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Theoretical Analysis for Fatigue Life Improvement of Corrected Spur Gear

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Abstract— ***Spur gears have wide range of applications in all over the world. It is obvious that improvement in fatigue life of spur gears can be beneficial. Corrected gears are one of the advanced concept used to solve variety of problems occur in gears. This paper represents theoretical design of spur gears by means of corrected gears to improve fatigue life. Problem states that a gear component mating with a pinion component for speed reduction purpose. Focus is mainly on improvement of fatigue life of spur gears by keeping centre distance and gear ratio exact, by involvement of corrected gears. Lundberg-Palmgren theory is used to define fatigue life of spur gears as well as corrected gears. In the end of paper, comparison is made for fatigue life between spur gears and corrected spur gears.***

***Keywords*— Spur gear; Fatigue life; Corrected gear; Centre distance; Gear ratio; Lundber-Palmgren (L-P) theory**

I. Introduction

In engineering, the term "gear" is defined as a machine element used to transmit a motion and power between rotating shafts by means of progressive engagement of projections called teeth. Spur gears are the most recognized style of gear. Spur gears are used exclusively to transmit rotary motion between parallel shafts, while maintaining uniform speed and torque. We can define fatigue life of gear as, "Number of cycles gear performed without any occurrence of failure with surrounding conditions and induced stresses."[1] The life model proposed by Lundberg and Palmgren is the commonly accepted theory for predicting the pitting fatigue life of rolling-element bearings, because the fatigue failure mechanism is similar for both gears and rolling element bearings, as in [2], [3]. The Lundberg-Palmgren model for bearings has been adapted to predict gear life. In actual practice, the gear causing the interference will have a tendency to jam on the flank of the pinion. Unless, of course, the pinion tooth root has already been undercut making room to provide free movement of the gear teeth. Besides due to interference and in the absence of an undercut, the mating gear will try to scoop out metal from the interfering portion. If the situation warrants, a pinion might have to be designed with the number of teeth less than the minimum number stipulated to avoid undercutting. In such cases, the practice which is now universally adopted is what is known as the "profile correction" of gear tooth. It can be also termed as"addendum modification", "profile shift". Such gears are called as "corrected gears" [4].

II. Fatigue Life and Lundberg Palmgren Theory

As in [5], ASTM (American Society for Testing and Materials) defines fatigue life, as the number of stress cycles of a specified character that a specimen sustains before failure of a specified nature occurs. The fatigue life model proposed in 1947 by Lundberg is the commonly accepted theory for determining the fatigue life of rolling-element bearings. The basic theory of Lundberg-Palmgren was applied to obtain the surface fatigue life of a single spur gear tooth. Following equation gives the life tor a 90% probability of survival η of a single tooth on a driver or driven gear of a mesh as,

$$\eta = J^{4.3} f^{3.9} \sum \rho^{-5}\, l^{-0.4} Q^{-4.3} \qquad (1)$$

The life of the complete driver gear (all teeth) L1 in terms of driver gear rotations is

$$L_1 = N_1^{\frac{-1}{e}} \eta \qquad (2)$$

Experimental research on MSI 9310 steel spur gears has shown gear fatigue to follow the Weibull failure distribution with e =2.5.

The Life of the complete driven gear L_2in terms of driver gear rotations is,

$$L_2 = \left(\frac{N_1}{N_2}\right) N_2^{\frac{-1}{e}} \eta \qquad (3)$$

The mesh life (both driver and driven gears) *Lm* in terms of driver gear rotations is given by,

$$L_m = \left(L_1^{-e} + L_2^{-e}\right)^{\frac{-1}{e}} \qquad (4)$$

To adapt the current gear life model for predictions based on gear tooth dynamic loads, the tooth was divided into intervals. The use of intervals allowed the current gear life model to account for load and curvature sums varying with contact position. The complete gear tooth life was determined from the interval lives and methods of probability and statistics. The details are as follows. When a pair of external spur gears is in mesh, the line tangent to the base circles of both the driver and driven gears is called the line of action. The gears begin contact when the outside radius of the driven gear intersects the line of action. As the gears rotate, the contact point occurs on the line of action. The contact ends when the outside radius of the driver gear intersects the line of action. The point at which the pitch circles of the driver and driven gears intersect is called the pitch point.

The distance along the line of action from the pitch point to the start of contact is,

$$Z_1 = \sqrt{R_{o2}^2 - R_{b2}^2} - (R_{p2} \times sin\emptyset) \qquad (4)$$

The distance along the line of action from the pitch point to the start of contact is,

$$Z_2 = \sqrt{R_{o1}^2 - R_{b1}^2} - (R_{p1} \times sin\emptyset) \qquad (5)$$

The contact length Z is defined as summation of Z_1 and Z_2,

$$Z = Z_1 + Z_2 \qquad (6)$$

Dividing the contact length into equal size intervals of length Δx gives,

$$\Delta x = \frac{Z}{J} \qquad (7)$$

$$x_i = -Z_1 + (i-1)\Delta x$$

The value of *x* is negative when contact is before pitch point, zero when at the pitch point and positive when after the pitch point. The life of each interval for a 90% probability of survival is given from equation (1),

$$\eta_j = B^{4.3} f^{3.9} (\overline{\Sigma\rho_j})^{-5} l_j^{-0.4} Q_j^{-4.3} \qquad (8)$$

Where, *B* and *f* do not change from interval to interval. Both curvature sum and involute length, however, change with contact position. At the i_{th} contact position the radii of curvature of the driver and driven gears given by,

$$R_{1i} = R_{p1} sin\emptyset + x_i$$

$$R_{2i} = R_{p2} sin\emptyset - x_i$$

At the curvature sum at i_{th} contact position is,

$$\Sigma\rho_i = \frac{1}{R_{1i}} + \frac{1}{R_{2i}} \qquad (9)$$

At the j_{th} interval the average curvature sum used in the life model is,

$$\overline{\Sigma\rho_j} = \frac{\Sigma p_i + \Sigma p_{i-1}}{2} \qquad (10)$$

The curvature sum varied slightly with contact position. (The contact position was made dimensionless by dividing by the base pitch P_b.) The plot shows the curvature sum to be symmetric about the pitch point (x =0). This was true only because the driver and driven gears of the example were the same size. The involute surface lengths of the driver and driven gears for the j_{th}interval are,

$$l_{1j} = \left(\frac{\Delta x}{R_{b1}}\right)x_i + (\Delta x \times tan\emptyset)$$

$$l_{2j} = -\left(\frac{\Delta x}{R_{b2}}\right)x_i + (\Delta x \times tan\emptyset)$$

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The involute length is a linear function of contact position. By using intervals the life model considers load that can vary with contact position. For the j_{th} interval the average load used in the life model is,

$$\overline{Q_J} = \frac{Q_i + Q_{i-1}}{2} \qquad (11)$$

The static load variation with contact position depends on the number of teeth in contact. As a pair of teeth begins contact, the preceding pair of mating teeth is also in contact. Toward the end of contact, double-tooth-pair contact again occurs as the following pair of mating teeth begins contact. As before, it is assumed that half the load is transferred per contact. The life of a complete gear tooth, η_t is determined from the interval lies and methods of probability and statistics where

$$\eta_t = (\sum_{j=1}^{J} \eta_j^{-e})^{\frac{-1}{e}}$$

The complete tooth life was always shorter than the lives of the shortest-lived intervals. Also, intervals with larger applied loads had much more influence on gear tooth life than intervals with smaller loads.

III. Bending Stress Theory

The theoretical bending stress according to Wilfred Lewis formula is given by [6], [7],

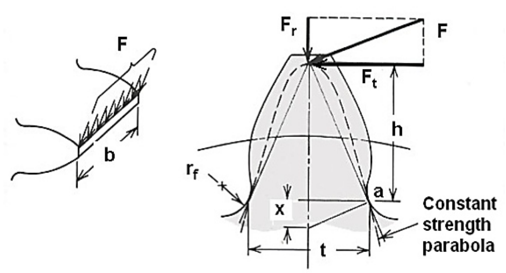

Fig. 1 Gear teeth as a cantilever beam

$$\sigma_{th} = \frac{F_t \times k_t}{f \times m \times Y} = \frac{F_t}{f \times m \times J} \qquad (12)$$

Where,

$$J = \frac{Y}{k_t}$$

From equation (12),

$$Y = \frac{1}{m \times [\frac{\cos\alpha_a}{\cos\alpha}] \times [\frac{6h}{t^2} \pm \frac{\tan\alpha_a}{t}]} \qquad (13)$$

The gear parameter at the critical section like thickness(t), effective moment arm (h), radius of curvature (r_f) are calculated analytically from the geometry of gear and according to AGMA stress correction factor (K_t) is obtained from empirical relation arrived from experiments conducted by Dolan & Brohamer for is given as,

$$k_t = 0.18 + [\frac{t}{R_f}^{0.15} \times \frac{t}{h}^{0.45}]$$

IV. Design Equations for Spur Gears and Corrected Spur Gears

TABLE I
Design Equations For Spur Gears

No.	Parameter	Equation
1	Centre distance (a)	$a = \frac{m}{2}(Z_1 + Z_2)$
2	Pitch circle diameter (d_p)	$d_p = mZ_1 = mZ_2$
3	Diametral pitch (P_d)	$P_d = \frac{Z}{d_p}$
4	Circular pitch, (P_c)	$P_c = \pi m$
5	Tooth thickness,(S)	$S = \frac{\pi m}{2}$
6	Base diameter (d_b)	$d_b = d_p \cos\emptyset$
7	Root diameter (d_r)	$d_r = d_p - (2 \times 1.25m)$
8	Outer diameter (d_0)	$d_o = \frac{Z+2}{P_d}$
9	Module	m
10	Total depth	2.25m
11	Working depth	2m
12	Addendum	1m

13	Dedendum	1.25m
14	Face width	5m
15	Clearance	0.25m
16	Fillet radius	0.3m

TABLE II
DESIGN EQUATIONS FOR CORRECTED SPUR GEARS

No.	Parameter	Equation
1	Pitch circle diameter (d_p)	$d_{p'} = d_p \frac{cos\emptyset}{cos\emptyset_w}$
2	Tooth thickness,(S)	$S' = \frac{p_c}{2} + 2xmtan\emptyset$
3	Base diameter (d_b)	$d_{b'} = d_{p'}cos\emptyset$
4	Root diameter (d_r)	$d_{r'} = d_{p'} - 2(1.25 - x)m$
5	Outer diameter (d_0)	$d_{o'} = 2(a + m - xm) - d_p$

TABLE III
DESIGN EQUATIONS OF FORCES, STRESSES AND LIFE

No.	Parameter	Equation
1	Tangential Force (F_t)	$F_t = 1000 \times \frac{P(KW)}{v(m/s)}$
2	Normal Force (F_N)	$F_N = \frac{F_t}{cos\emptyset}$
3	Radial Force (F_r)	$F_r = F_t tan\emptyset$
4	Tooth Bending Stress (σ_b)	$\sigma_b = \frac{F_t}{bYm}$
5	Dynamic Load (F_d),	$F_d = K_0 \times K_v \times b \times v$
6	Total Dynamic Load(F_{td})	$F_{td} = (F_d + F_t)$
7	Hertz Contact Stress(P_p)	$P_p = y_m y_p \sqrt{\frac{F_t}{bd_{p1}} \frac{u+1}{u}}$
8	Life of Pinion(L_1)	$L_1 = N_1^{\frac{-1}{e}} \eta$
9	Life of Gear(L_2)	$L_2 = (\frac{N_1}{N_2}) N_2^{\frac{-1}{e}} \eta$
10	Life of Mesh(L)	$L_m = (L_1^{-e} + L_2^{-e})^{\frac{-1}{e}}$

V. GEARS USED AS A RESEARCH OBJECT

A Spur gear pair used for speed reduction purpose has following data.

TABLE III
PROBLEM SPECIFICATIONS

No.	Parameter	Data
1	Module	10
2	Pressure Angle	20°
3	Gear Ratio	1.5
4	Centre Distance	210 mm
5	Power to be transmitted	100 KW
6	Speed of pinion	750 rpm
7	Speed of gear	500 rpm

TABLE V
MATERIAL SPECIFICATIONS

No.	Parameter	Data
1	Material	Flame or Induction hardening steel (C 45) IS No : 1570
2	Ultimate tensile strength, (σ_{ut})	640 N/mm^2
3	Yield stress, (σ_y)	390 N/mm^2
4	Endurance limit, (σ_e)	360 N/mm^2
5	Permissible bending stress, (σ_{bp})	180 N/mm^2
6	Factor of Safety	2
7	Permissible surface fatigue strength (P_{sc})	1640 N/mm^2

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8	Brinell hardness (HB)	$5840\ N/mm^2$
9	Heat treatment	Hardened
10	Young's modulas, (E)	$2.06 \times 10^5 N/mm^2$

It is necessary that values of gear ratio and centre distance are to be kept exact. Material for gear and pinion component is same.

A. Sample Calculation for Spur Gears

TABLE VI
DESIGN CALCULATIONS FOR SPUR GEARS

No.	Parameter	Data	
		Pinion	**Gear**
1	Centre distance (a)	$210\ mm$	$210\ mm$
2	Pitch circle diameter (d_p)	$160\ mm$	$240\ mm$
3	Diametral pitch (P_d)	$0.1\ mm$	$0.1\ mm$
4	Circular pitch, (P_c)	$31.4\ mm$	$31.4\ mm$
5	Tooth thickness,(S)	$15.7\ mm$	$15.7\ mm$
6	Base diameter (d_b)	$150.35\ mm$	$225.53\ mm$
7	Root diameter (d_r)	$135\ mm$	$215\ mm$
8	Outer diameter (d_0)	$180\ mm$	$260\ mm$
9	Total depth	$22.5\ mm$	$22.5\ mm$
10	Working depth	$20\ mm$	$20\ mm$
11	Addendum	$10\ mm$	$10\ mm$
12	Dedendum	$12.5\ mm$	$12.5\ mm$
13	Face width	$50\ mm$	$50\ mm$
14	Clearance	$2.5\ mm$	$2.5\ mm$
15	Fillet radius	$3\ mm$	$3\ mm$

B. Sample Calculation for Corrected Spur Gears

TABLE VII
DESIGN CALCULATIONS FOR CORRECTED SPUR GEARS

No.	Parameter	Data	
		Pinion	**Gear**
1	Pitch circle diameter (d_p)	$167.711\ mm$	$251.167\ mm$
2	Tooth thickness,(S)	$19.814\ mm$	$20.052\ mm$
3	Base diameter (d_b)	$150.35\ mm$	$225.53\ mm$
4	Root diameter (d_r)	$146.304\ mm$	$226.956\ mm$
5	Outer diameter (d_0)	$188.044\ mm$	$268.696\ mm$

VI. COMPARISON OF FORCES-STRESSES AND THEORETICAL LIFE OF SPUR GEARS AND CORRECTED SPUR GEARS

As it is obvious that material for gear component and pinion component is same. According to Lewis formula, design should be prepared according to weaker component. As pinion is weaker, forces-stresses calculation is to be made for pinion component. Theoretical life is calculated using Lundberg and Palmgren theory for gear, pinion and meshing.

TABLE VIII
FORCES-STRESSESCALCULATION FOR SPUR GEAR AND CORRECTED SPUR GEAR

No.	Parameter	Data	
		Spur Gears	**Corrected Spur Gears**
1	Tangential Force (F_t)	$15923.57\ N$	$15314\ N$
2	Normal Force (F_N)	$16945.51\ N$	$17082.24\ N$
3	Radial Force (F_r)	$5795.71\ N$	$7568.65\ N$
4	Tooth Bending Stress (σ_b)	$115.35\ N/mm^2$	$92.31\ N/mm^2$
5	Dynamic Load (F_d),	$4000\ N$	$4057.23\ N$
6	Total Dynamic Load(F_{td})	$19{,}923.57\ N$	$19{,}371.23\ N$
7	Hertz Contact Stress(P_p)	$862.68\ N/mm^2$	$743.30\ N/mm^2$

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TABLE IX
THEORETICAL LIFE CALCULATION FOR SPUR GEAR AND CORRECTED SPUR GEAR

No.	Parameter	Data	
		Spur Gears	**Corrected Spur Gears**
1	Life of Pinion(L_1)	37.30 $\times 10^7$ $No.of\ cycles$ (8290 $hours$)	43.46 $\times 10^7$ $No.of\ cycles$ (9660 $hours$)
2	Life of Gear(L_2)	65.81 $\times 10^7$ $No.of\ cycles$ (21935 $hours$)	76.66 $\times 10^7$ $No.of\ cycles$ (25650 $hours$)
3	Life of Mesh(L)	34.21 $\times 10^7$ $No.of\ cycles$ (9120 $hours$)	39.85 $\times 10^7$ $No.of\ cycles$ (10625 $hours$)

VII. RESULTS

Results achieved after calculating forces, stresses and life for spur gears and corrected spur gears are represented graphically as below:

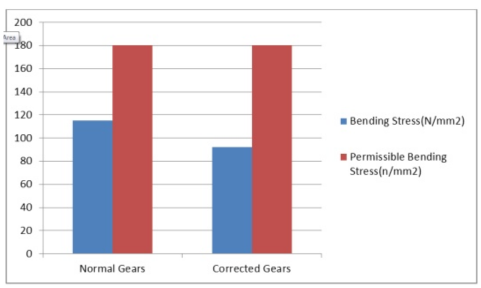


Fig. 2 Comparison of bending stresses for spur gears and corrected spur gears

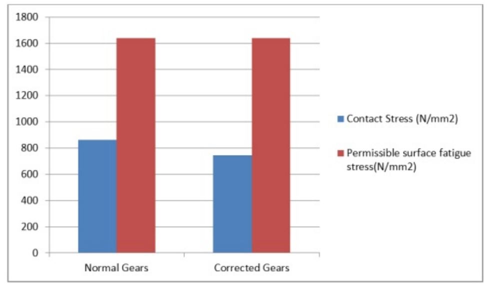


Fig. 3 Comparison of contact stresses for spur gears and corrected spur gears

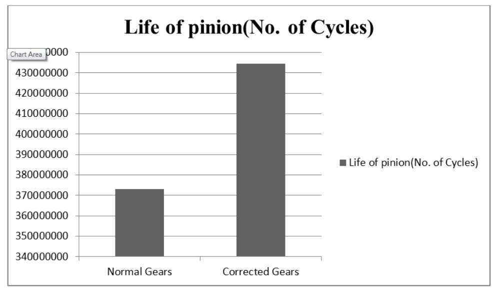


Fig. 4 Life Comparison for pinion between spur gears and corrected spur gears

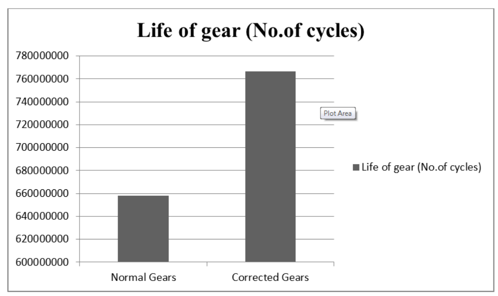


Fig. 5 Life Comparison for gear between spur gears and corrected spur gears

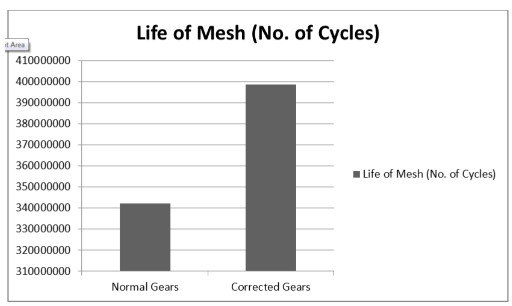


Fig. 6 Life Comparison for mesh between spur gears and corrected spur gears

VIII. CONCLUSIONS

The presented work gives the specific way to improve fatigue life of spur gears using a methodology corrected gears with the help of Lundberg and Palmgren theory. The proposed methodology of corrected gears improves fatigue life of conventional spur gears. It also reduces the effect of bending stresses and contact stresses induced during operating conditions. A methodology of corrected gears also improves the strength of spur gears at root and flank. It also provides smooth running operation.

REFERENCES

[1] Fatigue and fatigue life, (2013) Wikipedia website. [Online]. Available: http://www.wikipedia.org/fatigue/

[2] John J. Coy and Erwin V. Zaretsky, "Life analysis of helical gear sets using Lundberg and Palmgrem theory", in *NASA Technical Note,* August 1975, p. 3

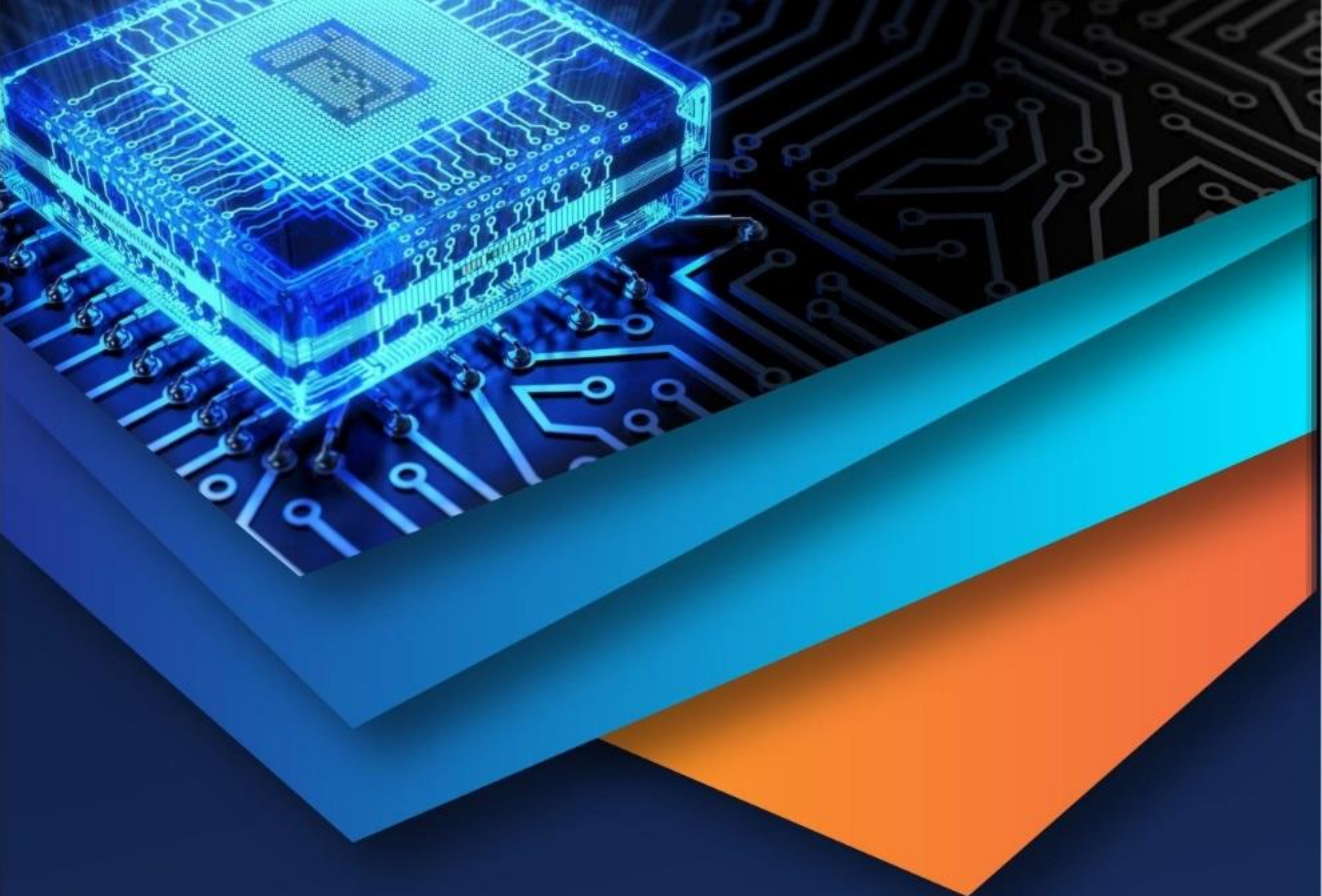
[3] Douglas A. Rohn and SruartH and John J. Coy, "Simplified fatigue life analysis for traction drive contacts, *NASA Technical Memorandum*, August 1980

[4] Gitin M. Maitra, *Handbook of gear Design*, 2nd edition, Tata MacGrwhill Publishing Company Limited, New Delhi, 2009, Ch.-2

[5] Gear Technology, The Journal of gear manufacturing, Vol. 6, No. 2, pp. 10-13, March-April 1989

[6] V. B. Bhandari, *Design of Machine Elements*, Third Edition, Tata MacGrawhill Education Private Limited, New Delhi, 2007, Chapter-17

[7] S.P.Ganesan and G. MuthuveerappanBending, "Geometry Factor for Profile Corrected Involute Gear Tooth With Trochoidal Fillet", pp. 3-4



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