iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 **Issue:** IX **Month of publication:** September 2018

DOI:

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 6 Issue IX, Sep 2018- Available at www.ijraset.com

Fractional Calculus of Generalized Mittag-Leffeler Function with Jacobi Polynomial

Aakankasha Vyas[1], Dr. P. K. Mishra[2]
[1]*Department of Mathematics, Mewar University, Chittorgarh Rajsathan*
[2]*Department of Mathematics, Sobhasaria Group of Institutions, Sikar*

Abstract: The paper is devoted to the study of generalized fractional calculus of the generalized Mittag-Leffler function $E^{\delta}{}_{\upsilon,\rho}(z)$ which is an entire function of the form

$$E^{\delta}{}_{\upsilon,\rho}(z) = \sum_{s=0}^{\infty} \frac{(\delta)_s z^s}{\Gamma(\upsilon s+\rho)\, s!}$$

Where $\upsilon > 0$ and $\rho > 0$. For $\delta = 1$, it is reduces to Mittag-Leffler function $E_{\upsilon,\rho}(z)$. We have shown that the generalized fractional calculus operators transform such function with power multipliers in to generalized Wright function. Some elegant results obtained by Kilbas and Saigo [9], Saxena and Saigo [21] are the special cases of the result derived in this paper.

2010 Mathematics Subject Classifications: 26A33, 33E12, Secondary 33C45.
Keywords: Generalized Mittag-Leffler function, Generalized Fractional Calculus, Generalized Wright function, Jacoby Polynomial.

I. INTRODUCTION & PRILIMINARIES

The function $E_{\upsilon}(z)$ is defined by the series representation

$$E_{\upsilon}(z) = \sum_{s=0}^{\infty} \frac{z^s}{\Gamma(\upsilon s+1)\, s!}, \upsilon > 0, z \in C \tag{1.1}$$

Mittag-Leffler [16, 17], Wiman [22, 23], Agarwal [1], Humbert and Agarwal [9], investigated the generalization of the above function $E_{\upsilon}(z)$ in the following manner; see [4, Section 18.1]

$$E^{\delta}{}_{\upsilon,\rho}(z) = \sum_{s=0}^{\infty} \frac{(\delta)_s z^s}{\Gamma(\upsilon n+\rho)\, s!} \quad \upsilon > 0, \rho > 0, z \in C \tag{1.2}$$

Where C be the set of complex numbers. For a detailed study of various properties, generalizations and applications of this function we can refer to papers of Dzherashyan [2], Kilbas and Saigo [7, 8, 9 and 10], Kilbas, Saigo and Saxena [13], Gorenflo and Mainardi [6], Gorenflo, Kilbas and Rogosin [4] and Gorenflo, Luchko and Rogosin [5].

A more generalized form of (1.2) is introduced by Prabhakar [18] as:

$$E^{\delta}{}_{\upsilon,\rho}(z) = \sum_{s=0}^{\infty} \frac{(\delta)_s z^s}{\Gamma(\upsilon n+\rho)\, s!} \tag{1.3}$$

Where $\upsilon, \rho, \delta \in C$ $(Re(\upsilon) > 0)$ and $E^{\delta}{}_{\upsilon,\rho}(z)$ is an entire function of order $[Re(\upsilon)]^{-1}$, [18, p.7]. For various properties other detail of (1.3), see [12].

The generalized Wright function ${}_p\Psi_q\,(z)$defined for $z \in C, a_i, b_j \in C$ and $\alpha_i, \beta_j \in R(\alpha_i, \beta_j \neq 0, i = 1,2, \ldots \ldots, p, j = 1,2, \ldots \ldots, q)$ is given by the series by Chaursia and Pandey [15]

$${}_p\Psi_q\,(z) = {}_p\Psi_q\left[\left(\begin{matrix}(a_i,\alpha_i)_{(1.p)} \\ (b_j,\beta_j)_{(1.q)}\end{matrix}\middle| z\right)\right] = \sum_{s=0}^{\infty} \frac{\prod_{i=1}^{p}\Gamma(a_i+\alpha_i s)\, z^s}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j s)\, s!} \tag{1.4}$$

Where C is the set of complex number and $\Gamma(z)$ is the Euler gamma function [3, section 1.1] and the function (1.4) was introduces by Wright [25] and known as generalized Wright function. Condition for the existence of the generalized Wright function (1.4) together with its representation in terms of Mellins-Barnes integral and in terms of H-function were established in [11].

Some particular cases of generalized Wright function (1.4) were presented in [11, section6]. Wright in [24], [27] investigated by "steepest descent" method, the asymptotic expansion of the function $\Phi(\alpha, \beta, z)$ for the large value of $z$in the cases $\alpha > 0$ and $-1 < \alpha < 0$, respectively. In [28] Wright indicated the application of the obtained results to the asymptotic theory of partitions. In

[25], [26], [28] Wright extended the last result to the generalized Wright function ${}_p\Psi_q\,(z)$ for all values of the argument z under the condition

$$\beta_n = \sum_{j=1}^{q} \beta_j - \sum_{i=1}^{p} \alpha_i > -1 \tag{1.5}$$

For the detailed study of various properties, generalizations and applications of Wright function and generalized Wright function, we refer to papers of Wright [24, 25, 26, 27 and 28], Luchko [14] and Kilbas [11].

II. FRACTIONAL CALCULUS OPERATORS AND GENERALIZED FRACTIONAL CALCULUS OPERATORS

The left and right-sided Riemann-Liouville fractional calculus operators are defined by Samko, Kilbas and Marichev [20, section5.1], for $\alpha \in C, Re(\alpha) > 0$

$$(I_{0+}^{\alpha} f)(x) = \frac{1}{\Gamma(\alpha)} \int_0^x \frac{f(t)}{(x-t)^{1-\alpha}}\, dt\,; (x > 0) \tag{2.1}$$

$$(I_{0-}^{\alpha} f)(x) = \frac{1}{\Gamma(\alpha)} \int_x^\infty \frac{f(t)}{(t-x)^{1-\alpha}}\, dt\,; (x > 0) \tag{2.2}$$

An interesting and useful generalization of the Riemann-Liouville and Eardly-Kober fractional integral operator has been introduced by Saigo [19] in terms of Gauss hypergeometric function as given below. Let $\alpha, \beta, \gamma \in C$ and $x \in R_+$, then the generalized fractional integration operators associated with Gauss hypergeometric function are defined as follows:

$$(I_{0+}^{\alpha,\beta,\gamma} f)(x) = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1}\, {}_2F_1\left(\alpha+\beta, -\gamma\,; \alpha;\, 1-\frac{t}{x}\right) f(t)\, dt\,; (R(\alpha) > 0) \tag{2.3}$$

$$(I_{-}^{\alpha,\beta,\gamma} f)(x) = \frac{1}{\Gamma(\alpha)} \int_x^\infty (x-t)^{\alpha-1} t^{-\alpha-\beta}\, {}_2F_1\left(\alpha+\beta, -\gamma\,; \alpha;\, 1-\frac{t}{x}\right) f(t)\, dt\,; (R(\alpha) > 0) \tag{2.4}$$

Lemma 1. Let $\alpha, \beta, \gamma \in C, (R(\alpha) > 0)$ and $\rho \in C$

(a) If $R(\rho) > \max[0, Re(\beta-\gamma)]$ then

$$(I_{0+}^{\alpha,\beta,\gamma} f)(x) = \frac{\Gamma(\rho)\Gamma(\rho-\beta+\gamma)}{\Gamma(\rho-\beta)\,\Gamma(\rho+\alpha+\beta+\gamma)}\, x^{\rho-\beta-1} \tag{2.5}$$

(b) If $R(\rho) > \max[Re(-\beta), Re(-\gamma)]$ then

$$(I_{-}^{\alpha,\beta,\gamma} f)(x) = \frac{\Gamma(\rho+\beta)\Gamma(\rho+\gamma)}{\Gamma(\rho)\,\Gamma(\rho+\alpha+\beta+\gamma)}\, x^{\rho-\beta} \tag{2.6}$$

III. LEFT-SIDED GENERALIZED FRACTIONAL INTEGRATION OF THE MITTAG-LEFFLER FUNCTION USING JACOBI POLYNOMIAL

In this section we consider the left-sided generalized fractional integral formula of the generalized Mittag-Leffler function using Jacobi Polynomial.

We are introducing here the Jacobi Polynomial which is defined via the hypergeometric function as

$$P_n^{(\alpha,\beta)}(z) = \frac{(\alpha+1)_n}{n!}\, {}_2F_1\left[-n, \alpha+\beta+n+1; \alpha+1; \frac{1}{2}(1-z)\right] \tag{3.1}$$

Where $(\alpha+1)_n$ is Pochhammer's symbol, on taking $z = \frac{t}{x}$ and solved

We are replacing the Gauss hypergeometric function by the Jacobi Polynomial in equation (2.3) it becomes

$$\left(I_{0+}^{\alpha+1,-\alpha-1-n,-(\alpha+\beta+n+1)} \frac{(\alpha+1)_n\, 2^n}{n!} f\right)(x) =$$

$$\frac{(\alpha+1)_n\, 2^n}{n!} \frac{x^n}{\Gamma(\alpha+1)} \int_0^x (x-t)^{\alpha+1-1}\, {}_2F_1\left[-n, \alpha+\beta+n+1; \alpha+1; \frac{1}{2}\left(1-\frac{t}{x}\right)\right] f(t)\, dt \tag{3.2}$$

Theorem 1. Let $\alpha, \beta, \gamma, \rho, \delta \in C$ be complex numbers such that $R(\alpha) > 0, R(\rho-\beta-n) > 0, \upsilon > 0$ and $a \in R$. If the condition (1.5) satisfied and $I_{0+}^{\alpha,\beta,\gamma}$ by equation (2.3) be the left-side operator of the generalized fractional integration associated with Jacobi Polynomial, then there holds the following relationship

$$\left(I_{0+}^{\alpha+1,-\alpha-1-n,(\alpha+\beta+n+1)}\left(\frac{(\alpha+1)_n}{n!}2^n t^{\rho-1}E_{\upsilon,\rho}^{\delta}[at^{\upsilon}]\right)\right)(x)$$

$$= \frac{x^{n+\alpha+\rho}(\alpha+1)_n}{n!\Gamma(\delta)}\ {}_2\psi_2\left\{\begin{matrix}\left(\frac{\rho}{2}-\frac{\beta}{2}-\frac{n}{2}+1,\frac{\upsilon}{2}\right) & (\delta,1)\\ (\alpha+\rho-n+1,\upsilon) & \left(\frac{\rho}{2}-\frac{\beta}{2}+\frac{n}{2}+1,\frac{\upsilon}{2}\right)\end{matrix}\middle| ax^{\upsilon}\right\} \quad (3.3)$$

Provided each member of the equation (3.3) exists.

Proof. By using the definition of generalized Mittag-Leffler function (1.3) and fractional integral formula (2.3), we have

$$\Omega = I_{0,+}^{\alpha+1,-\alpha-1-n,-(\alpha+\beta+n+1)}\left(\frac{(\alpha+1)_n}{n!}2^n t^{\rho-1}E_{\upsilon,\rho}^{\delta}[at^{\upsilon}]\right)(x)$$

$$= \frac{(\alpha+1)_n}{n!}2^n\frac{x^n}{\Gamma(\alpha+1)}\int_0^{\infty}(x-t)^{\alpha+1-1}\ {}_2F_1\left[-n,\alpha+\beta+n+1;\alpha+1;\frac{1}{2}\left(1-\frac{t}{x}\right)\right]\left(t^{\rho-1}E_{\upsilon,\rho}^{\delta}[at^{\upsilon}]\right)dt$$

By the use of Jacobi Polynomial $P_n^{(\alpha,\beta)}(z)$ (3.1), series form of generalized Mittag-Leffler function (1.3), interchanging the order of integration and summation and evaluating the inner integral by the use of known formula of Beta integral. Finally by the virtue of Gauss summation theorem, we have,

$$\Omega = \frac{x^{n+\alpha+\rho}(\alpha+1)_n}{n!\,\Gamma(\delta)}\sum_{s=0}^{\infty}\frac{\Gamma(\delta+s)\Gamma\left(\frac{\rho}{2}-\frac{\beta}{2}-\frac{n}{2}+1+\frac{\upsilon}{2}s\right)}{\Gamma(\alpha+\rho-n+1+\upsilon s)\Gamma\left(\frac{\rho}{2}-\frac{\beta}{2}+\frac{n}{2}+1+\frac{\upsilon}{2}s\right)}\frac{(ax^{\upsilon})^s}{s!}$$

or

$$\Omega = \frac{x^{n+\alpha+\rho}(\alpha+1)_n}{n!\Gamma(\delta)}\ {}_2\psi_2\left[\left(\begin{matrix}\left(\frac{\rho}{2}-\frac{\beta}{2}-\frac{n}{2}+1,\frac{\upsilon}{2}\right) & (\delta,1)\\ (\alpha+\rho-n+1,\upsilon) & \left(\frac{\rho}{2}-\frac{\beta}{2}+\frac{n}{2}+1,\frac{\upsilon}{2}\right)\end{matrix}\middle| ax^{\upsilon}\right)\right]$$

Interchanging the order of integration and summations, which is permissible under the conditions, stated with the theorem due to convergence of the integrals involved in the process. This completes the proof of the theorem.

Corollary 1: For $Re(\alpha+1)>0, Re(\rho-\beta-n)>0, \vartheta>0$ *and* $a\in R$. If the condition (1.5) is satisfied, then there holds the formula

$$\left(I_{0+}^{\alpha+1,-\alpha-1-n,(\alpha+\beta+n+1)}\left(\frac{(\alpha+1)_n}{n!}2^n t^{\rho-1}E_{\upsilon,\rho}[at^{\upsilon}]\right)\right)(x)$$

$$= \frac{x^{n+\alpha+\rho}(\alpha+1)_n}{n!}\ {}_2\psi_2\left\{\begin{matrix}\left(\frac{\rho}{2}-\frac{\beta}{2}-\frac{n}{2}+1,\frac{\upsilon}{2}\right) & (1,1)\\ (\alpha+\rho-n+1,\upsilon) & \left(\frac{\rho}{2}-\frac{\beta}{2}+\frac{n}{2}+1,\frac{\upsilon}{2}\right)\end{matrix}\middle| ax^{\upsilon}\right\} \quad (3.4)$$

provided that each member of equation (3.4) makes sense.

IV. RIGHT-SIDED GENERALIZED FRACTIONAL INTEGRATION OF THE MITTAG-LEFFLER FUNCTION USING JACOBI POLYNOMIAL

In this section we have discussed the right-sided generalized fractional integral formula of the generalized Mittag-Leffler function using Jacobi Polynomial.

Theorem 2. Let $\alpha,\beta,\gamma,\rho,\delta\in C$ be complex numbers such that $R(\alpha)>0$, $R(\alpha+\rho)>\max[-R(\beta),-R(\gamma)]$, with the condition R($\beta$)$\neq R(\gamma), \upsilon>0$ and $a\in R$. If the condition (1.5) satisfied and $I_{0-}^{\alpha,\beta,\gamma}$ be the Right-side operator of the generalized fractional integration associated with Jacobi Polynomial, then there holds the following relationship

$$(I_{0,-}^{\alpha+1,-\alpha-1-n,-(\alpha+\beta+n+1)}\left(\frac{(\alpha+1)_n}{n!}2^n t^{-\alpha-1-\rho}E_{\upsilon,\rho}^{\delta}[at^{-\upsilon}]\right)(x)$$

$$= \frac{(\alpha+1)_n}{n!}\frac{x^{n-\rho}}{\Gamma(\delta)}\ {}_3\Psi_3\left[\left(\begin{matrix}\left(\frac{\rho}{2}-\frac{\beta}{2}-n+1,\frac{\upsilon}{2}\right) & (\rho-n,\upsilon) & (\delta,1)\\ (\alpha+\rho-2n+1,\upsilon) & \left(\frac{\rho}{2}-\frac{\beta}{2}+1,\frac{\upsilon}{2}\right) & (\rho,\upsilon)\end{matrix}\middle| ax^{-\upsilon}\right)\right] \quad (4.1)$$

Provided both the sides of (4.1) exists.

Proof. By using the definition of generalized Mittag-Leffler function (1.3) and generalized fractional integral formula (2.4), we have

$$\Lambda=(I_{0,-}^{\alpha+1,-\alpha-1-n,-(\alpha+\beta+n+1)}\left(\frac{(\alpha+1)_n}{n!}2^n t^{-\alpha-1-\rho}E_{\upsilon,\rho}^{\delta}[at^{-\upsilon}]\right)(x))$$

$$=\frac{(\alpha+1)_n}{n!}2^n\frac{x^n}{\Gamma(\alpha+1)}\int_0^\infty(t-x)^{\alpha+1-1}\,{}_2F_1\left[-n,\alpha+\beta+n+1;\alpha+1;\frac{1}{2}\left(1-\frac{t}{x}\right)\right]\left(t^{-\alpha-\rho-1}E_{\upsilon,\rho}^{\delta}[at^{-\upsilon}]\right)dt$$

$$=\frac{(\alpha+1)_n}{n!}\frac{x^{n-\rho}}{\Gamma(\delta)}\sum_{s=0}^{\infty}\frac{\Gamma\left(\frac{\rho}{2}-\frac{\beta}{2}-n+1+\frac{\upsilon s}{2}\right)\Gamma((\rho-n+\upsilon s))\Gamma(\delta+s)}{\Gamma(\alpha+\rho-2n+1+\upsilon s)\Gamma\left(\frac{\rho}{2}-\frac{\beta}{2}+1+\frac{\upsilon s}{2}\right)\Gamma(\rho+\upsilon s)}\frac{(ax^{-\upsilon})}{s!}$$

$$=\frac{(\alpha+1)_n}{n!}\frac{x^{n-\rho}}{\Gamma(\delta)}\,{}_3\Psi_3\left[\left(\left.\begin{matrix}\left(\frac{\rho}{2}-\frac{\beta}{2}-n+1,\frac{\upsilon}{2}\right) & (\rho-n,\upsilon) & (\delta,1)\\ (\alpha+\rho-2n+1,\upsilon) & \left(\frac{\rho}{2}-\frac{\beta}{2}+1,\frac{\upsilon}{2}\right) & (\rho,\upsilon)\end{matrix}\right|ax^{-\upsilon}\right)\right] \tag{4.2}$$

Provided each member of the equation (4.2) makes sense.

Corollary 2 : For $Re(\alpha+1)>0, Re(\rho+\beta+n)>0, \vartheta>0$ and $a\in R$. If the condition (1.5) is satisfied, then there holds the formula

$$(I_{0,-}^{\alpha+1,-\alpha-1-n,-(\alpha+\beta+n+1)}\left(\frac{(\alpha+1)_n}{n!}2^n t^{-\alpha-1-\rho}E_{\upsilon,\rho}[at^{-\upsilon}]\right)(x)$$

$$=\frac{(\alpha+1)_n}{n!}x^{n-\rho}\,{}_3\Psi_3\left[\left(\left.\begin{matrix}\left(\frac{\rho}{2}-\frac{\beta}{2}-n+1,\frac{\upsilon}{2}\right) & (\rho-n,\upsilon) & (1,1)\\ (\alpha+\rho-2n+1,\upsilon) & \left(\frac{\rho}{2}-\frac{\beta}{2}+1,\frac{\upsilon}{2}\right) & (\rho,\upsilon)\end{matrix}\right|ax^{-\upsilon}\right)\right] \tag{4.3}$$

provided that each member of equation (4.3) makes sense.

REFERENCES

[1] R.P. Agarwal, A propos d'une Note M. Pierre Humbert. C. R. Acad. Sci. Paris 236(1953), 2031-2032.
[2] M. M. Dzherbashyan, Integral Transforms and Representations of Functions in Complex Domain (Russian). Nauka, Moscow (1966).
[3] A. Erdlyi, W. Magnus, F. Oberhettinger and F.G. Tricomi, Higher Transcendental Functions, Vol. I. McGraw-Hill, New York-Toronto-London (1953).
[4] A. Erdlyi, W. Magnus, F. Oberhettinger and F.G. Tricomi, Higher Transcendental Functions, Vol. III. McGraw-Hill, New York (1955).
[5] R. Gorenflo, A.A. Kilbas and S.V. Rogosin, On the generalized Mittag-Leffler type function. Integral Transforms Spec. Funct. 7 (1998), 215-224.
[6] R. Gorenflo, Yu. Luchko and S.V. Rogosin, Mittag-Leffler type functions, notes on growth properties and distribution of zeros. Preprint No. A 04-97, Freie Universitt Berlin, Serie A Mathematik, Berlin (1997).
[7] R. Gorenflo and F. Mainardi, The Mittag-Leffler type function in the Riemann-Liouville fractional calculus. In: Boundary Value Problems, Special Functions and Fractional Calculus (Proc. Int. Conf. Minsk1996) Belarusian State Univ., Minsk (1996), 215-225.
[8] P. Humbert and R. P. Agarwal, Sur la function de Mittag-Leffler et quelquesunes de ses generalizations. Bull. Sci. Math. (2) 77 (1953), 180-185.
[9] A. A. Kilbas and M. Saigo, On solution of integral equations of Abel-Volterra type. Differential and Integral Equations 8 (1995), 993-1011.
[10] A.A. Kilbas and M. Saigo, Fractional integrals and derivatives of Mittag-Leffler function (Russian). Dokl. Akad. Nauk Belarusi 39, No 4(1995), 22-26.
[11] A. A. Kilbas and M. Saigo, On Mittag-Leffler type function, fractional calculus operators and solutions of integral equations. Integral Transform. Spec. Function. 4 (1996), 355-370.
[12] A.A. Kilbas and M. Saigo, Solution in closed form of a class of linear Differential equation of fractional order (Russian). Differentsialnye Uravnenija 33 (1997), 195-204; Translation in: Differential Equations 33 (1997), 194-204.
[13] A. A. Kilbas, M. Saigo and J.J. Trujillo, On the generalized Wright function. Fract. Calc. Appl. Anal. Vol. 5, No 4 (2002), 437-460.
[14] A. A. Kilbas, M. Saigo and R.K. Saxena, Generalized Mittag-Leffler function and generalized fractional calculus operators. Integral Transform. Spec. Funct. 15 (2004), 31-49.
[15] A.A. Kilbas, M. Saigo and R.K. Saxena, Solution of Volterra integro-differential equations with generalized Mittag-Leffler function in the kernels. J. Integral Eq. Appl.14 (2002), 377-396.
[16] Yu.F. Luchko, Asymptotics of zeros of the Wright function. Z. Anal. Anwendungen 19, No 2 (2000), 583-595.
[17] V. B. L. Chaurasa, S. C. Pandey, On the fractional calculus of generalized Mittag-Leffler function Scientia, Series A: mathematical Science, Vol. 20 (2010), 113-122.
[18] A. M. Mathai and R. K. Saxena, The H-Function with Applications in Statistics and Other Disciplines. John Wiley and Sons, New York-London-Sydney (1978).
[19] G.M. Mittag-Leffler, Sur la nouvelle fonction E_(x), C. R. Acad. Sci. Paris,137 (1903), 554-558.
[20] G.M. Mittag-Leffler, Sur la representation analytiqie d'une function monogene (cinquieme note). Acta Math. 29(1905), 101-181.
[21] T. R. Prabhakar, A singular integral equation with a generalized Mittag-Leffler function in the kernel. Yokohama Math. J. 19(1971), 7-15.
[22] M. Saigo, A Remark on integral operators involving the Gauss hypergeometric functions, Math .Rep. Kyushu Univ.11 (1978), 135-143.
[23] S. G. Samko, A. A. Kilbas and O.I. Marichev, Fractional Integrals and Derivatives. Theory and Applications. Gordon and Breach, Yverdon et al. (1993).
[24] R. K. Saxena and Megumi Saigo, Certain properties of the fractional calculus operators associated with generalized Mittag-Leffler function. Fract. Calc. Appl.Anal.Vol.8, No.2 (2005), 141-154.
[25] H. M. Srivastava and P.W. Karlsson, Multiple Gaussian hypergeometric series. Ellis Horwood, Chichester [John Wiley and Sons], New York (1985).
[26] A. Wiman, ber den Fundamental satz in der Theorie der Functionen E_(x). Acta Math. 29 (1905), 191-201.
[27] A. Wiman, ber die Nullstellun der Funktionen E_(x) . Acta Math .29 (1905), 217-234.
[28] E. M. Wright, The asymptotic expansion of the generalized Bessel function. Proc. London Math. Soc. (2) 38 (1934), 257-270.

[29] E. M. Wright, The asymptotic expansion of the generalized hypergeometric functions. J. London Math. Soc. 10(1935), 286-293.
[30] E.M. Wright, The asymptotic expansion of the generalized hypergeometric functions. Proc. London Math. Soc. (2) 46(1940), 389-408.
[31] E.M. Wright, The generalized Bessel functions of order greater than one. Quart. J. Math. Oxford Ser. 11 (1940), 36-48.
[32] E.M. Wright, The asymptotic expansion of integral functions defined by Taylor Series. Philos. Trans. Roy. Soc. London, Ser. A 238 (1940), 423-451.