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Fractional Calculus of Generalized Mittag-Leffler Function with Jacobi Polynomial

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Abstract: The paper is devoted to the study of generalized fractional calculus of the generalized Mittag-Leffler function $E_{v,\rho}^\delta(z)$ which is an entire function of the form

$$E_{v,\rho}^\delta(z) = \sum_{s=0}^{\infty} \frac{(\delta)_s z^s}{\Gamma(v s + \rho) s!}$$

Where $v > 0$ and $\rho > 0$. For $\delta = 1$, it reduces to Mittag-Leffler function $E_{v,\rho}(z)$. We have shown that the generalized fractional calculus operators transform such function with power multipliers in to generalized Wright function. Some elegant results obtained by Kilbas and Saigo [9], Saxena and Saigo [21] are the special cases of the result derived in this paper.

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I. INTRODUCTION & PRILIMINARIES

The function $E_v(z)$ is defined by the series representation

$$E_v(z) = \sum_{s=0}^{\infty} \frac{z^s}{\Gamma(v s + 1) s!}, v > 0, z \in \mathbb{C} \tag{1.1}$$

Mittag-Leffler [16, 17], Wiman [22, 23], Agarwal [1], Humbert and Agarwal [9], investigated the generalization of the above function $E_v(z)$ in the following manner; see [4, Section 18.1]

$$E_{v,\rho}^\delta(z) = \sum_{s=0}^{\infty} \frac{(\delta)_s z^s}{\Gamma(v n + \rho) s!} \quad v > 0, \rho > 0, z \in \mathbb{C} \tag{1.2}$$

Where \mathbb{C} be the set of complex numbers. For a detailed study of various properties, generalizations and applications of this function we can refer to papers of Dzherashyan [2], Kilbas and Saigo [7, 8, 9 and 10], Kilbas, Saigo and Saxena [13], Gorenflo and Mainardi [6], Gorenflo, Kilbas and Rogosin [4] and Gorenflo, Luchko and Rogosin [5].

A more generalized form of (1.2) is introduced by Prabhakar [18] as:

$$E_{v,\rho}^\delta(z) = \sum_{s=0}^{\infty} \frac{(\delta)_s z^s}{\Gamma(v n + \rho) s!} \tag{1.3}$$

Where $v, \rho, \delta \in \mathbb{C}$ ($Re(v) > 0$) and $E_{v,\rho}^\delta(z)$ is an entire function of order $[Re(v)]^{-1}$, [18, p.7]. For various properties other detail of (1.3), see [12].

The generalized Wright function ${}_p\Psi_q(z)$ defined for $z \in \mathbb{C}$, $a_i, b_j \in \mathbb{C}$ and $\alpha_i, \beta_j \in \mathbb{R}$ ($\alpha_i, \beta_j \neq 0, i = 1, 2, \dots, p, j = 1, 2, \dots, q$) is given by the series by Chaurasia and Pandey [15]

$${}_p\Psi_q(z) = {}_p\Psi_q \left[\left[\begin{matrix} (a_i, \alpha_i)_{(1,p)} \\ (b_j, \beta_j)_{(1,q)} \end{matrix} \middle| z \right] \right] = \sum_{s=0}^{\infty} \frac{\prod_{i=1}^p \Gamma(a_i + \alpha_i s) z^s}{\prod_{j=1}^q \Gamma(b_j + \beta_j s) s!} \tag{1.4}$$

Where \mathbb{C} is the set of complex number and $\Gamma(z)$ is the Euler gamma function [3, section 1.1] and the function (1.4) was introduces by Wright [25] and known as generalized Wright function. Condition for the existence of the generalized Wright function (1.4) together with its representation in terms of Mellins-Barnes integral and in terms of H-function were established in [11].

Some particular cases of generalized Wright function (1.4) were presented in [11, section6]. Wright in [24], [27] investigated by “steepest descent” method, the asymptotic expansion of the function $\Phi(\alpha, \beta, z)$ for the large value of z in the cases $\alpha > 0$ and $-1 < \alpha < 0$, respectively. In [28] Wright indicated the application of the obtained results to the asymptotic theory of partitions. In

[25], [26], [28] Wright extended the last result to the generalized Wright function ${}_p\Psi_q(z)$ for all values of the argument z under the condition

$$\beta_n = \sum_{j=1}^q \beta_j - \sum_{i=1}^p \alpha_i > -1 \tag{1.5}$$

For the detailed study of various properties, generalizations and applications of Wright function and generalized Wright function, we refer to papers of Wright [24, 25, 26, 27 and 28], Luchko [14] and Kilbas [11].

II. FRACTIONAL CALCULUS OPERATORS AND GENERALIZED FRACTIONAL CALCULUS OPERATORS

The left and right-sided Riemann-Liouville fractional calculus operators are defined by Samko, Kilbas and Marichev [20, section 5.1], for $\alpha \in \mathbb{C}, \text{Re}(\alpha) > 0$

$$(I_{0+}^\alpha f)(x) = \frac{1}{\Gamma(\alpha)} \int_0^x \frac{f(t)}{(x-t)^{1-\alpha}} dt; (x > 0) \tag{2.1}$$

$$(I_{0-}^\alpha f)(x) = \frac{1}{\Gamma(\alpha)} \int_x^\infty \frac{f(t)}{(t-x)^{1-\alpha}} dt; (x > 0) \tag{2.2}$$

An interesting and useful generalization of the Riemann-Liouville and Eardly-Kober fractional integral operator has been introduced by Saigo [19] in terms of Gauss hypergeometric function as given below. Let $\alpha, \beta, \gamma \in \mathbb{C}$ and $x \in \mathbb{R}_+$, then the generalized fractional integration operators associated with Gauss hypergeometric function are defined as follows:

$$(I_{0+}^{\alpha, \beta, \gamma} f)(x) = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} {}_2F_1\left(\alpha + \beta, -\gamma; \alpha; 1 - \frac{t}{x}\right) f(t) dt; (R(\alpha) > 0) \tag{2.3}$$

$$(I_{0-}^{\alpha, \beta, \gamma} f)(x) = \frac{1}{\Gamma(\alpha)} \int_x^\infty (x-t)^{\alpha-1} t^{-\alpha-\beta} {}_2F_1\left(\alpha + \beta, -\gamma; \alpha; 1 - \frac{t}{x}\right) f(t) dt; (R(\alpha) > 0) \tag{2.4}$$

Lemma 1. Let $\alpha, \beta, \gamma \in \mathbb{C}, (R(\alpha) > 0)$ and $\rho \in \mathbb{C}$

(a) If $R(\rho) > \max[0, \text{Re}(\beta - \gamma)]$ then

$$(I_{0+}^{\alpha, \beta, \gamma} f)(x) = \frac{\Gamma(\rho)\Gamma(\rho-\beta+\gamma)}{\Gamma(\rho-\beta)\Gamma(\rho+\alpha+\beta+\gamma)} x^{\rho-\beta-1} \tag{2.5}$$

(b) If $R(\rho) > \max[\text{Re}(-\beta), \text{Re}(-\gamma)]$ then

$$(I_{0-}^{\alpha, \beta, \gamma} f)(x) = \frac{\Gamma(\rho+\beta)\Gamma(\rho+\gamma)}{\Gamma(\rho)\Gamma(\rho+\alpha+\beta+\gamma)} x^{\rho-\beta} \tag{2.6}$$

III. LEFT-SIDED GENERALIZED FRACTIONAL INTEGRATION OF THE MITTAG-LEFFLER FUNCTION USING JACOBI POLYNOMIAL

In this section we consider the left-sided generalized fractional integral formula of the generalized Mittag-Leffler function using Jacobi Polynomial.

We are introducing here the Jacobi Polynomial which is defined via the hypergeometric function as

$$P_n^{(\alpha, \beta)}(z) = \frac{(\alpha+1)_n}{n!} {}_2F_1\left[-n, \alpha + \beta + n + 1; \alpha + 1; \frac{1}{2}(1-z)\right] \tag{3.1}$$

Where $(\alpha + 1)_n$ is Pochhammer's symbol, on taking $z = \frac{t}{x}$ and solved

We are replacing the Gauss hypergeometric function by the Jacobi Polynomial in equation (2.3) it becomes

$$\left(I_{0+}^{\alpha+1, -\alpha-1-n, -(\alpha+\beta+n+1)} \frac{(\alpha+1)_n 2^n}{n!} f \right)(x) = \frac{(\alpha+1)_n 2^n}{n!} \frac{x^n}{\Gamma(\alpha+1)} \int_0^x (x-t)^{\alpha+1-1} {}_2F_1\left[-n, \alpha + \beta + n + 1; \alpha + 1; \frac{1}{2}\left(1 - \frac{t}{x}\right)\right] f(t) dt \tag{3.2}$$

Theorem 1. Let $\alpha, \beta, \gamma, \rho, \delta \in \mathbb{C}$ be complex numbers such that $R(\alpha) > 0, R(\rho - \beta - n) > 0, v > 0$ and $a \in \mathbb{R}$. If the condition (1.5) satisfied and $I_{0+}^{\alpha, \beta, \gamma}$ by equation (2.3) be the left-side operator of the generalized fractional integration associated with Jacobi Polynomial, then there holds the following relationship

$$\begin{aligned}
 & \left(I_{0+}^{\alpha+1, -\alpha-1-n, (\alpha+\beta+n+1)} \left(\frac{(\alpha+1)_n}{n!} 2^n t^{\rho-1} E_{\nu, \rho}^\delta [at^\nu] \right) \right) (x) \\
 &= \frac{x^{n+\alpha+\rho} (\alpha+1)_n}{n! \Gamma(\delta)} {}_2\Psi_2 \left\{ \begin{matrix} \left(\frac{\rho}{2} - \frac{\beta}{2} - \frac{n}{2} + 1, \frac{\nu}{2} \right) & (\delta, 1) \\ (\alpha + \rho - n + 1, \nu) & \left(\frac{\rho}{2} - \frac{\beta}{2} + \frac{n}{2} + 1, \frac{\nu}{2} \right) \end{matrix} \middle| ax^\nu \right\} \quad (3.3)
 \end{aligned}$$

Provided each member of the equation (3.3) exists.

Proof. By using the definition of generalized Mittag-Leffler function (1.3) and fractional integral formula (2.3), we have

$$\begin{aligned}
 \Omega &= I_{0,+}^{\alpha+1, -\alpha-1-n, (\alpha+\beta+n+1)} \left(\frac{(\alpha+1)_n}{n!} 2^n t^{\rho-1} E_{\nu, \rho}^\delta [at^\nu] \right) (x) \\
 &= \frac{(\alpha+1)_n}{n!} 2^n \frac{x^n}{\Gamma(\alpha+1)} \int_0^\infty (x-t)^{\alpha+1-1} {}_2F_1 \left[-n, \alpha + \beta + n + 1; \alpha + 1; \frac{1}{2} \left(1 - \frac{t}{x} \right) \right] (t^{\rho-1} E_{\nu, \rho}^\delta [at^\nu]) dt
 \end{aligned}$$

By the use of Jacobi Polynomial $P_n^{(\alpha, \beta)}(z)$ (3.1), series form of generalized Mittag-Leffler function (1.3), interchanging the order of integration and summation and evaluating the inner integral by the use of known formula of Beta integral. Finally by the virtue of Gauss summation theorem, we have,

$$\Omega = \frac{x^{n+\alpha+\rho} (\alpha+1)_n}{n! \Gamma(\delta)} \sum_{s=0}^{\infty} \frac{\Gamma(\delta+s) \Gamma\left(\frac{\rho}{2} - \frac{\beta}{2} - \frac{n}{2} + 1 + \frac{\nu}{2}s\right)}{\Gamma(\alpha + \rho - n + 1 + \nu s) \Gamma\left(\frac{\rho}{2} - \frac{\beta}{2} + \frac{n}{2} + 1 + \frac{\nu}{2}s\right)} \frac{(ax^\nu)^s}{s!}$$

or

$$\Omega = \frac{x^{n+\alpha+\rho} (\alpha+1)_n}{n! \Gamma(\delta)} {}_2\Psi_2 \left[\left(\begin{matrix} \left(\frac{\rho}{2} - \frac{\beta}{2} - \frac{n}{2} + 1, \frac{\nu}{2} \right) & (\delta, 1) \\ (\alpha + \rho - n + 1, \nu) & \left(\frac{\rho}{2} - \frac{\beta}{2} + \frac{n}{2} + 1, \frac{\nu}{2} \right) \end{matrix} \middle| ax^\nu \right) \right]$$

Interchanging the order of integration and summations, which is permissible under the conditions, stated with the theorem due to convergence of the integrals involved in the process. This completes the proof of the theorem.

Corollary 1: For $Re(\alpha + 1) > 0, Re(\rho - \beta - n) > 0, \vartheta > 0$ and $a \in R$. If the condition (1.5) is satisfied, then there holds the formula

$$\begin{aligned}
 & \left(I_{0+}^{\alpha+1, -\alpha-1-n, (\alpha+\beta+n+1)} \left(\frac{(\alpha+1)_n}{n!} 2^n t^{\rho-1} E_{\nu, \rho}^\delta [at^\nu] \right) \right) (x) \\
 &= \frac{x^{n+\alpha+\rho} (\alpha+1)_n}{n!} {}_2\Psi_2 \left\{ \begin{matrix} \left(\frac{\rho}{2} - \frac{\beta}{2} - \frac{n}{2} + 1, \frac{\nu}{2} \right) & (1, 1) \\ (\alpha + \rho - n + 1, \nu) & \left(\frac{\rho}{2} - \frac{\beta}{2} + \frac{n}{2} + 1, \frac{\nu}{2} \right) \end{matrix} \middle| ax^\nu \right\} \quad (3.4)
 \end{aligned}$$

provided that each member of equation (3.4) makes sense.

IV. RIGHT-SIDED GENERALIZED FRACTIONAL INTEGRATION OF THE MITTAG-LEFFLER FUNCTION USING JACOBI POLYNOMIAL

In this section we have discussed the right-sided generalized fractional integral formula of the generalized Mittag-Leffler function using Jacobi Polynomial.

Theorem 2. Let $\alpha, \beta, \gamma, \rho, \delta \in C$ be complex numbers such that $R(\alpha) > 0, R(\alpha + \rho) > \max[-R(\beta), -R(\gamma)]$, with the condition $R(\beta) \neq R(\gamma), \nu > 0$ and $a \in R$. If the condition (1.5) satisfied and $I_{0,-}^{\alpha, \beta, \gamma}$ be the Right-side operator of the generalized fractional integration associated with Jacobi Polynomial, then there holds the following relationship

$$\begin{aligned}
 & \left(I_{0,-}^{\alpha+1, -\alpha-1-n, (\alpha+\beta+n+1)} \left(\frac{(\alpha+1)_n}{n!} 2^n t^{-\alpha-1-\rho} E_{\nu, \rho}^\delta [at^{-\nu}] \right) \right) (x) \\
 &= \frac{(\alpha+1)_n}{n!} \frac{x^{n-\rho}}{\Gamma(\delta)} {}_3\Psi_3 \left[\left(\begin{matrix} \left(\frac{\rho}{2} - \frac{\beta}{2} - n + 1, \frac{\nu}{2} \right) & (\rho - n, \nu) & (\delta, 1) \\ (\alpha + \rho - 2n + 1, \nu) & \left(\frac{\rho}{2} - \frac{\beta}{2} + 1, \frac{\nu}{2} \right) & (\rho, \nu) \end{matrix} \middle| ax^{-\nu} \right) \right] \quad (4.1)
 \end{aligned}$$

Provided both the sides of (4.1) exists.

Proof. By using the definition of generalized Mittag-Leffler function (1.3) and generalized fractional integral formula (2.4), we have

$$\begin{aligned}
 \Lambda &= (I_{0,-}^{\alpha+1,-\alpha-1-n,-(\alpha+\beta+n+1)}) \left(\frac{(\alpha+1)_n}{n!} 2^n t^{-\alpha-1-\rho} E_{\nu,\rho}^\delta [at^{-\nu}] \right) (x) \\
 &= \frac{(\alpha+1)_n}{n!} 2^n \frac{x^n}{\Gamma(\alpha+1)} \int_0^\infty (t-x)^{\alpha+1-1} {}_2F_1 \left[-n, \alpha + \beta + n + 1; \alpha + 1; \frac{1}{2} \left(1 - \frac{t}{x} \right) \right] (t^{-\alpha-\rho-1} E_{\nu,\rho}^\delta [at^{-\nu}]) dt \\
 &= \frac{(\alpha+1)_n}{n!} \frac{x^{n-\rho}}{\Gamma(\delta)} \sum_{s=0}^\infty \frac{\Gamma\left(\frac{\rho-\beta}{2}-n+1+\frac{vs}{2}\right)\Gamma((\rho-n+vs))\Gamma(\delta+s)}{\Gamma(\alpha+\rho-2n+1+vs)\Gamma\left(\frac{\rho-\beta}{2}+1+\frac{vs}{2}\right)\Gamma(\rho+vs)} \frac{(ax^{-\nu})^s}{s!} \\
 &= \frac{(\alpha+1)_n}{n!} \frac{x^{n-\rho}}{\Gamma(\delta)} {}_3\Psi_3 \left[\left(\begin{matrix} \left(\frac{\rho-\beta}{2}-n+1, \frac{\nu}{2}\right) & (\rho-n, \nu) & (\delta, 1) \\ (\alpha+\rho-2n+1, \nu) & \left(\frac{\rho-\beta}{2}+1, \frac{\nu}{2}\right) & (\rho, \nu) \end{matrix} \middle| ax^{-\nu} \right) \right] \quad (4.2)
 \end{aligned}$$

Provided each member of the equation (4.2) makes sense.

Corollary 2 : For $Re(\alpha + 1) > 0, Re(\rho + \beta + n) > 0, \vartheta > 0$ and $a \in R$. If the condition (1.5) is satisfied, then there holds the formula

$$\begin{aligned}
 &(I_{0,-}^{\alpha+1,-\alpha-1-n,-(\alpha+\beta+n+1)}) \left(\frac{(\alpha + 1)_n}{n!} 2^n t^{-\alpha-1-\rho} E_{\nu,\rho}^\delta [at^{-\nu}] \right) (x) \\
 &= \frac{(\alpha+1)_n}{n!} x^{n-\rho} {}_3\Psi_3 \left[\left(\begin{matrix} \left(\frac{\rho-\beta}{2}-n+1, \frac{\nu}{2}\right) & (\rho-n, \nu) & (1, 1) \\ (\alpha+\rho-2n+1, \nu) & \left(\frac{\rho-\beta}{2}+1, \frac{\nu}{2}\right) & (\rho, \nu) \end{matrix} \middle| ax^{-\nu} \right) \right] \quad (4.3)
 \end{aligned}$$

of equation (4.3) makes sense.

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