Comparative Study of Rigorous Analysis and Superposition Principle for Floating Granular Piles Group

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Abstract: In the modern time, where growth is the keyword in everyone’s life and the natural resources are exhausted continuously by the man kind including the land resource, the ground improvement is becoming the need of the hour, especially civil engineers, for areas consisting of soft soils. The most economical and feasible solution seems to be the use of granular piles (GP) in such cases, which are normally composed of boulders, sand etc. In the present paper a comparative analysis of a group of three and four partially stiffened GPs is presented revealing the comparison between rigorous and superposition methods for settlement interaction factor for floating group of piles.

Keywords: Relative stiffness of granular pile, Stiffening factor, Percentage length of stiffening, Settlement interaction factor.

Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>GP</td>
<td>Granular pile</td>
</tr>
<tr>
<td>L</td>
<td>Length of granular pile</td>
</tr>
<tr>
<td>n</td>
<td>Total number of elements of GP</td>
</tr>
<tr>
<td>d</td>
<td>Diameter of granular pile</td>
</tr>
<tr>
<td>L/d</td>
<td>Relative length of GP</td>
</tr>
<tr>
<td>P</td>
<td>Load on each granular pile of two pile group</td>
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<tr>
<td>P_b</td>
<td>Load on the base of the GP</td>
</tr>
<tr>
<td>E_s</td>
<td>Deformation modulus of soil</td>
</tr>
<tr>
<td>E_b</td>
<td>Deformation modulus of bearing stratum</td>
</tr>
<tr>
<td>\nu_b</td>
<td>Poisson’s Ratio of bearing stratum</td>
</tr>
<tr>
<td>\nu_s</td>
<td>Poisson’s Ratio of soil</td>
</tr>
<tr>
<td>K_{gp} = (E_{gp}/E_s)</td>
<td>Relative stiffness of granular pile</td>
</tr>
<tr>
<td>p_b</td>
<td>Pile base pressure</td>
</tr>
<tr>
<td>s</td>
<td>Spacing between center to center of the piles</td>
</tr>
<tr>
<td>s/d</td>
<td>Normalized spacing center to center between piles</td>
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<tr>
<td>\tau</td>
<td>Shear stress</td>
</tr>
<tr>
<td>L_s</td>
<td>Length of the pile stiffened from the top of the pile</td>
</tr>
<tr>
<td>\eta = L_s/L</td>
<td>Percentage length of stiffening</td>
</tr>
<tr>
<td>\chi</td>
<td>Stiffening factor</td>
</tr>
<tr>
<td>\rho</td>
<td>Normalized displacement of GP along its length</td>
</tr>
<tr>
<td>\alpha_{3F}</td>
<td>Settlement interaction factor for a group of three floating piles obtained by rigorous analysis</td>
</tr>
<tr>
<td>\alpha_{4F}</td>
<td>Settlement interaction factor for a group of four floating piles obtained by rigorous analysis</td>
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<tr>
<td>\alpha_{3FS}</td>
<td>Settlement interaction factor for a group of three floating piles obtained by superposition analysis</td>
</tr>
<tr>
<td>\alpha_{4FS}</td>
<td>Settlement interaction factor for a group of four floating piles obtained by superposition analysis</td>
</tr>
<tr>
<td>E_b/E_s</td>
<td>Relative stiffness of bearing stratum on which the piles are resting</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

In the present paper, the comparative analysis of a group of three and four partially stiffened floating granular pile, each axially loaded with a load, ‘P’, partially stiffened is carried out using elastic continuum approach. Stiffening means that the elastic properties of the material of GP are improved as the replacement of the conventional material of GP is done by a material having better engineering material, may be geo-textile, SCD etc.

Major Assumptions Made During Analysis

(i) Stress strain behavior is assumed to be linear. (ii) Soil is considered to be homogeneous, isotropic and linearly elastic. (iii) The base of stone column/granular pile is taken to be smooth across which the load is uniformly distributed. (iv) The present study has been done by assuming no slip or yield condition.

II. LITERATURE REVIEW

Since 1969 with pioneer study of [2] in the area of granular piles as ground improvement technique, number of researchers has worked in this area, giving new heights to this area. [3] studied two types of column i.e. plain and reinforced (encased) and propose an upper bound analysis of the settlement of the foundation system with stone column inclusions, considering the non-linear behavior of the surrounding soft clay. [4] conducted the finite element method (FEM) studies to study the effect of encasing stone columns with geo-synthetic material for improvement in the load carrying capacity of the stone columns. The stone columns and the soft soils were modeled using hyperbolic non-linear elastic models, while the geo-synthetic encasement around the stone column was modeled as a linear elastic material. Acceleration of consolidation rate by stone columns was analyzed by, [5] within the framework of a basic unit cell i.e., a cylindrical soil body around a column. [6] presented the, analytical solutions based on the unit-cell concept to predict deformation behaviors of geo-textile-encased stone columns at any depth below the top plane of the columns. [7] studied that for a given reinforcement condition, the percentage improvement in load-carrying capacity was found to be higher for un-drained conditions than for drained conditions. The variation in the magnitude of the percentage improvement in strength with area replacement ratio indicated that, for drained conditions, there could be an upper bound area replacement ratio beyond which the benefits of increasing the area ratio become economically unjustifiable. [8] investigates the effects of encasement stiffness and strength on the response of individual geo-textile encased granular columns embedded in soft soil through model tests. Similarity analysis was first executed to determine the suitable properties of the constituents used in the model tests to ensure that the prototype-scale and model-scale geo-textile encased granular columns exhibit comparable behavior.

III. PROBLEM DEFINITION AND METHOD OF ANALYSIS

Fig. 1 (a) and (b) show the system of three GPs at equal spacing in a group. Fig. 2 (a) and (b) shows the system of four granular piles at a spacing of, ‘s’ in a group. The analysis is similar to that of a three GP group except for the influence of one more GP at a distance of, √2s. The soil displacements at the periphery of GP nodes are obtained for the influence of elemental shear stresses on own and all three adjacent GPs.

A. Soil Displacements

For three floating granular piles

\[
\{s\} = \left[\begin{array}{c}
1 IF^S + 2 IF^S + 3 IF^S
\end{array}\right] \left[\begin{array}{c}
\tau
\end{array}\right]
\]

(1)

Where \([1IF^S], [2IF^S] and [3IF^S]\) are soil displacement square matrices of size ‘(n+1)’ each due to influence of elemental shear stresses of self and correspondingly adjacent two symmetric GPs in three pile group as shown in Fig 1(b). Due to symmetry of positions of granular piles, 2 and 3, \([1IF^S] = [3IF^S]\) and thus soil displacement equations are

\[
\{s\} = \left[\begin{array}{c}
1 IF^S + 2 IF^S
\end{array}\right] \left[\begin{array}{c}
\tau
\end{array}\right]
\]

(2)

where \(\{s\}\) and \(\{\tau\}\) are soil displacement and normalized soil displacement vectors of size ‘(n+1)’ respectively; \(\{\tau\}\) is a column vector of size, ‘(n+1)’, for GP-soil interface shear stresses including the base pressure; \([1IF^S]+2x[3IF^S]\) is a combined square matrix for soil displacement influence coefficients of size ‘(n+1)’, for a floating granular pile

Similarly for four floating GPs-
\[
\begin{align*}
\left\{ \frac{\rho^s}{d} \right\} = \left\{ \frac{s^s}{d} \right\} = \left[ \begin{array}{c}
IF^s_1 \\
2IF^s_2 \\
3IF^s_3 \\
4IF^s_4 \\
\end{array} \right] \left[ \begin{array}{c}
\tau \\
\end{array} \right] \\
\left[ \begin{array}{c}
E \\
\end{array} \right]
\end{align*}
\]

(3)

where \([IF^s_1], [IF^s_2], [IF^s_3]\) and \([IF^s_4]\) are soil displacement square matrices of size (n+1) each due to influence of elemental shear stresses of own (first), second (at spacing s), third (at spacing s) and fourth GP (at spacing \(\sqrt{2}s\)) respectively as shown in Fig 2(a). All the other terms of equation are already defined in the analysis. Due to symmetry of positions of granular piles, 2 and 3, \([IF^s_3] = [IF^s_2]\) and thus soil displacement equations are

\[
\begin{align*}
\left\{ \frac{\rho^s}{d} \right\} = \left\{ \frac{s^s}{d} \right\} = \left[ \begin{array}{c}
1IF^s_2 \\
2IF^s_2 \\
4IF^s_4 \\
\end{array} \right] \left[ \begin{array}{c}
\tau \\
\end{array} \right] \\
\left[ \begin{array}{c}
E \\
\end{array} \right]
\end{align*}
\]

(4)

Fig.1 (a) Plan of group of three symmetrically placed floating partially stiffened GP (b) Sectional Elevation at X-X of a group of three symmetrically placed floating partially stiffened GPs
Fig. 2(a) Plan of group of four symmetrically placed floating, partially stiffened GPs, 2(b) Sectional Elevation at X-X of a, group of four symmetrically placed floating partially stiffened floating GPs

B. Granular Pile Displacements
Vertical displacements of elements of a single GP are evaluated based on a generalized stress-strain relationship as
\[ \varepsilon_v = \frac{\sigma_v}{E_{gp}} \quad (5) \]

Where \( \varepsilon_v \) is the axial strain of an element, \( \sigma_v \) is the axial stresses on the element respectively. \( E_{gp} \) is the deformation modulus of the granular pile.

C. Vertical Gp Displacements

The vertical displacements of granular pile at different level are evaluated from displacement of the top of granular pile \( \rho_t \) in the downward direction, by subtracting the displacement of first node from top displacement \( \rho_t \). The settlement of the first element of GP is:

\[ \rho_1^P = \frac{S_1^P}{d} = \rho_t - \varepsilon_{v1} \frac{\Delta z}{2d} \quad (6) \]

Where \( \varepsilon_{v1} \) is the axial strain of the first element of GP and \( \Delta z = (L/n) \) is element length. \( S_1^P \) and \( \rho_1^P \) is the displacement and normalized displacements of the first node respectively.

The vertical displacements of GP nodes in terms shaft shear stresses are

\[ \{ \rho_s \} = \rho_t \{ l \} + [MD] \left\{ \frac{\tau}{E_s} \right\} \quad (7) \]

Where \([MD]\) is a square matrix of size, \((n+1) = [MB][MA]\).

D. Compatibility Of Displacements Of Soil And Gp

For getting the solutions in terms of interface shear stresses and base pressure, apply the compatibility condition of displacements of the granular pile and the soil as described below. For three granular piles group (Equations (2) and (7))

\[ \{ \rho_s \} = \{ \rho_P \} \quad \text{or} \quad [AMC] \left\{ \frac{\tau}{E_s} \right\} = \rho_t \{ l \} \quad (8) \]

Where \([AMC] = \left[ \begin{array}{c} \mathbf{I} \mathbf{F}_s^S + 2 \mathbf{F}_s^S \end{array} \right] - [MD] \), of size \((n+1) \times (n+1)\).

For four granular piles group (Equations (3) and(7))

\[ \{ \rho_s \} = \{ \rho_P \} \quad \text{or} \quad [AMD] \left\{ \frac{\tau}{E_s} \right\} = \rho_t \{ l \} \quad (9) \]

Where \([AMD] = \left[ \begin{array}{c} \mathbf{I} \mathbf{F}_s^S + 2 \mathbf{F}_s^S + 4 \mathbf{F}_s^S \end{array} \right] - [MD] \), of size \((n+1) \times (n+1)\).

The parameter \( \alpha \) (settlement interaction factor) as defined by Mattes and Poulos (1971) is used (originally defined for un-stiffened group of two granular piles), and now defined as (for partially stiffened three/four granular piles)
For group of three granular pile partially stiffened, floating granular pile

\[ \alpha_{3F} = \frac{\text{settlement of a GP in a group of three partially stiffened GP} - \text{settlement of a single partially stiffened GP}}{\text{settlement of a single partially stiffened GP}} \]

For group of four granular pile partially stiffened, floating granular pile

\[ \alpha_{4F} = \frac{\text{settlement of a GP in a group of four partially stiffened GP group} - \text{settlement of a single partially stiffened GP}}{\text{settlement by a single partially stiffened GP}} \]

Interaction factors for a three and four GP group are also obtained from the principle of superposition as

For a group of three GPs

\[ \alpha_{3FS} = 2 \times \alpha_{2F} \] (10)

For a group of four GPs

\[ \alpha_{4FS} = 2 \times \alpha_{2F} \text{ (for spacing, s)} + \alpha_{2F} \text{ (for spacing, } \sqrt{2} \text{ s)} \] (11)

Where \( \alpha_{2F} \) is settlement interaction factor for group of two floating granular pile. The settlement interaction factors and top settlement influence factor are evaluated by rigorous method and superposition principle and compared with the variations of various parameters already listed.

IV. RESULTS AND DISCUSSION

![Diagram](image.png)

Fig.3 Variation of settlement interaction factor \( \alpha_{3F}, \alpha_{3FS} \), with relative stiffness of GP, \( K_{gp} \) Effect of stiffening factor, \( \chi \), on a GP in a group of three partially stiffened floating pile GPs (\( s/d=3, L/d=10, \eta=30\% \)) Rigorous and Superposition

Fig.3 depicts the comparison between rigorous and superposition values of, \( \alpha_{3F}, \alpha_{3FS} \), i.e. for a group of three GPs with the effect of stiffening factor, \( \chi \), i.e. for, \( L/d=10, s/d=3, \eta=30\% \& K_{gp}=10 \) at \( \chi=1, 2, 4 \) and 8, the value of, \( \alpha_{3F} \) for rigorous analysis are, 0.44, 0.53, 0.61 and 0.67, while for superposition analysis the value of, \( \alpha_{3FS} \), are 0.45, 0.54, 0.62 and 0.68, so there is a percentage difference of, 2.2, 1.8, 1.6 and 1.4 is observed while for \( K_{gp}=100 \) at \( \chi=1, 2, 4 \) and 8, the value of, \( \alpha_{3F} \) for rigorous analysis are, 0.76,
0.82, 0.85 and 0.87, while for superposition analysis the value of, $\alpha_{3FS}$, are, 0.77, 0.83, 0.86 and 0.88, so there is a percentage change of, 1.3, 1.2, 1.1 and 1.1, is observed. Hence more variation between rigorous and superposition values is at lower values of $K_{gp}$.

The Fig. 4 reveals the comparison between rigorous and superposition values of, $\alpha_{3F}$, $\alpha_{3FS}$, i.e. for a group of three GPs with the effect of percentage length of stiffening, $\eta$, on a GP in a group of three partially stiffened floating GPs ($s/d=2$, $L/d=20$, $\chi=2$) Rigorous and Superposition

![GROUP OF THREE PARTIALLY STIFFENED FLOATING GPs](image)

**Fig. 4 Variation of settlement interaction factor, $\alpha_{3F}$, $\alpha_{3FS}$, with relative stiffness of GP, $K_{gp}$ – Effect of % length of stiffening, $\eta$, on a GP in a group of three partially stiffened floating GPs ($s/d=2$, $L/d=20$, $\chi=2$) Rigorous and Superposition**

Although it may be noted that variation is not significant.

The Fig. 5 shows the comparison between rigorous and superposition values of, $\alpha_{4F}$, $\alpha_{4FS}$, i.e. for a group of four GPs with the effect of stiffening factor, $\chi$. It has been observed that for, $L/d=10$, $s/d=3$, $\eta=30\%$ & $K_{gp}=10$ at, $\chi=1$, 2, 4 and 8, the value of, $\alpha_{4F}$, for rigorous analysis are, 0.60, 0.73, 0.84 and 0.92, while for superposition analysis the value of, $\alpha_{4FS}$, are, 0.61, 0.74, 0.85 and 0.93, so there is a percentage change of, 1.6, 1.3, 1.1 and 1.0, is observed while for, $K_{gp}=100$ at, $\chi=1$, 2, 4 and 8, the value of, $\alpha_{4F}$, for rigorous analysis the value are, 1.07, 1.15, 1.20 and 1.22, while for superposition analysis the value of, $\alpha_{4FS}$, are, 1.08, 1.16, 1.21 and 1.23, so there is a percentage increase of, 0.9, 0.8, 0.8 and 0.8, occurs.

![GROUP OF FOUR PARTIALLY STIFFENED FLOATING GPs](image)

**Fig. 5 Variation of settlement interaction factor, $\alpha_{4F}$, $\alpha_{4FS}$, with relative stiffness of GP, $K_{gp}$ Effect of stiffening factor, $\chi$, on a GP in a group of four partially stiffened floating pile GPs ($s/d=3$, $L/d=10$, $\eta=30\%$) Rigorous and Superposition**
GROUP OF FOUR PARTIALLY STIFFENED FLOATING GPs

Fig. 6 Variation of settlement interaction factor, $\alpha_{4F}$, with relative stiffness of GP, $K_{gp}$ – Effect of % length of stiffening, $\eta$, on a GP in a group of four partially stiffened floating GPs ($s/d=2$, $L/d=20$, $\chi=2$) Rigorous and Superposition

Fig. 6 is showing the comparison between rigorous and superposition values of, $\alpha_{4F}$, $\alpha_{4FS}$, i.e. for a group of four GPs with the effect of percentage length of stiffening, $\eta$. It may be noted that for, $L/d=20$, $s/d=2$, $\chi=2$ & $K_{gp}=10$ at $\eta=10$, 20, 30 and 40%, the value of, $\alpha_{4F}$, for rigorous analysis are, 0.96, 0.97, 0.95 and 0.93, while for superposition analysis the value of, $\alpha_{4FS}$, are, 1.00, 1.01, 1.00 and 0.98, so there is a percentage increase of, 4, 3.9, 5 and 5.1, while for, $K_{gp}=100$ at $\eta=10$, 20, 30 and 40%, the value of, $\alpha_{4F}$, for rigorous analysis are, 1.26, 1.31, 1.34 and 1.36, while for superposition analysis the value of, $\alpha_{4FS}$, are, 1.30, 1.35, 1.38 and 1.40 thereby causing a percentage increase of, 3, 2.9, 2.8 and 2.8.

V. CONCLUSIONS

This analysis is based on elastic continuum approach, in which the Mindlin’s equations are used with introduction of stiffening parameters viz. stiffening factor, percentage length of stiffening. The analysis reveals that both rigorous and superposition analysis although are in close agreement but a slight variation occurs, which even decreases at higher values of relative stiffness of GP.

VI. ACKNOWLEDGEMENT

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REFERENCES