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Explication of Dio 3 -Tuples from Truncated Octahedral Number-I

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Abstract: We attempt to explicate dio 3-tuples from truncated octahedral number of different ranks. We also present 4 sets of dio 3-tuples under 3 cases with some numerical examples.

Keywords: Dio 3-tuples, Truncated octahedral number.

I. INTRODUCTION

The study of Diophantine equation goes back to ancient Greeks. The most famous example from that time is $x^2 + y^2 = z^2$. Diophantine equations have over centuries provided fertile ground for mathematicians. The earliest work of importance have been on the problem of determining Pythagorean triples to the equation $x^2 + y^2 = z^2$. All triples are given by $x = d(a^2 - b^2)$, $y = 2abd$, $z = d(a^2 + b^2)$ where $a, b, d \in \mathbb{Z}$ with $\gcd(a, b) = 1$. The name Diophantine equation is in honour of the mathematician Diophantus, who lived in Alexandria around 300 AD. His work arithmetica was one of the ancient texts that went missing. It originally consisted of 13 books of which only 6 has survived into modern era.

[1-3] has been referred for multitudinal ideas on number theory. [4-19] has been studied for numerous ideas on diophantine triples. In this paper we attempt to explicate dio 3-tuples from truncated octahedral number of different ranks. We also present 4 sets of dio 3-tuples under 3 cases with some numerical examples.

II. METHOD OF ANALYSIS

A. Case (i)

Let $a = TO_{n-1} = 16n^3 - 81n^2 + 138n - 79$ = Truncated octahedral number of rank n-1

$b = TO_n = 16n^3 - 33n^2 + 24n - 6$ = Truncated octahedral number of rank n

We then have

$$ab + 544n^4 - 686n^3 - 2057n^2 + 5604n - 150 = \alpha^2$$

$$\text{where } \alpha = 16n^3 - 57n^2 + 80n + 18 \quad (1)$$

Let 'c' be any non-zero integer such that

$$ac + 544n^4 - 686n^3 - 2057n^2 + 5604n - 150 = \beta^2 \quad (2)$$

$$bc + 544n^4 - 686n^3 - 2057n^2 + 5604n - 150 = \gamma^2 \quad (3)$$

Introducing the linear transformation $\beta = a + \alpha$, $\gamma = b + \alpha$ in (2) and (3) we get $c = a + b + 2\alpha$

$$\text{ie., } c = 64n^3 - 228n^2 + 322n - 49 = 4TO_n - 96n^2 + 226n - 25$$

\therefore The triples (a, b, c) are dio 3-tuples with the property $D(544n^4 - 686n^3 - 2057n^2 + 5604n - 150)$

Some numerical examples are presented below in table I

Table I

N	(a ,b ,c)	D(n)
1	(-6,1,109)	3255
2	(1,38,195)	6046
3	(38,201,593)	23691
4	(201,586,1687)	84714
5	(586,1289,386)	230695

Below we present 3 sets of dio 3-tuples with their corresponding properties

Table II

S. No	(a, b, c)	D(n)
1	$(TO_{n-1}, TO_n, 4TO_n - 96n^2 + 226n - 21)$	$544n^4 - 622n^3 - 2285n^2 + 5924n - 74$
2	$(TO_{n-1}, TO_n, 4TO_n - 96n^2 + 226n - 23)$	$544n^4 - 654n^3 - 2171n^2 + 5764n - 113$
3	$(TO_{n-1}, TO_n, 4TO_n - 96n^2 + 226n - 27)$	$544n^4 - 718n^3 - 1943n^2 + 5444n - 185$

B. Case (ii)

Here we take $a = TO_{n-2} = 16n^3 - 129n^2 + 220n - 314$ = Truncated octahedral number of rank n-2

$b = TO_n = 16n^3 - 33n^2 + 24n - 6$ = Truncated octahedral number of rank n

Proceeding as in case (i) we've $c = 64n^3 - 324n^2 + 404n - 234 = 4TO_n - 192n^2 + 308n - 210$

∴ The triples (a, b, c) are dio 3-tuples with the property $D(960n^4 + 3892n^3 - 16982n^2 + 15736n - 35)$

Some numerical examples are presented below in tableIII

Table III

n	(a, b, c)	D(n)
1	(-207,1,-90)	3571
2	(-262,38,-210)	10005
3	(-383,201,-210)	77179
4	(-474,586,294)	286045
5	(-439,1289,1686)	740595

Below we present 3 sets of dio 3-tuples with their corresponding properties in tableIV

Table IV

S. No	(a, b, c)	D(n)
1	$(TO_{n-2}, TO_n, 4TO_n - 192n^2 + 308n - 204)$	$960n^4 + 3988n^3 - 17468n^2 + 16216n + 232$
2	$(TO_{n-2}, TO_n, 4TO_n - 192n^2 + 308n - 206)$	$960n^4 + 3956n^3 - 17306n^2 + 16056n + 141$
3	$(TO_{n-2}, TO_n, 4TO_n - 192n^2 + 308n - 208)$	$960n^4 + 3924n^3 - 17144n^2 + 15896n + 52$

C. Case (iii)

Here we take

$$a = TO_{n-2} = 16n^3 - 129n^2 + 220n - 314 = \text{Truncated octahedral number of rank } n-2$$

$$b = TO_{n-1} = 16n^3 - 81n^2 + 138n - 79 = \text{Truncated octahedral number of rank } n-1$$

Proceeding as in earlier cases we've $c = 64n^3 - 420n^2 + 438n - 93 = 4TO_n - 228n^2 + 342n - 69$

∴ The triples (a, b, c) are dio 3-tuples with the property $D(-3872n^4 + 38310n^3 - 95885n^2 + 72712n - 2306)$

Some numerical examples are listed below in tableV

Table V

n	(a, b, c)	D(n)
1	(-207, -6, -11)	8959
2	(-262, 1, -385)	4106
3	(-383, 38, -831)	73603
4	(-474, 201, -965)	214990
5	(-439, 586, -403)	332879

Below we present 3 sets of dio 3-tuples with their respective properties in tableVI

Table VI

s.no	(a, b, c)	D(n)
1	$(TO_{n-2}, TO_{n-1}, 4TO_n - 228n^2 - 342n - 63)$	$(-3872n^4 + 38406n^3 - 96515n^2 + 72952n - 1397)$
2	$(TO_{n-2}, TO_{n-1}, 4TO_n - 228n^2 - 342n - 65)$	$(-3872n^4 + 3834n^3 - 96305n^2 + 72872n - 1702)$
3	$(TO_{n-2}, TO_{n-1}, 4TO_n - 228n^2 - 342n - 67)$	$(-3872n^4 + 38342n^3 - 96095n^2 + 72792n - 2005)$

III.CONCLUSION

In this paper we have explicated dio 3-tuples under 3 cases from Truncated octahedral number of different ranks. One may also search for similar type of dio 3-tuples from some other special numbers with suitable properties.

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