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# Sensitivity Analysis of a Class of Serial Manipulators

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**Abstract:** *The sensitivity analysis of serial manipulator, which varies in different geometrical parameters along with other manipulators with respect to areas, dexterity and sensitivity is analyzed. The sensitivity of serial manipulator is shown algebraically. They consist of two sensitivity indices, first related to the orientation and the second is position, it can be compared with other serial manipulators. Sensitivity of serial manipulator is analyzed and four different case studies are presented in this paper.*

**Keywords:** *sensitivity analysis, serial manipulator, poses*

## I. INTRODUCTION

A system which controls several degree of freedom of the end-effector via a mechanical system is known as Robot.

A fundamental robotics task is to plan collision free motion for complex bodies from a start to end position. The total degree of freedom of a robot is six. The position and orientation of the robots end effector (here called its pose) can be described by its generalized coordinate's. These are usually the coordinates of a specific point of the end-effector and the angles that define its orientation.

Now-a-days robots are making considerable remarks on many modern applications. It includes industries, healthcare, transportation and excavations in the deep sea. The manipulators are classified into different types and used in different environments. Suitable actuators are used for their actuations. Sensors play vital role in detecting robot's position, speed, force, etc. They provide feedback to the system. These are collectively helpful for programming, path planning and controlling. The challenges always occur in designing link and joint structures, as well as actuation to obtain the desired position of the robot

### A. Manipulators

Manipulator consists of links and joints. Links are known as the rigid sections that make up the mechanism and joints are known as the connection between two links. At an end a component is attached to the manipulator which interacts with its environment to perform tasks, it is called end-effectors.

### B. Serial Manipulators

A serial manipulator fig 1.4 consists of a fixed base, a series of links connected by joints, and ending at a free end carrying the tool or the end-effector. In contrast to parallel manipulators, there are no closed loops. By actuating the joints, one can position and orient the end-effector in a plane or in three-dimensional (3D) space to perform desired tasks with the end-effector. One end of the manipulator is attached to the ground and other end is free to move in space. For this reason a serial manipulator is sometimes called as an open-loop manipulator. We call the fixed link the base and the free end where a gripper or a mechanical hand is attached to the end effector. Serial manipulators are widely used in industrial applications, where quick, precise positioning and alignment are essential

### C. Different Types Of Joints

- 1) *Revolute:* The revolute joint fig1.7 is represented as —R— and sometimes referred to colloquially as a hinge or pin joint. The surfaces are the same except one of them is an external surface, convex in any plane normal to the axis of revolution, and one is an internal surface, concave in any plane normal to the axis. The surfaces may not be solely in the form of right circular cylinders, since surfaces of that form do not provide any constraint on axial sliding. A revolute joint permits only rotation of one of the bodies joined relative to the other. The position of one body relative to the other may be expressed as the angle between two lines normal to the joint axis, one fixed in each body. Thus, the joint has one degree of freedom.

- 2) *Prismatic*: Prismatic joint, fig 1.8 often abbreviated as —P| and sometimes referred to colloquially as a sliding joint. These may not be right circular cylindrical surfaces. A general cylindrical surface is obtained by extruding any curve in a constant direction. Again, one surface is internal and the other is an external surface. A prismatic joint permits only sliding of one of the members joined relative to the other along the direction of extrusion. The position of one body relative to the other is determined by the distance between two points on a line parallel to the direction of sliding, with one point fixed in each body. Thus, this joint also has one degree of freedom.
- 3) *Helical*: The most general form of a helical joint, fig1.9 often abbreviated as —H| and sometimes referred to colloquially as a screw joint. The simple example is a screw and nut wherein the basic generating curve is a pair of straight lines. The angle of rotation about the axis of the screw joint  $\theta$  is directly related to the distance of displacement of one body relative to the other along that axis  $d$  by the expression  $d = h\theta$ , where the constant  $h$  is called the pitch of the screw.
- 4) *Cylindrical*: A cylindrical joint fig 1.10, often abbreviated as —C|, is a lower pair formed by contact of two congruent right circular cylinders, one an internal surface and the other an external surface. It permits both rotation about the cylinder axis and sliding parallel to it. Therefore, it is a joint with two degrees of freedom. Modeling a single cylindrical joint as a combination of a prismatic and revolute joint requires the addition of a virtual link between the two with zero mass and zero length. The massless link can create computational problems.
- 5) *Spherical*: A spherical joint fig 1.11, often abbreviated as —S|, is a lower pair formed by contact of two congruent spherical surfaces. Once again, one is an internal surface, and the other is an external surface. A spherical joint permits rotation about any line through the center of the sphere. Thus, it permits independent rotation about axes in up to three different directions and has three degrees of freedom. A spherical joint is easily replaced by a kinematically equivalent compound joint consisting of three revolute that have concurrent axes. They do not need to be successively orthogonal, but often they are implemented that way. The arrangement is, in general, kinematically equivalent to a spherical joint, but it does exhibit a singularity. When the revolute joint axes become coplanar. This is as compared to the native spherical joint that never has such a singularity. Likewise, if a spherical joint is modeled in simulation as three revolute, computational difficulties again can arise from the necessary inclusion of massless virtual links having zero length.
- 6) *Planar*: A planar joint fig 1.12 is formed by planar contacting surfaces. Like the spherical joint, it also has three degrees of freedom. A kinematically equivalent compound joint consisting of a serial chain of three revolute with parallel axes can replace a planar joint. As was the case with the spherical joint, the compound joint exhibits a singularity when the revolute axes become coplanar.

#### D. Forward Kinematics

A manipulator is composed of serial links which are affixed to each other revolute or prismatic joints from the base frame through the end-effector. Calculating the position and orientation of the end-effector in terms of the joint variables is called as forward kinematics. In order to have forward kinematics for a robot mechanism in a systematic manner, one should use a suitable kinematics model. Denavit-Hartenberg method that uses four parameters is the most common method for describing the robot kinematics. These parameters  $a$ ,  $\alpha$ ,  $d$  and  $\theta$  are the link length, link twist, link offset and joint angle, respectively. A coordinate frame is attached to each joint to determine DH parameters.  $Z_i$  axis of the coordinate frame is pointing along the rotary or sliding direction of the joints. As shown in Figure 2, the distance from  $Z_{i-1}$  to  $Z_i$  measured along  $X_{i-1}$  is assigned as  $a_{i-1}$ , the angle between  $Z_{i-1}$  and  $Z_i$  measured along  $X_i$  is assigned as  $\alpha_{i-1}$ , the distance from  $X_{i-1}$  to  $X_i$  measured along  $Z_i$  is assigned as  $d_i$  and the angle between  $X_{i-1}$  to  $X_i$  measured about  $Z_i$  is assigned as  $\theta_i$ . The general transformation matrix  ${}^{i-1}T_i$  for a single link can be obtained as follows. Where  $R_x$  and  $R_z$  present rotation,  $D_x$  and  $Q_i$  denote translation, and  $c\theta_i$  and  $s\theta_i$  are  $\cos\theta_i$  and  $\sin\theta_i$ , respectively. The forward kinematics of the end-effector with respect to the base frame is determined by multiplying all of the  ${}^{i-1}T_i$  matrices.

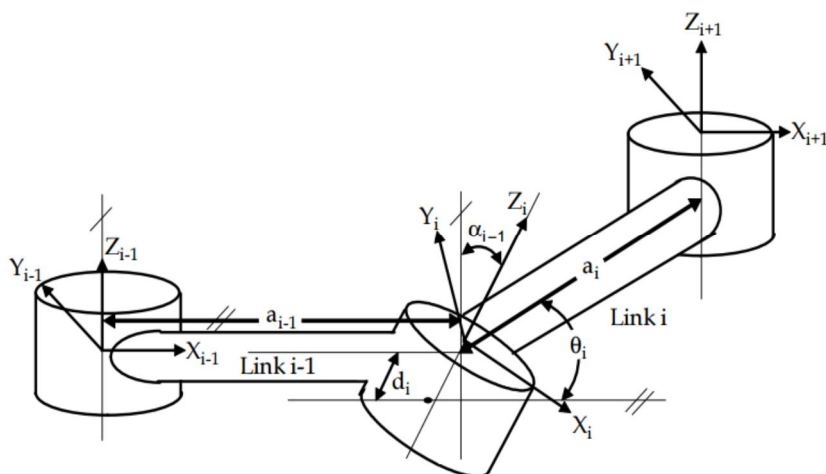


Fig 1: Forward Kinematics

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_{i-1} & -s\alpha_{i-1} & 0 \\ 0 & s\alpha_{i-1} & c\alpha_{i-1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{base}_{end\_effector} T = {}^0_1 T {}^1_2 T \dots {}^{n-1}_n T$$

$${}^{base}_{end\_effector} T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_6 T = {}^0_1 T(q_1) {}^1_2 T(q_2) {}^2_3 T(q_3) {}^3_4 T(q_4) {}^4_5 T(q_5) {}^5_6 T(q_6)$$

## II. METHODOLOGY

- A. The main objective of this project is using sensitivity analysis to figure out which input parameters are effecting the unexpected output parameters
- B. Unexpected output parameters would not let the programmer to code the robots to work at full speed due to safety issues; also the robots are placed far apart from each other to avoid collusion. So by tweaking the input parameters we can adjust the output values as required



C. Once that is attained the robots can work in tandem much more effectively at higher speed in a compact space ,increasing the production capacity

$$D. P=a1(\cos(B1)+\sin(B1))+b1(\cos(B2)+\sin(B2))+c1(\cos(B3)+\sin(B3))+c1(\cos(B4)+\sin(B4))$$

$$E. \partial p=a1\partial B1D_1+b1\partial B2E_1+c1\partial B3F_1+c1\partial B4G_1$$

$$F. l2v2\partial p=l2v2a1\partial B1D_1+l2v2b1\partial B2E_1+l2v2c1\partial B3F_1+l2v2c1\partial B4G_1$$

$$G. \begin{bmatrix} \partial p \\ \partial F \end{bmatrix} = A^{-1}x_1\partial B1+A^{-1}y_1\partial B3$$

$$H. \text{ Let } A^{-1}x_1=T_a$$

$$I. A^{-1}y_1=T_b$$

$$J. \begin{bmatrix} \partial p \\ \partial F \end{bmatrix} = T_a\partial B1+T_b\partial B3$$

$$K. J = \begin{bmatrix} -l_1s_{12} - l_2s_{12} - l_3s_{123} - l_3s_{1234} & -l_2s_{12} - l_3s_{123} - l_3s_{1234} & -l_3s_{123} - l_3s_{1234} & -l_3s_{1234} \\ l_1c_{12} + l_2c_{12} + l_3c_{123} + l_3c_{1234} & l_2c_{12} + l_3c_{123} + l_3c_{1234} & l_3c_{123} + l_3c_{1234} & l_3c_{1234} \end{bmatrix}$$

L. Variations in Cartesian coordinates

$$\begin{bmatrix} \partial l_{1x} \\ \partial l_{1y} \end{bmatrix} = \begin{bmatrix} \cos(B1) & -l_1\sin(B1) \\ \sin(B1) & l_1\cos(B1) \end{bmatrix} \begin{bmatrix} \partial l_1 \\ \partial B1 \end{bmatrix}$$

$$\begin{bmatrix} \partial l_{2x} \\ \partial l_{2y} \end{bmatrix} = \begin{bmatrix} 0 & -l_2\sin(B2) \\ 0 & l_2\cos(B2) \end{bmatrix} \begin{bmatrix} \partial l_2 \\ \partial B2 \end{bmatrix}$$

$$\begin{bmatrix} \partial l_{3x} \\ \partial l_{3y} \end{bmatrix} = \begin{bmatrix} \cos(B3) & -l_3\sin(B3) \\ \sin(B3) & l_3\cos(B3) \end{bmatrix} \begin{bmatrix} \partial l_3 \\ \partial B3 \end{bmatrix}$$

M. Aggregate sensitivity indices

$$N. \begin{bmatrix} \partial F \\ \partial P \end{bmatrix} = J_e \begin{bmatrix} \partial l_1 \\ \partial l_2 \\ \partial l_3 \end{bmatrix}$$

O. Where  $J_e = [J_A \ J_B \ J_C]$

$$P. \partial l_1 [l_{1x} \ l_{1y}]$$

$$Q. \partial l_2 [l_{2x} \ l_{2y}]$$

$$R. \partial l_3 [l_{3x} \ l_{3y}]$$

$$S. V_F = \|J_{sF}\|_2 / n_v$$

$$T. V_P = \|J_{sP}\|_2 / n_v$$

### III. RESULTS

TABLE-1

| ORIENTATION (T <sub>1</sub> ) | POSITION (T <sub>2</sub> ) |
|-------------------------------|----------------------------|
| 37.899457                     | 35.060128                  |
| 37.973358                     | 35.175731                  |
| 38.171274                     | 35.484571                  |
| 38.430107                     | 35.886847                  |
| 38.668659                     | 36.256017                  |
| 38.813380                     | 36.479261                  |
| 34.940532                     | 33.671756                  |
| 35.008664                     | 33.782669                  |

|           |           |
|-----------|-----------|
| 35.191127 | 34.078984 |
| 35.429753 | 34.464958 |
| 35.649680 | 34.819180 |
| 35.783102 | 35.033389 |
| 26.987806 | 29.638806 |
| 27.040430 | 29.735500 |
| 27.181363 | 29.993924 |
| 27.365676 | 30.330732 |
| 27.535546 | 30.640002 |
| 27.638601 | 30.827101 |
| 21.902453 | 27.140281 |
| 21.861701 | 27.112646 |
| 21.751794 | 27.039413 |
| 21.606343 | 26.945901 |
| 21.470538 | 26.862942 |
| 21.387316 | 26.814670 |
| 31.847820 | 32.535349 |
| 31.788564 | 32.496425 |
| 31.628750 | 32.391582 |
| 31.417253 | 32.253138 |
| 31.219782 | 32.124192 |
| 31.098772 | 32.045327 |
| 37.911357 | 35.476748 |
| 37.840819 | 35.434150 |
| 37.650577 | 35.319404 |
| 37.398813 | 35.167869 |
| 37.163746 | 35.026716 |
| 37.019697 | 34.940377 |

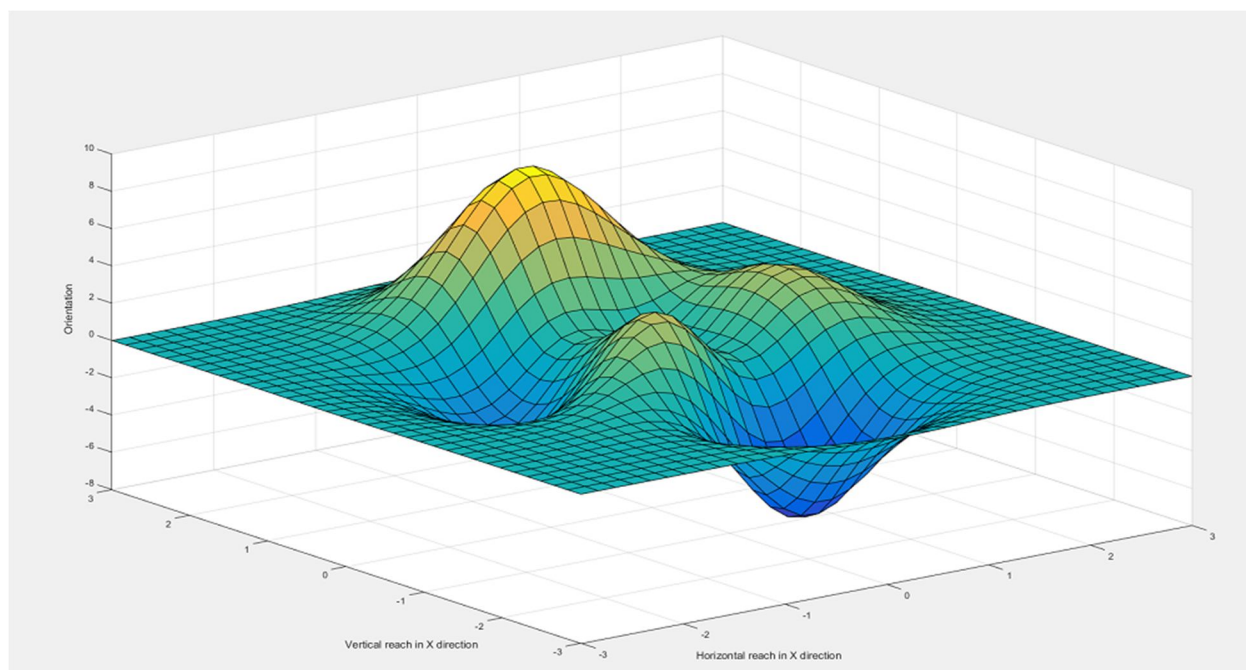


Fig 2: Surface plot for sensitivity Index of orientation of a serial manipulator to Variations in the Geometric Parameters.

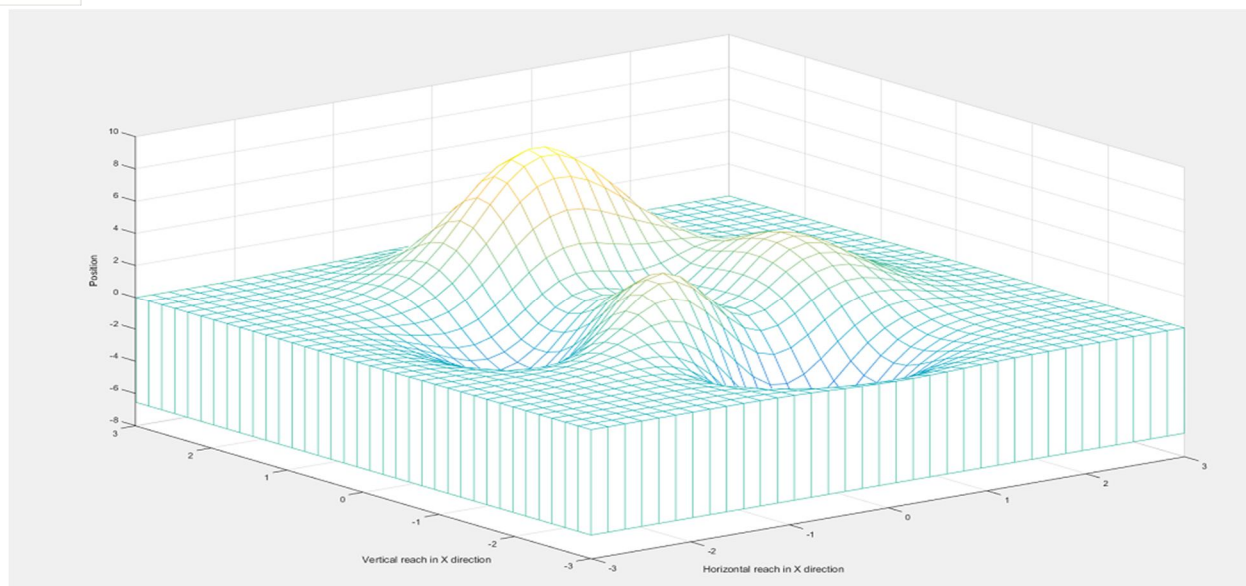


Fig 3: Surface plot for sensitivity Index of position of a serial manipulator to Variations in the Geometric Parameters

#### IV. CONCLUSIONS

This work deals with sensitivity analysis of serial manipulators. First, the sensitivity coefficients of the pose of the serial manipulator is calculated. Moreover, two aggregate sensitivity indices were determined one related to the orientation of the serial manipulator and another one related to its position. The variations in the geometric parameters such as manufacturing tolerances and installation errors are unavoidable. Out of the four manipulator structures the best structure is the one with link lengths  $a_1=a_2=a_3=3$  at the revolute joints. In this structure for the configuration defined by the set of angles  $A_1=30$ ,  $B_1=60$ ,  $C_1=45$ ,  $x=30$ ,  $x_1=45$ ,  $y=45$ ,  $w_1=45$ , the lowest sensitivity index in position and orientation is obtained. The designer can use this structure for positioning the object in the work cell with the greatest accuracy.

#### V. FUTURE SCOPE

Only four structures are considered in the analysis. The problem can be extended by considering it as a multi objective optimization, like genetic algorithm or simulated annealing, by taking various lengths and orientations.

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