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Determination of M- Power Soft Subgroup Structures

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Abstract: In this paper, we introduce the order of the soft structure of the group theory of power of the soft sets related with soft group. We also characterise the m-power soft subgroups and its determination.
Keywords: soft set, soft group, m-power set, m-power group, determination, soft subset, union, intersection.

I. INTRODUCTION

The theory of soft sets, introduced by Moldtsov [1], is an extension of set theory for the study of intelligent systems characterized by insufficient and incomplete information. Maji et al. [2] give an application of soft set theory in a decision making problem by using the rough sets and they conducted a theoretical study on soft sets in a detailed way [3]. Chen et al. [4] proposed a reasonable definition of parameterizations reduction of soft sets and compared them with the concept of attributes reduction in rough set theory. The algebraic structures of set theories which deal with uncertainties have been studied by some authors. Rosenfeld [5] proposed fuzzy groups to establish results for the algebraic structures of fuzzy sets.

Fuzzification of algebraic structures was studied by many authors [5,6,7]. Many papers on soft algebras have been published since Aktas¸ and Ca gman[8]introducedthenotionofasoftgroupin2007.Recently,Junetal.[9]studiedsoftidealsandidealisticsoftBCK/BCI-algebras.Acaretal.[10]introducedinitial concepts of soft rings. Aygunoglu and Aygun[11]introduced the concept of fuzzy soft group and, in the meantime, they studied its properties and structural characteristics. Atagun and Sezgin [12] introduced and studied the concepts of soft subrings, soft ideal of a ring, and soft sub fields of a field. In this paper, we introduce the order of the soft structure of the group theory of power of the soft sets related with soft group. We also characterise the m-power soft subgroups and its determination.

II. PRELIMINARIES AND BASIC CONCEPTS

1) *Definition 2.1 [D.Molodtsov]:* A pair K_A is called a soft set over U, where F is a mapping given by K : A → P(U). In other words, a soft set over U is a parameterised family of subsets of the universe U.
2) *Definition 2.2 [Maji et.al]:* For two soft sets K_A and G_B over U, K_A is called a soft subset of G_B, if
   (1) $A \leq B$ and
   (2) For all e ε A ; K(e) and G(e) are identical approximations.

It is denoted by $K_A \leq G_B$. K_A is called a soft super set of G_B if G_B is a subset of K_A. It is denoted by $K_A \leq G_B$.

3) *Definition 2.3[Maji et.al]:* Union of two soft sets of K_A and G_B over U is the soft set H_C, where C = A U B and e ε C,

$$H_C = \begin{cases} K(e), & \text{if } e \in A-B, \\ G(e), & \text{if } e \in B-A, \\ K(e) \cup G(e) & \text{if } e \in A\cap B. \end{cases}$$

It is denoted by $K_A \cup G_B = H_C$.

4) *Definition 2.4[Maji et.al]:* Intersection of two soft sets of K_A and G_B over U is the soft set H_C, where C = A ∩ B and e ε C,

$$H_C = \begin{cases} K(e), & \text{if } e \in A-B, \\ G(e), & \text{if } e \in B-A, \\ K(e) \cap G(e) & \text{if } e \in A\cup B. \end{cases}$$

It is denoted by $K_A \cap G_B = H_C$.

5) *Definition 2.5 [H.Aktas et.al]:* Let KA be a soft set over G. Then KA is said to be a soft group over G if and only if K(x) is a subgroup of G for all x ε A.
6) *Definition 2.6 [H.Aktas et.al]:* One considers the following.

(1) K_A is said to be an identity soft group over G if K(x) = {e} for all x ε A, where e is the identity element of G.

(2) K_A is said to be an absolute soft group over G if K(x) = G for all x ε A.

7) *Definition 2.7:* Let K_A be a soft set over G and K(x) ε K_A for x ε A. Then $K(x^m)$ = { a^m/ a ε K(x), m ε Z} is called m-power of K(x).

8) *Example 2.8:* Let K_A be a soft set over S_3, where A =S_3, and let

K_A = { K(e) = {e}, K(12) = {e, (12)}, K(13) = {e, (13)}, K(23) = {e, (23)}, K(123) = { e , (123), (132)} be a soft set over group S_3.
And the third power of K(123) is K(123)3 = {e, (123)3, (132)3} = {e,e,e} = {e}.
Of course , when the group is additive, the m^{th} power of K(x) will be denoted by mK(x) = { mK(x) = {ma / a ε K(x), m ε Z}.

III. PROPERTIES OF M-POWER SOFT SUBGROUPS

In this section, we shall derive some important results based on the above definitions.

1) *Theorem-3.1:* Let K_A be a soft set over G and K(x), K(y) ε KA for all x,y ε A. Then , for all n ε Z,

a) $(K(x) \cap K(y))^m \subset K(x)^m \cap K(y)^m$

b) $(K(x) \cup K(y))^m \subset K(x)^m \cup K(y)^m$

c) $(K(x) \times K(y))^m \subset K(x)^m \times K(y)^m$.

Proof: Let a^m ε $(K(x) \cap K(y))^m$, for m ε Z. From Definition 2.7 a ε (K(x) ∩ K(y)) and a m ε K(x) and a m ε K(y) . This means that am ε $K(x)^m \cap K(y)^m$. This completes the proof. Theorem -3.1 (2) and theorem-3.1 (3) can be proved similarly by definition 2.7.
In general, the converse of the above theorem-3.1 is not true. We illustrate an example of this situation.

2) *Example 3.2:* Let A = {0,1} and let K : A→ Q(Z) be a function such that K(0) = {3r / r ε Z} and K(1)= { 3r + 1/ r ε Z}. The intersection is K(0) ∩ K(1) is empty, so $(K(0) \cap K(1))^3$ is empty. On the other hand $K(0)^3 \cap K(1)^3$ is non-empty. Consquently $(K(x) \cap K(y))^m \neq K(x)^m \cap K(y)^m$.

3) *Definition 3.3:* Let K_A be a soft set over G and K(x) ε K_A. If there is a positive integer m such that $K(x)^m$ = {e}, then the least such positive integer m is called the order of K(x). If no such m exists, then K(x) has infinite order. The order of K(x) is denoted by ‖K(x)‖. If K_A is a soft group over G, then the order of K(x) ε K_A coincides with the order of K(x), which is a subgroup of G. Of course if there is any element x in A such that K(x) = {e}, then the order of K(x) is 1.

4) *Example 3.4:* In Example 2.8 the order of an element K(123) is 3.Let K_N be a soft group over group of integer numbers Z, where K is a mapping from natural numbers N to Q(Z) such that K(m) = mZ for all m ε N. There is no any positive integer such that m such that $K(x)^m$ = {0}, so K(m) has infinite order for all m ε N-{0}.

5) *Theorem 3.5:* Let G be a finite group and K_A a soft group over G. Then, the order of elements of K_A are finite.
Proof; It is straightforward.

6) *Theorem 3.6:* Let K_A be a soft set over finite group G and K(x) ε K_A for x ε A. Then, the order of K(x) is the least common multiple of order of elements of K(x). Proof: Let m be the order of K(x). Then K(x)m = {e}. This means that a^m= e for all a ε K(x). we know from classical group that ‖a‖ m, namely, a divides m for all a ε K(x). Thus, m is common multiple of elements of K(x). Let , m be another common multiple elements of K(x). Then, by reason of a^p = e for all a ε K(x), $K(x)^p$ = {e}. However, since m is the least number that satisfies the condition $K(x)^m$ = {e}, hence p/m. This completes the proof.

7) *Theorem 3.7:* Let G be finite group, K_A soft group over G, and K(x) and K(y) the elements of K_A. Then, for all x, y ε A, one has the following:

a) ‖K(x) ∩ K(y) ‖ ≤ Greatest common divisor of (‖K(x)‖, ‖K(y)‖) for all x,y ε A.

b) ‖K(x) U K(y) ‖ = Least common multiple of (‖K(x)‖, ‖K(y)‖) for all x,y ε A.

c) ‖K(x) × K(y) ‖ = ‖K(x)‖ ‖K(y)‖ for all x,y ε A.

Proof: (1) K(x) ∩ K(y) is a subgroup of K(x) and K(y), so ‖K(x) ∩ K(y) ‖ / ‖K(x)‖ and ‖K(x) ∩ K(y) ‖ / ‖K(y)‖. It follows ‖K(x) ∩ K(y) ‖ ≤ Greatest common divisor of (‖K(x)‖, ‖K(y)‖) for all x,y ε A.

(2)Let ‖K(x) U K(y) ‖ = r, ‖K(x)‖ = p, and ‖K(y)‖ = m. From theorem 3.1 $(K(x) \cap K(y))^r \subset K(x)^r \cap K(y)^r$ = {e}. This follows m / r and m /r. Thus r is common multiple of m and p. Consider $(K(x) \cap K(y))^t$ = K(x)t U $K(y)^t$ = {e} U {e}. Since r is the least positive integer that satisfied the condition $(K(x) \cap K(y))^r$ = {e}, p divides t. Hence p is least common multiple order of K(x) and K(y). This completes the proof.

(3)Since K(x) and K(y) are subgroups of G, it is seen easily.

8) *Definition 3.8:* Let G be a group and K_A soft set over G. The set = { $K(x)^m$/ x ε A, m ε Z} is called m^{th} power of soft set.

9) *Example 3.9:* Let K_A be a soft set over S_3 defined in Example 2.8. Then , the second power of K_A is that $K_A{}^3$ = {$K(e)^3$ = K(e), and $K(12)^3$ = K(e), $K(13)^3$ =K(e), $K(23)^3$ = K(e), $K(123)^3$= K(123)}.

10) *Theorem 3.10:* Let K_A and E_B be two soft sets over G. Then,

(1) $(K_A \vee E_B))^m = K_A{}^m \vee E_B{}^m$,

(2) If $A \subset B$ and for all a ε A, K(a) and E(a) are identical approximations, then $(K_A \wedge E_B)^m \subset K_A{}^m \wedge E_B{}^m$.

Proof: we consider the following

(1) Suppose that $K_A \vee E_B = (H, A^{\times} B)$ and $K_A{}^m \vee E_B{}^m = (T, A^{\times}B)$. Using Definition 3.8 and theorem 3.1 we have

$(H, A^{\times}B)^m = \{ H(a,b)^m / (a,b) \varepsilon A^{\times}B\}$

$= \{(K(a) \cup E(b))^m / (a,b) \varepsilon A^{\times}B\}$

$= \{K(a)^m \cup E(b)^m / (a,b) \varepsilon A^{\times}B\}$

$= K_A{}^m \vee E_B{}^m$.

(2) Suppose that $K_A \wedge E_B = (H, A^{\times}B)$ and $K_A{}^m \wedge E_B{}^m = (T, A^{\times}B)$. Using the same argument in (1), we have

$(H, A^{\times}B)^m = \{ H(a,b)^m / (a,b) \varepsilon A^{\times}B\}$

$= \{(K(a) \cap E(b))^m / (a,b) \varepsilon A^{\times}B\}$

$= \{K(a)^m \cap E(b)^m / (a,b) \varepsilon A^{\times}B\}$

$= \{T(a,b) / (a,b) \varepsilon A^{\times}B\}$

$= K_A{}^m \wedge E_B{}^m$.

11) *In Example- 2.8:*The order of K_A is 6 and in Example 3.4 if we choose $N= N_5=\{0,1,2,3,4,5\}$ and K(m) = mZ, for m ε N_5, then the order of the group $K_N{}^5$ is 6.

IV. CONCLUSION

we discuss the m-structures of couples subgroups.In this paper, we introduce the order of the soft structure of the group theory of power of the soft sets related with soft group. We also characterise the m-power soft subgroups and its determination.

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