounded solutions of (8) are oscillatory.

Proof:

For $\gamma > 1$, if $\{u_n\}$ is a bounded solution, and $\{u_n\} > 0$ for all n sufficiently large, Then there is a point c such that

$$\Delta^2 u_n \leq 0, \Delta u_n \geq 0, u_n \geq 0, \text{ for } n \geq c. \text{ then}$$

$$n \geq c, u_{n-m} \leq u_{(n-\sigma(n))} \text{ so the multiplying } \frac{n}{u_{(n-\sigma(n))}} \text{ by (8)}$$

and summing c to n

$$\sum_{s=c}^n \frac{s \Delta^2 u_s}{u_{s-m}} \leq - \sum_{s=c}^n s p_s \dots \dots \dots (9)$$

summing the fact that $\Delta u_{s-m} \geq \Delta u_s$ it is clear that (9) can be written as

$$\left[\frac{s u_s}{u_{s-m}} \right]_c^n - \sum_{s=c}^n u_{s-m} + \sum_{s=c}^n \frac{s \Delta u_s \Delta u_{s-m}}{[u_{s-m}]^2} \leq - \sum_{s=c}^n s p_s \dots \dots \dots (10)$$

Since $\{u_n\}$ is bounded, the left side of (10) is bounded below while the right tends $-\infty$

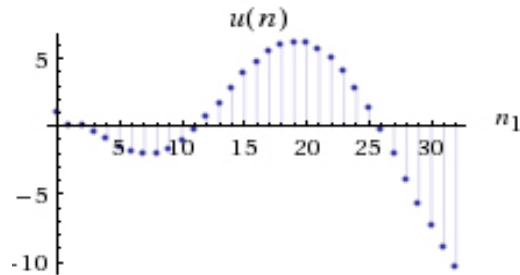
This contradiction completes the proof.

Example:1

Consider the difference equation

$$\Delta^2 u_n + \frac{n}{n^2 + 1} u_{n-1} = 0 \dots \dots \dots (11)$$

All condition to Theorem 1 to Theorem 4 satisfied and all solution of equation (11) is oscillatory.



REFERENCES

1. W.J. Coles, A Simple proof of a well-known oscillation theorems, *proc.Amer.Math.Soc.* 19(1968),507
2. YA.V.Bykov and E.I.Sevcov, Sufficient conditions for oscillation of solutions of non linear finite difference equations(i.e. Russian) *Differencial'nye Uravnenija* 9 (1973) 2241-2244.
3. YA.V.Bykov and L.VZivogladova, On the oscillation of solutions of non linear finite difference equations(i.e. Russian) *Differencial'nye Uravnenija* 9 (1973) 2080-2081
4. YA.V.Bykov , L.VZivogladova and E.I.Sevcov, Sufficient conditions for oscillation of solutions of non linear finite difference equations(i.e. Russian) *Differencial'nye Uravnenija* 9 (1973) 1543-1524
5. D.Hinton and R.Lewis, Spectral Analysis of second order difference equation *J.Math.Anal.Appl.*63 (1978), 421-438
6. A.G.Kartatos, Some therems on oscillations of certain nonlinear second order ordinary differential equations, *Arch.Math(Basel)*18 (1967) 425-429

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7. V.A.staikos and A.G.Petsoulas, Some oscillation criteria for second order nonlinear delay-differential equations, *J.Math.Anal.Appl.*30 (1970), 695-701
8. L. Berezansky, E. Braverman, and E. Liz, *Sufficient conditions for the global stability of nonautonomous higher order difference equations*, Journal of Difference Equations and Applications **11** (2005), no. 9, 785–798.
9. I. Györi and G. Ladas, *Oscillation Theory of Delay Differential Equations: With Applications*, Oxford Mathematical Monographs. Oxford Science Publications. The Clarendon Press Oxford University Press, New York, 1991..
10. P.Mohankumar and A.Ramesh, *Oscillatory Behaviour Of The Solution Of The Third Order Nonlinear Neutral Delay Difference Equation*, International Journal of Engineering Research & Technology (IJERT) ISSN: 2278-0181 Vol. 2 Issue 7,no.1164-1168 July – 2013
11. B.Selvaraj, P.Mohankumar and A.Ramesh, *On The Oscillatory Behavior of The Solutions to Second Order Nonlinear Difference Equations*, International Journal of Mathematics and Statistics Invention (IJMSI) E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759 Volume 1 Issue 1 | Aug. 2013| PP-19-21
12. P.Mohankumar and A.Ramesh *A logistic First Order Difference Equation of Periodic Chemotherapy Model*, American Journal of Pharmacy & Health Research 2013. 2013, Volume 1, Issue 8 ISSN : 2321–3647(online)
13. P.Mohankumar and A.Ramesh, *Rate of Memorization the School Mathematics Using A Difference Equation Model*, International Journal Of Scientific Research And Education Dec.2013 Volume1, Issue 8 Pages 200-203||2013 ISSN (e): 2321-7545
14. P.Mohankumar and A.Ramesh, *On the Oscillatory Behaviour for a certain of second order delay difference equations*, Proc. of National Conference Recent Advances in Mathematical Analysis and Application-2013 on 06th and 07th sep.2013 page:201-206 ISBN:978-93-82338-70-3



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