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Trajectory Planning of 6R Jointed Industrial Robot Manipulators

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Abstract: In this paper, the trajectory of 6R(6 Revolute) jointed industrial robot is planned by taking the constraints of displacement, velocity, acceleration and jerk. PUMA 560 robot manipulator is used as an example to plan the trajectory but it can be generalized for all major industrial robot manipulators. Three degree and Five degree polynomials are used to plan the trajectory, taking time as the optimal parameter and smoothness of the trajectory as the performance measure. The trajectory is simulated using MATLAB for both polynomials, and the result of the simulation shows that the five degree polynomial gives better and smoother trajectory and it also accommodates the additional constraint of jerk.
Keywords: Trajectory, Robot Manipulators, Revolute joints, Polynomials, Smoothness

I. INTRODUCTION

The advancement in the field of manufacturing is mainly due to the requirement of the high quality product. With the need of high quality product, we need a manufacturing system that can deal with manufacturing of products with very less tolerance limits, complex shapes,etc. In order to achieve this, a machine is needed that can repeat the same thing again and again and produce good product with high accuracy in its dimensions. The machine that can be used is industrial robot manipulators. Industrial robot manipulators are very widely used in the industries for welding, painting, assembling, etc. They are used because they give great results, for example a welding needs to be done for joining of two parts and n number of such pairs are to be welded, in this case if we use human hand to do the job, it can be done but the weld might not be same for all the joints. In contrast if we use robot to do the same weld, all the joints will be welded exactly the same. So robot manipulator not only bring accuracy but it also brings the consistency in the product manufacturing. Serial robot manipulator with six degree of freedom is generally used in the industry as its workspace can be any point in three dimensional space with any orientation. To do any kind of job, a robot manipulator need to move from one point to another. The path traced by the joints and end effector of the robot while moving from one point to another point considering the constraints is called the trajectory of the robot manipulator. The trajectory of the robot should be smoother and jerk free for the job to be done properly.

There are many approaches for planning the trajectory of the robot manipulator. Rajan, V.T. [4] 1985, proposed a method of first parameterizing the path in the Configuration space, then, given a path, using control theory to determine the minimum time trajectory subject to the actuator torque constraints, and finally searching among all possible paths to find the minimum time path. The minimum time path was assumed to be smooth and hence was parameterized by splines. Won et al.[5] 1991, introduced a point-to-point joint trajectory planning scheme which generated a smooth joint trajectory for a given joint velocity profile. The given joint velocity profile was approximated in terms of the Nth partial sum of Fourier series. Cao et. al [6] 1994, presented a suitable objective function to combine time-optimal and smooth trajectory planning. The optimization process involved, first obtaining smooth and time-suboptimal joint paths by minimizing the objective function, and then contracting travelling time by scaling the resulting time intervals for time-optimal joint paths so that the resulting velocities or accelerations or jerks at some knots of the paths are maximal within their limitations. Bobrow, J.E[7]1998, presented path planning technique which produced time-optimal manipulator motions in a workspace containing obstacles.The full nonlinear equations of motion were used in conjunction with the actuator limitations to produce optimal trajectories. The Cartesian path of the manipulator was represented with B-spline polynomials, and the shape of this path was varied in a manner that minimizes the traversal time. D. Constantinescu, E. A. Croft [8] 2000, presented a method for determining smooth and time-optimal path constrained trajectories for robotic manipulators and investigated the performance of these trajectories both through simulations and experiments. The desired smoothness of the trajectory was imposed through limits on the torque rates. Hiroaki et.al[9] 2003, described that a learning control algorithm with B-spline is effective to improve the trajectory tracking accuracy of an industrial robot and showed the results of simulation and experiment. The learning control method consists of two processes: Global Learning (GL) and Local Learning (LL). GL estimates the dynamics of a robot control system and obtains a learning gain matrix used in LL. LL decreases the tracking errors by iterative

trial movements and acquires satisfactory tracking accuracy. Khouki et.al[10] 2008,studied the problem of minimum-time trajectory planning for a three degrees-of-freedom planar manipulator using a hierarchical hybrid neuro-fuzzy system. A first neuro-fuzzy network named NeFIK was considered to solve the inverse kinematics problem. After a few pre-processing steps characterizing the minimum-time trajectory and the corresponding torques, a second neuro-fuzzy controller was built. Its purpose was to fit the robot dynamic behavior corresponding to the determined minimum-time trajectory with respect to actuators models, torque nominal values, as well as position, velocity, acceleration and jerk boundary conditions. Xiaoping Liao et.al[11] 2010,planned trajectory using an adaptive genetic algorithm while considering constraints of displacement, velocity, acceleration and jerk of each joint. According to the optimal time intervals generated, the results of simulation on robot kinematics showed that the method designed for robot trajectory planning can obtain the goal trajectory.
The above mentioned methods are useful in planning the trajectory of robotic manipulator but the methods are either too complex or they are fusion of multiple simple algorithms. This paper uses simple three degree and five degree polynomial to plan the trajectory and checks its feasibility by simulating in MATLAB. It also compares among the polynomials trajectories.

II. MATHEMATICAL MODEL OF ROBOT MANIPULATOR

The mathematical model of the robot manipulator can be established through denavit hartenberg(D-H) convention. This convention provides a way to represent the various joints and links of robot manipulator in cartesian coordinate and a table of D-H parameter can be made which can be used to do the kinematic analysis of the robot manipulator. The representation of PUMA 560 manipulator in D-H coordinate is shown below in figure 1. D-H parameter table of PUMA 560.

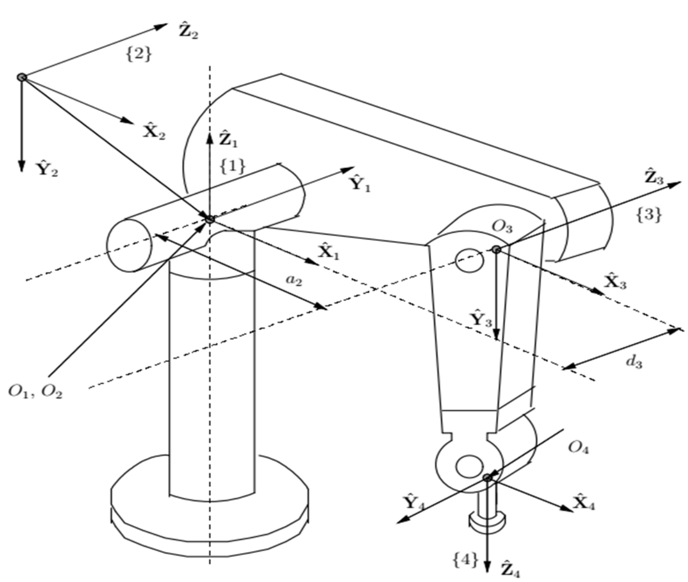


Figure 1. PUMA 560 in D-H Coordinate.

III.KINEMATIC ANALYSIS

A. Forward Kinematic Analysis

In this analysis,the coordinate of any joint with respect to the base coordinate is found. It is done by using the transformation matrix. The coordinate of one joint with respect to another can also be calculated. It can be calculated by using the following transformation matrix.

$$ {}^{i-1}_{i}T=\begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i\cos\alpha_{i-1} & \cos\theta_i\cos\alpha_{i-1} & -\sin\alpha_{i-1} & -\sin\alpha_{i-1}d_i \\ \sin\theta_i\sin\alpha_{i-1} & \cos\theta_i\sin\alpha_{i-1} & \cos\alpha_{i-1} & \cos\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1) $$

The above matrix can be used to calculate the coordinate of i^{th} frame with respect to $(i-1)^{th}$ frame. The coordinate of the end effector can be obtained by multiplying the individual matrices from frame 1 to frame 6.

B. Inverse Kinematic Analysis

The inverse kinematic solution for a robot manipulator gives the joint angle values for a particular position of end effector. For a given position of the end effector, the joint angle values are not unique and there are various ways to solve for inverse kinematics of robot manipulator. Algebraic method is used to find the inverse kinematic solution of PUMA 560 in this study.

The transformation matrix in (1) can be rewritten for PUMA 560 as

$$ {}^{0}_{6}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T\,{}^{4}_{5}T\,{}^{5}_{6}T \qquad (2) $$

The values of θ_i can be found by multiplying each side of direct kinematics equation by an inverse transformation matrix for separating out variables in search of solvable equation.The values of θ_i found are:

θ_1 =Atan2(p_y, p_x)-Atan2(d_3,$\pm\sqrt{p_x^2 + p_y^2 - d_3^2}$)

$\theta_2 = \theta_{23} - \theta_3$

θ_3 =Atan2(a_3,d_4)-Atan2(k,$\pm\sqrt{a_3^2 + d_4^2 - k^2}$)

θ_4 =Atan2($-r_{13}s_1 + r_{23}c_1$,$-r_{13}c_1c_{23} - r_{23}s_1c_{23} + r_{33}s_{23}$)

θ_5 =Atan2(s_5,c_5)

θ_6 =Atan2(s_6,c_6) (3)

where,

θ_{23}=Atan2[$(-a_3 - a_2c_3)p_z - (c_1p_x + s_1p_y)(d_4 - a_2s_3)$,$(a_2s_3 - d_4)p_z - (a_3 + a_2c_3)(c_1p_x + s_1p_y)$]

$$k = \frac{p_x^2 + p_y^2 + p_z^2 - a_2^2 - a_3^2 - d_3^2 - d_4^2}{2a_2}$$

$$s_{23} = \frac{(-a_3 - a_2c_3)p_z + (c_1p_x + s_1p_y)(a_2s_3 - d_4)}{p_z^2 + (c_1p_x + s_1p_y)^2}$$

$$c_{23} = \frac{(a_2s_3 - d_4p_z) - (a_3 + a_2c_3)(c_1p_x + s_1p_y)}{p_z^2 + (c_1p_x + s_1p_y)^2}$$

$$s_5 = -r_{13}(c_1c_{23}c_4 + s_1s_4) - r_{23}(s_1c_{23}c_4 - c_1s_4) + r_{33}(s_{23}c_4)$$

$$c_5 = r_{13}(-c_1s_{23}) + r_{23}(-s_1s_{23}) + r_{33}(-c_{23})$$

$$s_6 = -r_{11}(c_1c_{23}s_4 - s_1c_4) - r_{21}(s_1s_{23}s_4 + c_1c_4) + r_{31}(s_{23}s_4)$$

$$c_6 = r_{11}[(c_1c_{23}c_4 + s_1s_4)c_5 - c_1s_{23}s_5] + r_{21}[(s_1c_{23}c_4 - c_1s_4)c_5 - s_1s_{23}s_5] - r_{31}(s_{23}c_4\ \ c_5 + c_{23}s_5)$$

where

$$c_1 = cos\theta_1, s_1 = sin\theta_1$$

IV. TRAJECTORY PLANNING

The trajectory of robot manipulator is planned to estimate the path traced by each joint of the robot manipulator while moving from an initial point to final point under certain constraints. The trajectory is planned using two polynomials viz. three degree polynomial and five degree polynomial, in this study.

A. Three Degree Polynomial Trajectory Planning

The trajectory is planned using the general equation of three degree polynomial.Since the equation has four coefficients,only four constraints can be accommodated using this polynomial.The initial and final joint angles are assumed to be known and the initial and final velocity are assumed to be zero.

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 \quad (4)$$

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2 \quad (5)$$

Constraints are:

$$\theta(0) = \theta_0$$
$$\theta(t_f) = \theta_f$$
$$\dot{\theta}(0) = 0$$
$$\dot{\theta}(t_f) = 0 \quad (6)$$

Applying the constraints of (6) in (4) and (5),we get

$$c_0 = \theta_0$$
$$c_1 = 0$$
$$c_2 = \frac{3}{t_f^3}(\theta_f - \theta_0)$$
$$c_3 = \frac{-2}{t_f^3}(\theta_f - \theta_0) \quad (7)$$

The above equations (7) can be substituted into (4) to plan the trajectory.

B. Five Degree Polynomial Trajectory Planning

In this method as well, the general five degree polynomial is used to plan the trajectory. But in this case there are six coefficient in the general equation so six coefficients can be accommodated. In addition to previous constraints, the acceleration at initial and final points are considered to be zero.

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5 \quad (8)$$

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2 + 4c_4 t^3 + 5c_5 t^4 \quad (9)$$

$$\ddot{\theta}(t) = 2c_2 + 6c_3 t + 12c_4 t^2 + 20c_5 t^3 \quad (10)$$

$$\theta^{\ddot{}}(\dot{t}) = c_2 + 6c_3 + 24c_4 t + 60c_5 t^2 \quad (11)$$

Constraints are:

$$\theta(0) = \theta_0$$
$$\theta(t_f) = \theta_f$$
$$\dot{\theta}(0) = 0$$
$$\dot{\theta}(t_f) = 0$$
$$\ddot{\theta}(0) = 0$$
$$\ddot{\theta}(t_f) = 0 \quad (12)$$

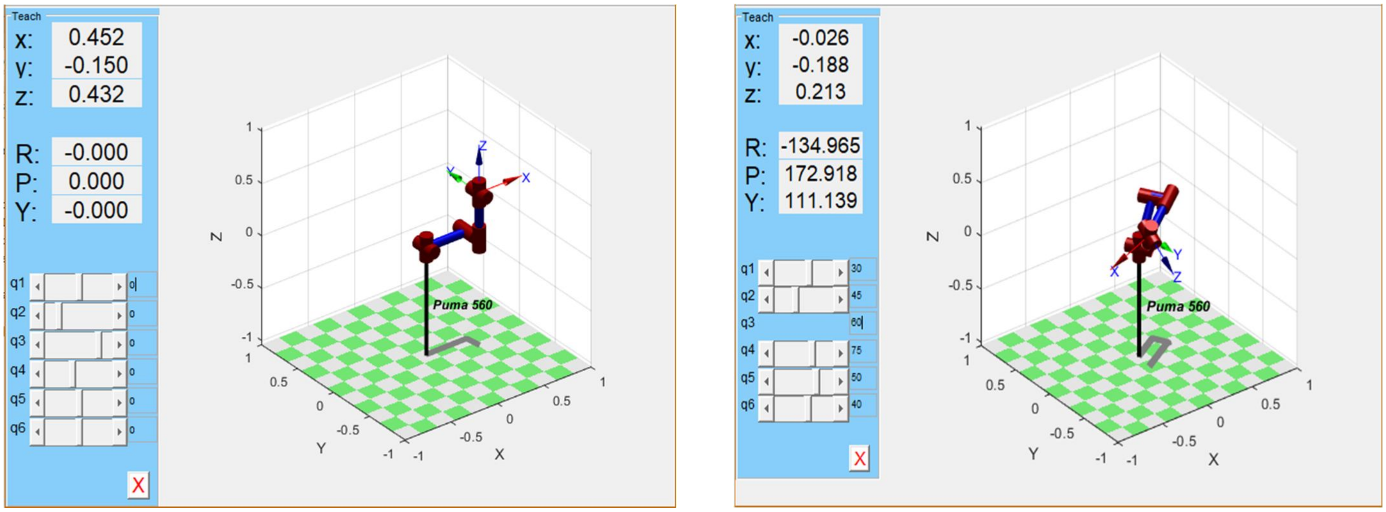
Applying the constraints of (12) in (8),(9) and (10) we get,

$$c_0 = \theta_0$$
$$c_1 = 0$$
$$c_2 = 0$$
$$c_3 = \frac{10(\theta_f - \theta_0)}{t_f^3}$$
$$c_4 = \frac{-15(\theta_f - \theta_0)}{t_f^4}$$
$$c_5 = \frac{6(\theta_f - \theta_0)}{t_f^5} \quad (13)$$

The above equations (13) can be substituted into (8) to plan the trajectory.

V. SIMULATION RESULTS

The simulation is done by using MATLAB software. Robotic System Toolbox is added to MATLAB to simulate for the robotic manipulator. The robot model is established using SerialLink function and the forward kinematics is done using fkine function and the inverse kinematics is done using ikine function. The initial joint angles are taken as 0° and the final joint angles are 30°,45°,60°,75°,50° and 40° for joint 1, joint 2, joint 3, joint 4, joint 5, and joint 6 respectively. The simulation is shown in the diagrams below.

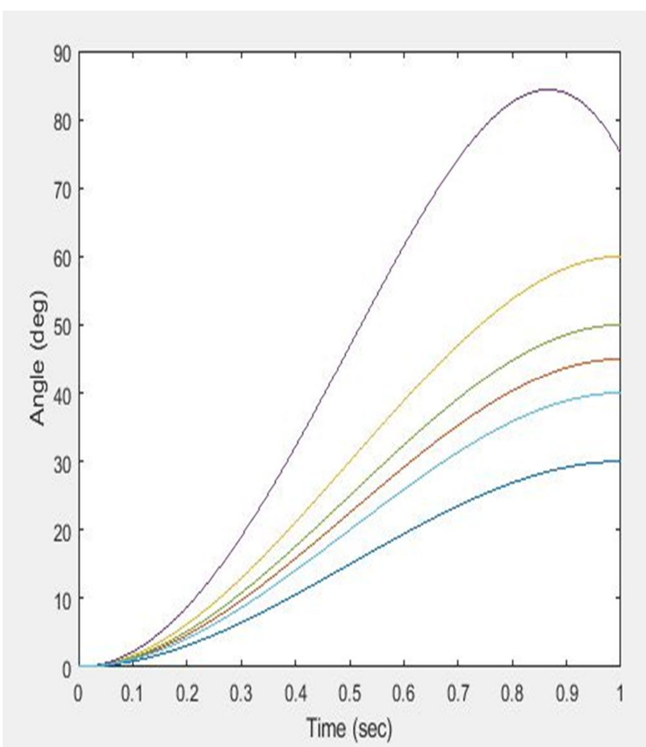


INITIAL POSITION FINAL POSITION

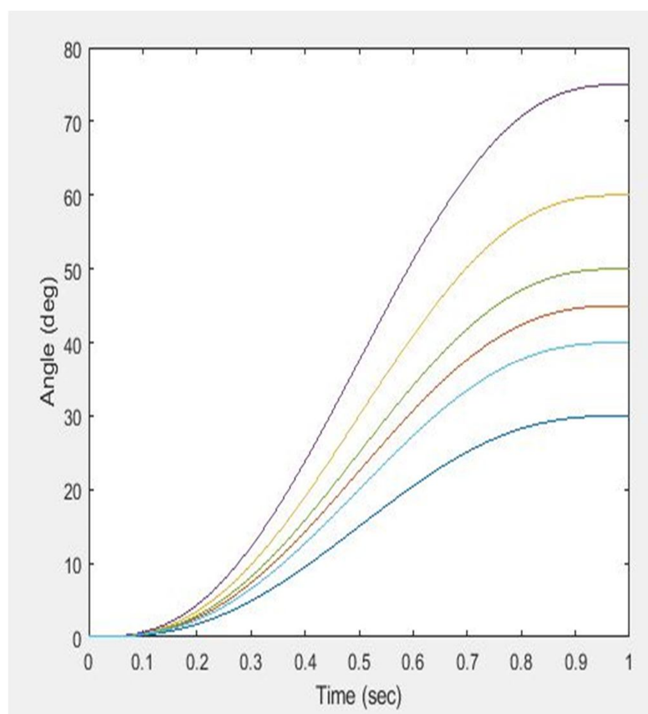
FIGURE II. POSITION OF JOINTS AT INITIAL AND FINAL POINTS

As shown in the figure (II),the initial X,Y and Z values are 0.452,-0.150 and 0.432 respectively and the Roll, Pitch and Yaw angles are zero initially. At the final position, the values of X,Y and Z are -0.026,-0.188 and 0.213 respectively and Roll, Pitch and Yaw angles are -134.965, 172.918 and 111.139 respectively.

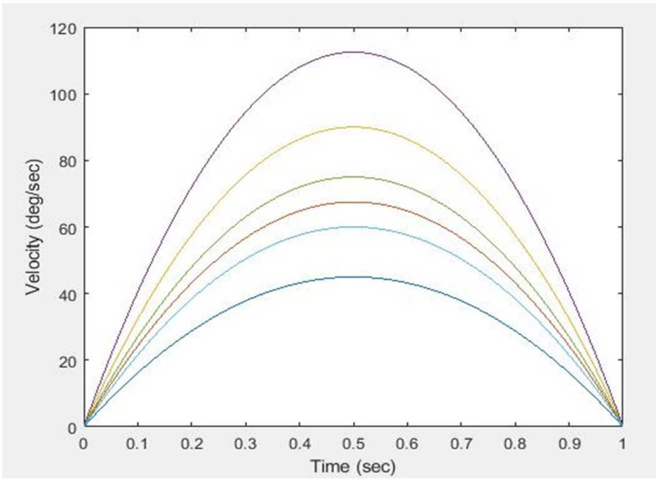
The movement of the robot manipulator is analysed with respect to displacement, velocity, and acceleration. In case of five degree polynomial, an additional constraint of jerk is accommodated. For this simulation the time is taken as one second. The graphs for all the joints for each parameter are put in one and shown below.



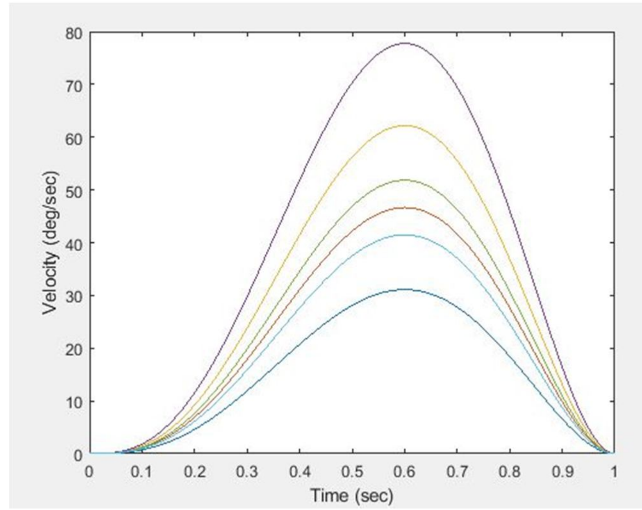
(A) DISPLACEMENT FOR THREE DEGREE POLYNOMIAL



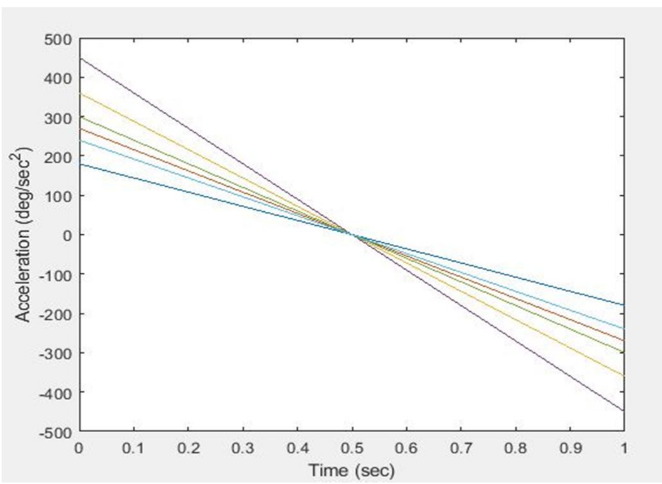
(B) DISPLACEMENT FOR FIVE DEGREE POLYNOMIAL



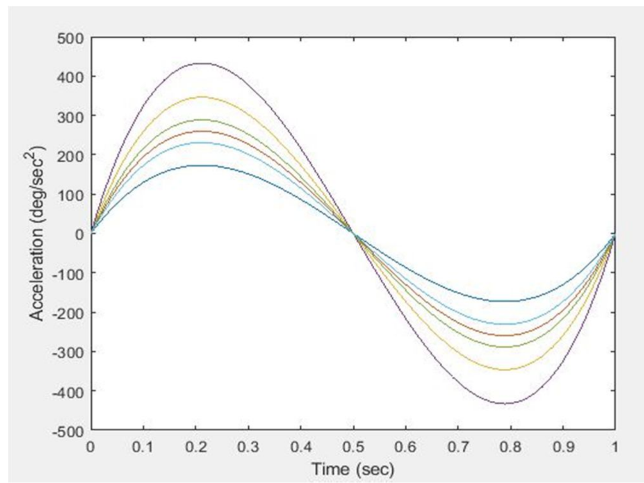
(C) VELOCITY FOR THREE DEGREE POLYNOMIAL



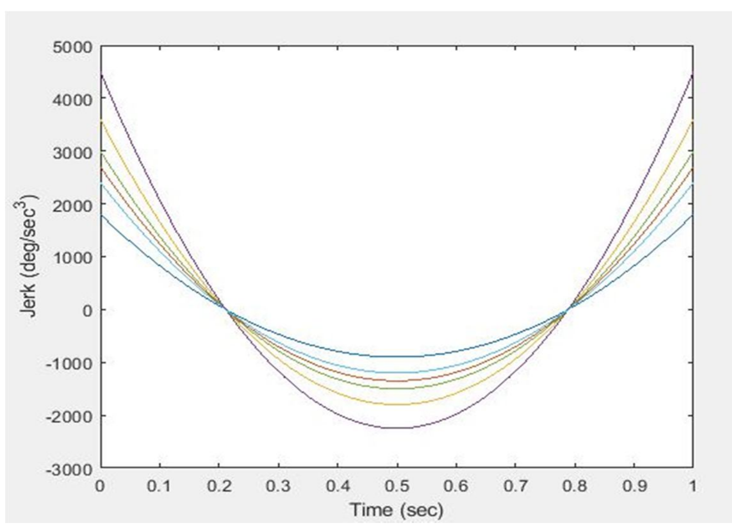
(D) VELOCITY FOR FIVE DEGREE POLYNOMIAL



(E) ACCELERATION FOR THREE DEGREE POLYNOMIAL



(F) ACCELERATION FOR FIVE DEGREE POLYNOMIAL



(G) JERK FOR FIVE DEGREE POLYNOMIAL

FIGURE III. JOINT CURVES FOR BOTH THREE DEGREE AND FIVE DEGREE POLYNOMIAL

The graphs shown in (A),(C) and (E) are curves for three degree polynomial which shows displacement, velocity and acceleration respectively. As taken in constraints the velocity increases from zero and then reaches to maximum and then again decreases to zero at the end. The acceleration is zero at half way from the initial to final position.

The graphs shown in (B),(D),(F) and (G) are curves for five degree polynomial which shows displacement, velocity, acceleration and jerk respectively. As taken in the velocity and acceleration increases from zero and is again zero at final position. The acceleration is zero along the half way as well. It has accommodated an additional constraint of jerk which is zero twice, one at half way of the first half and other at half way of the second half.

VI. CONCLUSIONS

From the study it is clear that both three degree and five degree polynomial are suitable to plan the trajectory of PUMA 560 as they satisfy the constraints taken initially. From the simulation graphs it can be seen that the curves for five degree are more smoother, here smoother means that the parameter like velocity, acceleration does not increase and decrease rapidly at the initial and the final position respectively. For both the case there is no rapid increase or decrease but for five degree polynomial, the increment and decrement are more smoother which gives better and jerk free trajectory. Five degree polynomial also gives the analysis of jerk along the trajectory which cannot be analysed in three degree polynomial. So five degree polynomial is better for trajectory planning. In the future, the dynamic analysis of trajectory can be done by analysing the effect of force on the trajectory.

REFERENCES

[1] Introduction to Robotics: Mechanics and Control, John J. Craig 2005.
[2] Robotics: Modelling, Planning and Control, Bruno Siciliano et al.
[3] Industrial Robotics: Technology Programming and Applications, Mikel P. Groover.
[4] Rajan,V. (1985). Minimum time trajectory planning. IEEE Proceedings of the International Conference on Robotics and Automation, St. Louis, 2, 759-764.
[5] Won, J-H., Choi, B-W & J Chung, M. (1991). Smooth joint trajectory planning for a point-to-point task. IEEE/RSJ International Workshop on Intelligent Robots and Systems IROS 91, Japan, 1299-1303
[6] Cao, B., Dodds, G. I., & Irwin, G. W. (1994). Time optimal and smooth constrained path planning for robot manipulators, Proceedings of the International Conference on Robotics and Automation, Minneapolis, Vol. 43, No.2, pp 1853-1858.
[7] Bobrow, J. E. (1988). Optimal robot path planning using the minimum time criterion. IEEE Transaction of Robotics and Automation, 43, 443-450.
[8] Constantinescu,*EA Croft,"Smooth and Time-Optimal Trajectory Planning for Industrial Manipulators along Specified Paths,"Journal of Robotic Systems ,voI.I7,No.5,2000,pp.233-249.
[9] Hiroaki Ozaki, Ken Hirano, Makoto Iwarnura, Chang-jnn Lin and Tetsuji Shimogawa,"Improvement of trajectory tracking for industrial robot arms by learning control with B-spline", Proc.IEEE Symp.lnternational Symposium on Assembly and Task Planning (lSATP 2003), IEEE Press, 2003,pp.264 - 269.
[10] Amar Khoukhi a* ,Luc Barona ,Marek Balazinskia ,Kudret Demirlib,"A hierarchical neuro-fuzzy system to near optimal-time trajectory planning of redundant manipulators,"Engineering Applications of Artificial Intelligence,voI.21,2008,pp.974-984.
[11] Xiaoping Liao,Weidong Wang, Yizhong Lin ,Changliang Gong, “Time-optimal trajectory planning for a 6R jointed welding robot using adaptive genetic algorithms”, C International Conference on Computer, Mechatronics, Control and Electronic Engineering (CMCE).