Comparative Study on PCA, LDA and their Fusion

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Abstract: Face recognition is one of the most commonly used biometric for person recognition. It is used for real time identification of surveillance video images. It is also used to provide law enforcement and information security. Principle Component Analysis (PCA) and Linear Discriminate Analysis (LDA) are the appearance based techniques which are mainly used for face recognition. PCA was used for feature extraction and dimension reduction. LDA was used to further improve the separability of samples. The PCA and LDA can be combined using wavelet fusion technique. The combination of these two can be obtained using wavelet fusion technique. It contains various levels to carry out fusion like feature extraction, matching score level, decision level, rank level.

Keywords: LDA, PCA, dimension reduction, feature extraction, Wavelet fusion.

I. INTRODUCTION

Face recognition has become one of the most successful application in the field of image analysis and understanding. The development and design of a camera based attendance system that could seen the faces of multiple students and could be successor to all the biometric devices that are being used till date are proposed here. Some of the preferred methods are principle component analysis (PCA) and linear discriminate analysis (LDA) [4]. Linear discriminate analysis which is also called fisherface is an appearance based technique used for dimensionality reduction and recorded a great performance in face recognition. This method works on the same principle as eigen face method (PCA). PCA performs dimensionality reduction by preserving as much of the class discriminatory information as possible. PCA projects data in an orthogonal subspace generated by the eigen vectors of the data co-variance matrix. Some recent advances in PCA based algorithm include multilinear subspace analysis, symmetrical PCA, two dimensional PCA, weighted modular PCA, kernel PCA and diagonal PCA. PCA can only capture these features contributing to the global characteristics of data because each principle component only contain some levels of global characteristics of data. PCA is based on the second order statistical property of signal to extract main components with correlation. But the image matrix is needed to be converted into image vector according to the row stacking or the column stacking when the PCA is used. And dimensions of the image vector is generally high I this way, which make the calculated amount consumed during the whole feature extraction process considerable[1]. LDA is used to find a linear combination of features which characterize or separate two or more classes of objects or events the resulting combination may used as a linear classifier. In computerized face recognition each face is represented by a large number of pixel values. LDA is primarily used here to reduce the number of features to a more manageable number before classification. Each of the new dimension is a linear combination of pixel values which form a template. The linear combination obtained using fishers linear discriminate are called fisher faces. LDA can obtain features which have optimal ability at identification, thus LDA is more adaptive to task of pattern classification[8]. A number of face recognition algorithm, along with their modification have been developed during several decades. Those algorithm are classified into two categories, appearance based algorithm and model based algorithm, through PCA and LDA are most widely used appearance based methods, they both have their own advantages and disadvantages. LDA select features that are most effective for class seperability.

PCA selects features important for class representation. PCA might outperform LDA when the number of samples per class is small. In the case of training set with large number of samples, LDA might outperform to get the benefit of both PCA and LDA. The proposed system uses PCA and LDA for feature extraction.

The proposed system gets the best features space by using two different feature extraction technique[5]. Both the acquired features can be combined using wavelet fusion technique. This kind of fusion is hence used in the existing face recognition systems. The wavelet fusion takes both feature extracted as input, combines them into one and produce a signal image. It improves quality of the proposed system. A.Ross and A.Jain have proposed various levels of fusion techniques:

A. Feature extraction level Fusion

As the features extracted from each classifier are independent of each other, they can be concatenated into a single new feature vector.
B. Matching score level fusion
Each classifier creates its feature vectors independently and compares them with the stored training learning templates.

C. Decision level fusion
Each classifier makes an individual recognition decision.

II. DESCRIPTION
Recognizing the human face is a simple task for a human being even after several years. But doing the same task by a computer is not simple because the computer will have problems in recognizing the human face if there is a chance in a facial image like lightning conditions, complex background, pose and occlusion. Still recognizing faces in images is an emerging trend in image processing streams. These recognizing of images can be done using LDA and PCA and the fusion of both[10].

A. Principle Component Analysis
It is a dimension reduction which is used for compression and recognition problems. It is also known as eigen face projection and closely related to popular signal processing technique. PCA is a method to efficiently represent a collection of sample points, reducing the dimensionality of the description by projecting the points on to the principle axis[1]. We’ll describe the PCA algorithm as proposed by M.TurketA. Pentland from Mit Media laboratory in 1991. Training phase: initialization of the system
Let the training set of M face images be \( \Gamma_1; \Gamma_2; \Gamma_3; \ldots; \Gamma_M \), each vector \( \Gamma_i \) represents an image of size \( (N\times M) \).
The average face of the training is defined by:

\[
\Psi = \frac{1}{M} \sum_{n=1}^{M} \Gamma_n
\]

Each face differs from the average face by the vector

\( \Gamma_i - \Psi \)

Where \( i = 1 \) to \( M \).
We rearrange then these vectors in a matrix \( \Xi = [\Gamma_1 \ldots \Gamma_M] \) of dimension \( N\times M \).
Matrix \( \Xi \) has zero-mean (mean value subtracted) vectorsofeach training face image in its columns; this is known as a translation of the origin to the mean face.
The second step is to find a set of the \( M-1 \) orthogonal vectors \( u_i \); which best describes the distribution of the input data in a least squares sense [9]. We find the covariance matrix C.

\[
C = \frac{1}{M} \sum_{n=1}^{M} \Phi_n \cdot \Phi_n
\]

The eigenvalues (\( \lambda_i \)) and the eigenvectors (\( u_i \)) are obtained from covariance matrix. C is real and symmetric. \( \Phi \) is a diagonal matrix with eigenvalues on its main diagonal. The eigenvectors \( u_i \) are sorted according to their corresponding eigenvalues. Larger value means that associated eigenvector captures more of the data variance. In PCA, we keep only the best k Eigen vectors (with the highest k eigenvalues) . The eigen faces are essentially the basis vectors of the eigen face decomposition . We can then reconstruct faces in the new space with the best eigenvectors (respectively eigenvalues) that verify the equation:

\[
[(\Gamma_1 - \Psi)]_{rc} = \sum_{k=1}^{k} [\Gamma_1 - \Psi]_{rc} \cdot u_{ik}
\]

\( [(\Gamma_1 - \Psi)]_{rc} \) means the reconstructed (projected) images. \( [(\Gamma_1 - \Psi)]_{re} \) means the real projection of initial images. For each class or person we compute the matrix of prototypes by calculating the mean image in eigen face space, the corresponding matrix \( \Omega \).

\[
\Omega_k = [\Gamma_{moy} \ldots \Gamma_{moyk}]
\]

B. Recognition Phase
After creating the eigenspace, we can proceed to recognition using eigenfaces. Given a new image of an individual(\( \Gamma_x \)), the pixels are concatenated as the same way as the training mage , where the mean image \( \Psi \) is subtracted\( \Gamma_x - \Psi \) and the result is projected onto the face space:

\[
\Gamma_{x projected} = \sum_{k=1}^{L} (\Gamma_x - \Psi) \cdot u_{ik} = \Omega_i
\]
This is the projection of an unknown face into the eigen face space.

\[ \Omega_i \] is then used to establish the predefined face classes best describes the new face. However, we will find the face class \( k \) that minimize the Euclidian distance

\[ \varepsilon_k = \sqrt{|| \Omega_i - \Omega_k ||^2} \]

Where \( \Omega_k \) is a vector describing the \( k \)th face class. A face is classified as belonging to certain class when the minimum \( \varepsilon_k \) is below some certain threshold.

C. Linear Discriminate Analysis

Steps for development of face recognition system using LDA let training database has \( M \) images, each of size \( N \times N \). Total number of pixels in each image is \( N^2 \). Let there are \( C \) number of persons. First reshape all 2-dimensional images of the training database into 1 dimensional column vectors. Then, put these 1-dimensional column vectors in rows to construct 2-dimensional matrix[8].

\[ \Psi = \frac{1}{M} \sum_{i=1}^{M} \Gamma_i \]

Find the deviation of each image from mean image as:

\[ \phi_n = \Gamma_i - \Psi \]

Calculate MxM matrix \( L \) as:

\[ L = \Lambda^T \Lambda \]

This gives \( M \) eigenvectors (i.e. \( v \)) corresponding to \( M \) eigen values. Using formula \( u = A^*v \), get most significant \( M \) eigenvectors of covariance matrix \( C = A \Lambda \Lambda^T \).

Project centered image vectors onto subspace formed using most significant eigenvectors of \( C \) (as done in eigenface method. Calculate the mean of each class in eigenspace. Within class scatter matrix \( S_w \) are calculated as:

\[ S_w = \sum_{j=1}^{C} \sum_{i=1}^{N_j} (\Gamma_i^j - \mu_j)(\Gamma_i^j - \mu_j)^T \]

Where \( \Gamma_i^j \) is the \( i^{th} \) sample of \( j^{th} \) class, \( N_j \) is the number of samples in class \( j \), \( C \) is the number of classes, \( \mu_j \) is the mean of class \( j \).

\[ S_b = \sum_{j=1}^{C} (\mu_j - \mu)(\mu_j - \mu)^T \]

Where the mean of all the classes is represented by \( \mu \).

Goal is to minimize \( S_w \) while maximizing \( S_b \). This can be achieved by maximizing the ratio \( \det(S_b) / \det(S_w) \). This ratio gets maximized when eigenvectors of \( S_b \) and \( S_w \) form the column vectors of the projection matrix i.e.,

\[ W = [w_1 \ w_2 \ldots \ w_{c-1}] \]

where \( \{w_i\} i=1,2,\ldots,C-1 \) are the eigenvectors of \( S_b \) and \( S_w \) corresponding to the set of decreasing eigenvalues \( \{ \lambda_i \} i=1,2,\ldots,C-1 \).

Euclidean distance is used as a similarity measure. Euclidean distance between vectors, \( x \) and \( y \), is defined as

\[ d_{l2}(x,y) = \| x - y \|^2 \]

D. Advantages of PCA and LDA:

1) Recognition of face using PCA is much simpler and efficient than other matching approaches.
2) Lack of redundancy of data.
3) Smaller database representation is required as only the training images are stored in the form of their projections
4) The fisher face projection approach is able to solve the illumination problem by maximizing the ratio of between class scatter to within class scatter.

E. Drawbacks of PCA and LDA
1) PCA has computing complexity associated with it. It is difficult to evaluate the covariance matrix in an accurate manner.
2) The method is highly sensitive to scale, therefore a low level preprocessing is required for scale normalizing.
3) It fails when all scatter matrices are singular.
4) Small sample size (SSS) problem is encountered in practice since there are often a large number of pixels available, but the total number of training samples is less than the dimensions of the feature space. These drawbacks of PCA and LDA can be overcome by fusion technique

F. Fusion of LDA and PCA Using Wavelet Technique
Wavelet based fusion techniques have been reasonably effective in combining perceptually important image features. Fusion techniques include the simplest method of pixel averaging to more complicated methods such as principal component analysis and wavelet transform fusion. Several approaches to image fusion can be distinguished, depending on whether the images are fused in the spatial domain or they are transformed into another domain, and their transforms fused. The most common form of transform image fusion is wavelet transform fusion. In common with all transform domain fusion techniques the transformed images are combined in the transform domain using a defined fusion rule then transformed back to the spatial domain to give the resulting fused image. Wavelet transform fusion is more formally defined by considering the wavelet transforms of the two registered input images together with the fusion rule[12]. Then, the inverse wavelet transform is computed, and the fused image is reconstructed.

In this section we present our methodology for fusing two appearance-based (or statistical) approaches to face recognition: the PCA representation (“eigenface” approach) and the LDA representation (“fisherface” approach). We already applied the fusion of LDA and PCA in the field of the face verification with good results. Figure 1 shows the overview of the proposed method. It is composed of the following steps:

1) Representation of the face according to the PCA and the LDA approaches;
2) The distance vectors $d^{PCA}$ and $d^{LDA}$ from all the N faces in the database are computed;
3) For the final decision, these two vectors are combined according to a given combination rule.

Many works analysed the differences between these two techniques, but very few work investigated the possibility of fusing them. In our opinion, the apparent strong correlation of LDA and PCA, especially when frontal views are used and PCA is applied before LDA, discouraged the fusion of such algorithms. However, it should be noted that LDA and PCA are not so correlated as one can think, as the LDA transformation applied to the principal components can generate a feature space significantly different from the PCA one[13]. Therefore, the fusion of LDA and PCA for face recognition and verification is worth of theoretical and experimental investigation.

First of all, we normalise the distance vectors $d^{PCA}$ and $d^{LDA}$ in order to reduce the range of these distances in the interval. The second step is to compute a combined distance vector $d$ that must contain both PCA and LDA informations. To this aim, we followed two ways:

First way, we obtained the combined distance vector by computing the mean vector:

$$d = \left\{ \frac{d^{PCA} + d^{LDA}}{2}, \ldots, \frac{d^{PCA} + d^{LDA}}{2} \right\}$$

Second way, we obtained the combined distance vector by appending the $d^{PCA}$ vector and $d^{LDA}$ vector:

$$d = \{d^{PCA}_1, \ldots, d^{PCA}_N, d^{LDA}_1, \ldots, d^{LDA}_N\}$$

where N is the number of images in the face database. In the following subsection we briefly describe the theoretical framework of the two face representations.
Fig 1. Overview of fusion methodology

III. COMPARATIVE STUDY

<table>
<thead>
<tr>
<th>Parameters</th>
<th>PCA</th>
<th>LDA</th>
<th>PCA+LDA</th>
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<tbody>
<tr>
<td>Recognition rate</td>
<td>91%</td>
<td>94%</td>
<td>95%</td>
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<tr>
<td>Equal error rate</td>
<td>2%</td>
<td>8.3%</td>
<td>Minimum of both</td>
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<td>Information ratio</td>
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<td>99.99% within 40 dimensions</td>
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<td>Time elapsed</td>
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<td>0.052781 seconds</td>
<td>-</td>
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<tr>
<td>Accuracy</td>
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<td>30.01%</td>
<td>More compared to PCA and LDA</td>
</tr>
<tr>
<td>Methods</td>
<td>Unsupervised</td>
<td>Supervised</td>
<td>-</td>
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IV. CONCLUSION

PCA algorithm is easy to implement because of its simplicity. There is high correlation between the training data and testing data so the accuracy of PCA algorithm depends on many things. PCA has better accuracy with frontal faces. PCA takes less processing time and efficient in storage. The accuracy of Eigen face is satisfactory. LDA is used for feature extraction. LDA is more complex than PCA in calculating the projection of face space. LDA has better accuracy with face expression. LDA can resolve the problem of the class mean of training samples deviates from the center of this class. The investigation effective fusion strategy based on rank level fusion to combine information presented by multiple face representation. Ranks of individual classifiers are combined using wavelet fusion technique. It is also observed that the system shows improved performance as compared to individual method.

REFERENCES

[10] Yuan Xie, Tao Zhang , A fault diagnosis approach using SVM with Data Dimension Reduction by PCA and LDA method