MHD and Thermal Radiation Effects on Rotating Fluid Past a Moving Vertical Plate with Heat and Mass Transfer in the Presence of Chemical Reaction, OHMIC Heating

K. laxmaiah¹, Dr. D. Raju², M. Chenna Krishna Reddy³

²Department of Mathematics, Vidya Jyothi Institute of Technology, Hyderabad-500 070
³Department of Mathematics, Osmania University, Hyderabad, Telangana – 500 007

Abstract: An analysis of MHD effects on unsteady free convection and mass transfer over a moving isothermal vertical plate in a rotating fluid in the presence of thermal radiation and viscous dissipation with Ohmic heating were presented. An exact solution is obtained for the axial and transverse components of the velocity by defining a complex velocity. The effects of velocity, temperature and concentration for different parameters like radiation parameter, rotation parameter, magnetic field parameter, Schmidt number, thermal Grashof number, mass Grashof number, Prandtl number and time on the plate are discussed.

Keywords: Magnetic field, Radiation, rotation, mass transfer, vertical plate, heat source.

I. INTRODUCTION

The study of hydromagnetic flow is called hydromagnetics or magnetohydrodynamics (MHD), which studies the dynamics of electrically conducting fluids. The set of equations which describe MHD are a combination of the Navier-Stokes equation of fluid dynamics and Maxwell’s equations of electromagnetism. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles. The effect of Coriolis force has wide applications in science and technology. Viskanta and Grosh [15] studied the transfer of energy in boundary layer flow of an incompressible and radiating medium over wedge. The Rosseland approximation for the radiant heat flux vector was used to simplify the energy equation. Arpaci [22] studied the interaction between thermal radiation and laminar convection of heated vertical plate in a stagnant radiating gas. Greenspan [9] discussed the theory of rotating fluids owing to its numerous applications in cosmical and geophysical fluid dynamics, meteorology and engineering. England and Emery [23] have presented a correlation between the analytical solution and the experimental results of the convection-radiation interaction upon a vertical flat plate for absorbing and non-absorbing gases. The flow past a horizontal plate has been studied by Debnath [10, 11], Puri and Kulshrestha [13]. Soundalgekar et al [20] presented an exact solution of flow past an impulsively started isothermal vertical plate under the action of transversely applied magnetic field. Again Soundalgekar et al [21] studied Stokes’ problem for an infinite vertical plate whose temperature varies with time in the presence of transverse magnetic field for an incompressible fluid. Srinathuni Lavanya and Chenna Kesavaiah [17] considered heat transfer to MHD free convection flow of a viscoelastic dusty gas through a porous medium with chemical reaction, Srinathuni Lavanya and Chenna Kesavaiah [18] have radiation effects on MHD natural convection heat transfer flow from spirally enhanced wavy channel through a porous medium.

Many research works have been done based on the action of a uniform transverse magnetic field either fixed to the fluid or to the plate. Heat and mass transfer on MHD flows have applications in Meteorology, solar physics, cosmic fluid dynamics, astrophysics and geophysics. Magneto convection plays an important role in various industrial applications such as magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors, electromagnetic pumps, controlled fusion research, crystal growing, MHD couples and bearings, plasma jets, chemical synthesis and underground nuclear waste storage sites. Singh [11] studied the effects of coriolis as well as magnetic force on the flow field of an electrically conducting fluid past an impulsively started infinite vertical plate. Bestman and Adjepong [2] studied the magnetohydrodynamic free convection flow,
with radiative heat transfer, past an infinite moving plate in rotating incompressible, viscous and optically transparent medium. Das et al. [19] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. An analysis of the unsteady magneto hydrodynamic flow of a viscous and electrically conducting fluid past a vertical flat plate by the presence of radiation was done by Raptis and Massalas [5]. Basant Kumar Jha [6] presented the effects of uniform transverse magnetic field on the transient free convective flow of an electrically conducting fluid in a vertical channel. Raptis and Perdikis [3] considered the effects of radiation and free convection flow past a moving vertical plate. Saravanan and Kandaswamy [16] have analyzed the effect of temperature dependent thermal conductivity on buoyancy in the presence of a uniform magnetic field. It was inferred that it is advantageous to use low Prandtl number liquid metals or alloys as coolants in fast reactors whose thermal conductivity does not depend much on the temperature. Raptis and Perdikis [4] investigated free convection and mass transfer effects on optically thin gray gas past an infinite moving vertical plate. The governing equations were solved analytically. Cai and Zhang [1] studied the heat and mass transfer effects of a finite vertical plate. Cavus and Karafistan [8] studied the effects of differential rotation in the lower convective region of the Sun, Ch Kesavaiah et.al. [7] has effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction, Bhavana et.al [12] investigated the Soret effect on free convective unsteady MHD flow over a vertical plate with heat source. However, MHD effects on a moving infinite vertical plate in a rotating fluid with mass transfer in the presence of thermal radiation is not studied in the literature. It is proposed to study MHD effects on flow past an impulsively started infinite isothermal vertical plate with uniform mass diffusion in a rotating fluid in the presence of thermal radiation and viscous dissipation with Ohmic heating. The dimensionless governing equations are solved by perturbation technique.

A. Mathematical Model

Three dimensional flow of a viscous, incompressible, electrically conducting fluid past an impulsively started infinite vertical isothermal plate with uniform mass diffusion in a rotating fluid [3, 6] is considered. On this plate, the x'-axis is taken along the plane in the vertically upward direction and the y'-axis is normal to the x'-axis in the plane of the plate and z'-axis is normal to it. Both the fluid and the plate are in a state of rigid rotation with uniform angular velocity \( \Omega' \) about the z'-axis. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. A transverse magnetic field \( B_0 \) of uniform strength is applied normal to the plane in the z' direction. The induced magnetic field and viscous dissipation is assumed to be negligible. Initially, the plate and fluid are at rest with the temperature \( T_w' \) and concentration \( C_w' \) everywhere. At time \( t' > 0 \), the plate is given an impulsive motion in the vertical direction against gravitational field with constant velocity \( u_0 \) in a fluid, in the presence of thermal radiation. At the same time the plate temperature is raised to \( T_w' \) and the concentration to \( C_w' \), which are there after maintained constant. Since the plate occupying the plane \( z' = 0 \) is of infinite extent, all the physical quantities depend only on \( z' \) and \( t' \). Then by Boussinesq’s approximation, the unsteady flow is governed by the following equations:

\[
\frac{\partial u'}{\partial t'} - 2 \Omega' v' = g\beta (T' - T_w') + g\beta' (C' - C_w') + \nu \frac{\partial^2 u'}{\partial z'^2} - \frac{B_0^2}{\rho} u'
\]  

(1)

Figure (1): Physical model of the problem
\[ \frac{\partial v}{\partial t} + 2\Omega u' = v \frac{\partial^2 v'}{\partial z'^2} - \frac{\sigma B_0^2}{\rho} v' \]  
(2)

\[ \rho C_p \frac{\partial T'}{\partial t} = k \frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q_l}{\partial z'} - Q_0(T' - T'_w) + \sigma B_0^2 u'^2 \]
(3)

\[ \frac{\partial C'}{\partial t} = D \frac{\partial^2 C'}{\partial z'^2} - Kr'(C' - C'_w) \]
(4)

The term \( \frac{\partial q_l}{\partial z'} \) represents the change in the radiative flux with distance normal to the plate with the following initial and boundary conditions:

\[ \begin{align*}
  t' \leq 0: & \quad u' = 0, \quad T' = T'_w, \quad C' = C'_w \quad \text{forall z'} \\
  t' > 0: & \quad u' = u_0, \quad T' = T'_w, \quad C' = C'_w \quad \text{at z'} = 0 \\
                  & \quad u = 0, \quad T \rightarrow T_w, \quad C' \rightarrow C'_w \quad \text{as z'} \rightarrow \infty.
\end{align*} \]
(5)

By Rosseland approximation [1, 13], radiative heat flux of an optically thin gray gas is expressed by

\[ \frac{\partial q_l}{\partial z'} = -4a^* \sigma (T'_w - T'^4) \]
(6)

It is assumed that the temperature differences within the flow are sufficiently small such that \( T'^4 \) may be expressed as a linear function of the temperature. This is accomplished by expanding \( T'^4 \) in a Taylor series about \( T'_w \) and neglecting higher-order terms, thus

\[ T'^4 \approx 4T'_w^3 T' - 3T'_w^4 \]
(7)

By using equations (6) and (7), equation (3) reduces to

\[ \rho C_p \frac{\partial T'}{\partial t} = k \frac{\partial^2 T'}{\partial z'^2} + 16a^* \sigma T'_w^3 (T'_w - T') - Q_0(T' - T'_w) + \sigma B_0^2 u'^2 \]
(8)

On introducing the following dimensionless quantities:

\[ (u,v) = \left( \frac{u', v'}{u_0} \right), \quad t = \frac{t' u_0^2}{v}, \quad z = \frac{z' u_0}{v}, \quad \theta = \frac{T' - T'_w}{T'_w - T'_w}, \quad C = \frac{C' - C'_w}{C'_w - C'_w} \]

\[ \begin{align*}
  Gr &= \frac{g \beta \nu (T'_w - T'_w)}{u_0^3}, \quad Sc = \frac{\nu D}{\kappa}, \quad Kr = \frac{v Kr'}{u_0^3}, \quad Gc = \frac{v g \beta (C'_w - C'_w)}{u_0^3} \\
  M &= \frac{\sigma B_0^2 u_0}{\rho}, \quad Pr = \frac{\mu C_p}{\kappa}, \quad \Omega = \frac{\nu v}{u_0^3}, \quad R = \frac{16a^* v^2 T'_w^3}{ku_0^3}, \quad Q = \frac{Q_0}{u_0^3 k}
\end{align*} \]
(9)

and the complex velocity \( q = u + iv, \quad i = \sqrt{-1} \) in equations (1) to (5), the equations relevant to the problem reduces to

\[ \frac{\partial q}{\partial t} + 2i\Omega i = Gr \theta + Gc C + \frac{\partial^2 q}{\partial z'^2} - M q, \]
(10)

\[ \frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial z'^2} - \frac{1}{Pr} \left( R + Q \right) \theta \]
(11)

\[ \frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial z'^2} - Kr C \]
(12)

where magnetic field parameter \( M \), rotation parameter \( \Omega \), radiation parameter \( R \), Schmidt number \( Sc \), heat source parameter \( Q \), chemical reaction parameter \( Kr \), thermal Grashof number \( Gr \) and mass Grashof number \( Gm \).
The initial and boundary conditions in non-dimensional form are
\[ q = 0, \quad \theta = 0, \quad C = 0, \quad \text{forall} \quad z \leq 0 \quad \& \quad t \leq 0 \]
\[ t > 0: \quad q = 1, \quad \theta = 1, \quad C = 1, \quad \text{at} \quad z = 0 \\
q = 0, \quad \theta \to 0, \quad C \to 0, \quad \text{as} \quad z \to \infty \] (13)

All the physical variables are defined in the nomenclature.

B. Method Of Solution

Equation (10) – (12) are coupled, non-linear partial differential equations and these cannot be solved in closed – form using the initial and boundary conditions (13). However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the fluid in the neighbourhood of the plate as

\[ q(z,t) = q_0(z) + \varepsilon e^{\alpha t} q_1(z) + O(\varepsilon^2) \] (14)
\[ \theta(z,t) = \theta_0(z) + \varepsilon e^{\alpha t} \theta_1(z) + O(\varepsilon^2) \] (15)
\[ C(z,t) = C_0(z) + \varepsilon e^{\alpha t} C_1(z) + O(\varepsilon^2) \] (16)

Substituting (13) in Equation (9) – (11) and equating the harmonic and non – harmonic terms, and neglecting the higher order terms of \( O(\varepsilon^2) \), we obtain

\[ q_0'' - Mq_0 = -Gr \theta_0 - Gc C_0 - 2\Omega \] (17)
\[ q_1'' - (M + n)q_1 = -Gr \theta_1 - Gc C_1 \] (18)
\[ \theta_0'' - (R + Q)\theta_0 = 0 \] (19)
\[ \theta_1'' - (R + Q + n Pr)\theta_1 = 0 \] (20)
\[ C_0'' - Kr Sc C_0 = 0 \] (21)
\[ C_1'' - (Kr + n) Sc C_1 = 0 \] (22)

The corresponding boundary conditions can be written as
\[ q = 0, \quad \theta = 0, \quad C = 0, \quad \text{forall} \quad z \leq 0 \quad \& \quad t \leq 0 \]
\[ t > 0: \quad q_0 = 1, \quad \theta_0 = 1, \quad C_0 = 1, \quad q_1 = 0, \quad \theta_1 = 0, \quad C_1 = 0 \quad \text{at} \quad z = 0 \\
q_0 \to 0, \theta_0 \to 0, C_0 \to 0, q_1 \to 0, \theta_1 \to 0, C_1 \to 0 \quad \text{as} \quad z \to \infty \] (23)

Solving Equations (17) - (22) under the boundary conditions (23) we obtain the velocity, temperature and concentration distributions in the boundary layer as

\[ q_0 = A_1 e^{m_0 z} + A_2 e^{m_2 z} + A_3 + A_4 e^{m_4 z} \]
\[ \theta_0 = e^{m_0 z} \]
\[ C_0 = e^{m_2 z} \]
\[ q_1 = 0, \theta_1 = 0, C_1 = 0 \]

In view of the above equations (14) – (16) becomes

\[ q(z,t) = A_1 e^{m_0 z} + A_2 e^{m_2 z} + A_3 + A_4 e^{m_4 z} \]
\[ \theta(z,t) = e^{m_0 z} \]
\[ C(z,t) = e^{m_2 z} \]
\[ m_2 = -\sqrt{Kr Sc} \quad m_4 = -\sqrt{(Kr+n)Sc}, \]
\[ m_6 = -\sqrt{R+Q}, \quad m_8 = -\sqrt{M}, \quad A_1 = -\frac{Gr}{m_6^2 + M}, \]
\[ A_2 = \frac{Gc}{m_6^2 + M}, \quad A_3 = \frac{2\Omega}{M}, \quad A_4 = (1 - A_1 - A_2 - A_3) \]

Using equation (16) we get the following expression for skin-friction components \( \tau_x \) and \( \tau_y \):
\[ \tau_x + i\tau_y = \left( \frac{\partial q}{\partial z} \right)_{z=0} = A_1 m_6 + A_2 m_2 + A_4 m_8 \]

In equation (16), (17) the argument of the complementary error function and error function is complex. Hence in order to obtain the \( u \) and \( v \) components of the velocity and skin-friction.

### II. RESULTS AND DISCUSSION

Using the above formula, expressions for \( u \) are obtained but they are omitted here to save the space. To get a physical view of the problem, these expressions are used to obtain the numerical values of velocity \( (u) \) for different values of the various parameter like Magnetic field parameter \( (M) \), rotation, Radiation parameter \( (R) \), Schmidt number \( (Sc) \), Heat Source parameter \( (Q) \), Chemical reaction \( (Kr) \), thermal Grashof number \( (Gr) \) and mass Grashof number \( (Gm) \).

#### A. Velocity profiles

Effects of radiation and rotation parameter on primary velocity; the primary velocity profiles of air for different values of the radiation parameter \( (R = 0.5, 1.0, 1.5, 2.0) \) and rotation parameter \( (\Omega = 0.1, 0.2, 0.3, 0.4) \) are shown in figure (2). It is observed that from figure 2(a) can be observed in the primary velocity decreases with increasing the radiation parameter \( (R) \) and with increasing the rotation parameter \( \Omega \) the primary velocity decreases in cooling of the plate observed in figure 2(b). This shows that primary velocity decreases in the presence of high thermal rotation. Effects of magnetic field and rotation parameter on primary velocity the primary velocity profiles of air for different values of the magnetic parameter \( (M = 0.5, 1.0, 1.5, 2.0) \) are shown in figure (3). It is observed that the primary velocity decreases with increasing magnetic parameter. This shows that primary velocity decreases in the presence of high magnetic field and rotation. In fact rotation has more influence than magnetic field on primary velocity. Effects of thermal and mass Grashof number on primary velocity the primary velocity profiles for different thermal Grashof number \( (Gr = 1.0, 2.0, 3.0, 4.0) \), mass Grashof number \( (Gc = 1.0, 2.0, 3.0, 4.0) \) are shown in figure 4(a) and 4(b). It is clear that the primary velocity increases with increasing thermal Grashof number or mass Grashof number. Figure (5) shows that primary velocity profiles for different values of chemical reaction parameter \( (Kr) \) it is clear that an increases chemical reaction parameter the velocity profiles decreases. The velocity profiles directed for various values of Schmidt number shown in figure (6), it is observed that increases in Schmidt number the result are decreases.

#### B. Temperature Distribution

The temperature profiles for different values of heat source parameter \( (Q = 0.5, 1.0, 1.5, 2.0) \) are shown in figure 7(a). The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature decreases with increasing heat source parameter. The temperature profiles for air \( (Pr = 0.71) \) are calculated for different values of thermal radiation parameter \( (R = 0.2, 2.0) \) are shown in figure 7(b). The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature decreases with increasing radiation parameter.
C. Concentration Profiles

Variations in the concentration profiles for different values of the chemical reaction parameter and Schmidt number are shown from figure 8(a) and 8(b); from these figures it is clear that an increasing values of the chemical reaction parameter and Schmidt number the concentration profiles decreases.

D. Skin friction

The skin-friction components for different parameter are calculated using Equation (16). The skin-friction components at the wall for different values of $Gc = 5.0$, $R = 2.0$, $M = 1.0$, $Kr = 1.0$, $n = 0.1$, $Q = 1.0$ and $\Omega$ are shown in table (1).

<table>
<thead>
<tr>
<th>$\Omega$</th>
<th>$Gr$</th>
<th>$\tau_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1.0</td>
<td>4.69</td>
</tr>
<tr>
<td>0.4</td>
<td>2.0</td>
<td>4.29</td>
</tr>
<tr>
<td>0.6</td>
<td>3.0</td>
<td>3.89</td>
</tr>
<tr>
<td>0.8</td>
<td>4.0</td>
<td>3.49</td>
</tr>
</tbody>
</table>

The effect of rotation on skin-friction decreases the component $\tau_x$ as $\Omega$ increases. As time advances the component $\tau_x$ increases.

Greater cooling of the plate, due to free-convection currents, lower rises $\tau_x$.

Using equation (15) we get the following expression for Nusselt number

$$N_u = \left( \frac{\partial \theta}{\partial z} \right)_{z=0} = m_v$$

<table>
<thead>
<tr>
<th>$R$</th>
<th>$t$</th>
<th>$N_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>1.1806</td>
</tr>
<tr>
<td>2.0</td>
<td>0.2</td>
<td>2.0693</td>
</tr>
<tr>
<td>5.0</td>
<td>0.4</td>
<td>3.0905</td>
</tr>
</tbody>
</table>

Nusselt number for different thermal radiation and time are calculated using equation (15), table (2) shows that as radiation increases Nusselt number increases, but the trend is reversed with time.

E. Sherwood Number

Using equation (14) we get the following expression for Sherwood number

$$S_h = \left( \frac{\partial \mu}{\partial z} \right)_{z=0} = m_z$$

<table>
<thead>
<tr>
<th>$Sc$</th>
<th>$Kr$</th>
<th>$Sh$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>0.00349300</td>
</tr>
<tr>
<td>1.0</td>
<td>2.0</td>
<td>0.00033550</td>
</tr>
<tr>
<td>1.5</td>
<td>3.0</td>
<td>$5.556 e^{-005}$</td>
</tr>
<tr>
<td>2.0</td>
<td>4.0</td>
<td>$1.22 e^{-005}$</td>
</tr>
</tbody>
</table>
Sherwood numbers for different Schmidt number and time are calculated using equation (16), table (2) shows that as Schmidt number increases Sherwood number increases, but the trend is reversed with time.

III. CONCLUSIONS

Theoretical analysis is performed to study magneto hydrodynamic effects on flow past an impulsively started infinite isothermal vertical plate with uniform mass diffusion, in the presence of thermal radiation in a rotating fluid. The dimensionless governing equations are solved by Laplace-transform technique. The conclusions of the study are as follows:

A. Concentration falls with the increase in Schmidt number.
B. Temperature is enhanced with the decreasing radiation parameter and increasing time.
C. The thermal radiation or rotation parameter on primary flow has a retarding effect for cooling of the plate.
D. The influence of rotation on primary flow is significant than magnetic field, but both has a retarding effect for cooling of the plate.
E. The secondary velocity is enhanced with the raise in thermal radiation or magnetic field parameter and opposite phenomenon occurs with the rotation parameter.
F. The effect of thermal Grashof number or mass Grashof number enhanced the primary flow but decreased the secondary flow.
G. The skin-friction components falls with increasing radiation parameter, thermal Grashof number and mass Grashof number on $\tau_x$ rises on $\tau_y$. But reverse phenomenon occurs with rotation parameter or magnetic parameter.
H. Sherwood number increases with increasing Schmidt number and the Nusselt number increases with increasing radiation parameter but both shows reverse trend with respect to time.

REFERENCES


Figures: