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On Cubic Equation With Four Unknowns $4\left(x^3+y^3\right)+12\left(s^2-1\right)(x+y)z^2=\left(3s^2+1\right)w^3, (s\neq\pm1)$

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Abstract: In this paper, the cubic equation with four unknown given by $4\left(x^3+y^3\right)+12\left(s^2-1\right)(x+y)z^2=\left(3s^2+1\right)w^3, (s\neq\pm1)$ ***is considered for determining its non-zero distinct integer solutions.***
Keywords: Cubic with four unknowns, homogeneous cubic, integer solutions.

I. INTRODUCTION

It is well-known that there are varieties of cubic equations with four unknowns to obtain integer solutions satisfying them [1-3]. In particular, different choices of cubic equations with four unknowns are presented in [4-12]. This paper has a different choice of cubic equation with four unknowns given by $4\left(x^3+y^3\right)+12\left(s^2-1\right)(x+y)z^2=\left(3s^2+1\right)w^3, (s\neq\pm1)$ to obtain its infinitely many non-zero distinct integer solutions.

II. METHOD OF ANALYSIS

The cubic equation with four unknowns to be solved for its non-zero distinct integer solutions is given by

$$4\left(x^3+y^3\right)+12\left(s^2-1\right)(x+y)z^2=\left(3s^2+1\right)w^3 \tag{1}$$

Introduction of the linear transformations

$$x=u+v, y=u-v, w=2u \quad (u\neq v\neq 0) \tag{2}$$

in (1) leads to

$$v^2=s^2u^2-\left(s^2-1\right)z^2 \tag{3}$$

Again, considering the linear transformations

$$u=X+\left(s^2-1\right)T \tag{4}$$

$$z=X+s^2T \tag{5}$$

in (3), it gives

$$X^2=\left(s^4-s^2\right)T^2+v^2 \tag{6}$$

The fundamental solution of (6) is

$$T_0=2v, X_0=\left(2s^2-1\right)v$$

To obtain the other solutions of (6), consider its pellian equation

$$X^2=\left(s^4-s^2\right)T^2+1$$

whose general solution $\left(\tilde{T}_n, \tilde{X}_n\right)$ is given by

$$\tilde{X}_n=\frac{1}{2}f_n, \tilde{T}_n=\frac{1}{2\sqrt{s^4-s^2}}g_n$$

where

$$f_n=\left(2s^2-1+2\sqrt{s^4-s^2}\right)^{n+1}+\left(2s^2-1-2\sqrt{s^4-s^2}\right)^{n+1}$$

$$g_n=\left(2s^2-1+2\sqrt{s^4-s^2}\right)^{n+1}-\left(2s^2-1-2\sqrt{s^4-s^2}\right)^{n+1}$$

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Applying the lemma of Brahmagupta between the solutions (T_0, X_0) and $(\tilde{T}_n, \tilde{X}_n)$, the other solutions to (6) are given by

$$\left.\begin{aligned} T_{n+1} &= vf_n + \frac{(2s^2-1)v}{2\sqrt{s^4-s^2}} g_n, \\ X_{n+1} &= \frac{(2s^2-1)v}{2} f_n + v\sqrt{s^4-s^2}\, g_n, n = 0,1,2,\ldots\ldots \end{aligned}\right\}. \tag{7}$$

Employing (4), (5) and (2), the sequence of solutions to (1) is represented as below:

$$x_{n+1} = \frac{(4s^2-3)v}{2} f_n + \frac{(4s^4-5s^2+1)}{2\sqrt{s^4-s^2}} v g_n + v,$$

$$y_{n+1} = \frac{(4s^2-3)v}{2} f_n + \frac{(4s^4-5s^2+1)}{2\sqrt{s^4-s^2}} v g_n - v,$$

$$z_{n+1} = \frac{(4s^2-1)v}{2} f_n + \frac{(4s^4-3s^2)}{2\sqrt{s^4-s^2}} v g_n,$$

$$w_{n+1} = (4s^2-3) v f_n + \frac{(4s^4-5s^2+1)}{\sqrt{s^4-s^2}} v g_n, n = -1,0,1,\ldots\ldots.$$

In addition to the above solutions, there are other choices of solutions to (1) that are illustrated below:
It is worth to note that (6) can be represented as the system of double equations as presented below in Table 1:

TABLE 1
System Of Double Equations

System	1	2	3
$X+v$	$s(s+1)T$	$s(s^2-1)T$	$s^2(s+1)T$
$X-v$	$s(s-1)T$	sT	$(s-1)T$

Solving each of the above systems, one obtains the values to X, v, T.

Substituting these values in (4), (5) and (2), the corresponding solutions to (1) are obtained and they are exhibited below:
Solutions to system 1:

$$x = (2s^2+s-1)T,\ y = (2s^2-s-1)T, z = 2s^2T, w = (4s^2-2)T$$

Solutions to system 2:

$$x = 2k(s^3+s^2-s-1), y = 2k(s^2+s-1), z = k(s^3+2s^2), w = 2k(s^3+2s^2-2)$$

Solutions to system 3:

$$x = k(2s^3+4s^2-2), y = k(2s^2+2s-4), z = k(s^3+3s^2+s-1), w = 2k(s^3+3s^2+s-3)$$

Further, (6) is written as

$$v^2 + (s^4-s^2)T^2 = X^2 * 1 \tag{8}$$

Assume

$$X = s^2A^2 + (s^4-s^2)s^2B^2 \tag{9}$$

write 1 as

$$1 = \frac{\left(s + i\sqrt{s^4-s^2}\right)\left(s - i\sqrt{s^4-s^2}\right)}{s^4} \tag{10}$$

Using (9) and (10) in (8) and employing the method of factorization, define

$$v+i\sqrt{s^4-s^2}T=\left(sA+i\sqrt{s^4-s^2}\,sB\right)^2\frac{\left(s+i\sqrt{s^4-s^2}\right)}{s^2}$$

On equating the real and imaginary parts, one obtains

$$\left.\begin{array}{l} v=s\left(A^2-\left(s^4-s^2\right)B^2\right)-2AB\left(s^4-s^2\right) \\ T=A^2-\left(s^4-s^2\right)B^2+2ABs \end{array}\right\} \quad (11)$$

Substituting (9) and (11) in (4), (5) and (2), the corresponding values of x, y, z and w satisfying (1) are given by

$$x=\left(2s^2+s-1\right)A^2+(1-s)\left(s^4-s^2\right)B^2+2AB\left(s^3-s-s^4+s^2\right)$$
$$y=\left(2s^2-s-1\right)A^2+(1+s)\left(s^4-s^2\right)B^2+2AB\left(s^3-s+s^4-s^2\right)$$
$$z=2s^2A^2+2ABs^3$$
$$w=\left(4s^2-2\right)A^2+2\left(s^4-s^2\right)B^2+4s\left(s^2-1\right)AB$$

III.CONCLUSIONS

In this paper, an attempt has been made to find non-zero distinct integer solutions to the cubic equation with four unknowns given by $4\left(x^3+y^3\right)+12\left(s^2-1\right)(x+y)z^2=\left(3s^2+1\right)w^3$, $(s\neq\pm1)$ in conclusion one may search for other sets of integer solutions to the considered cubic equation.

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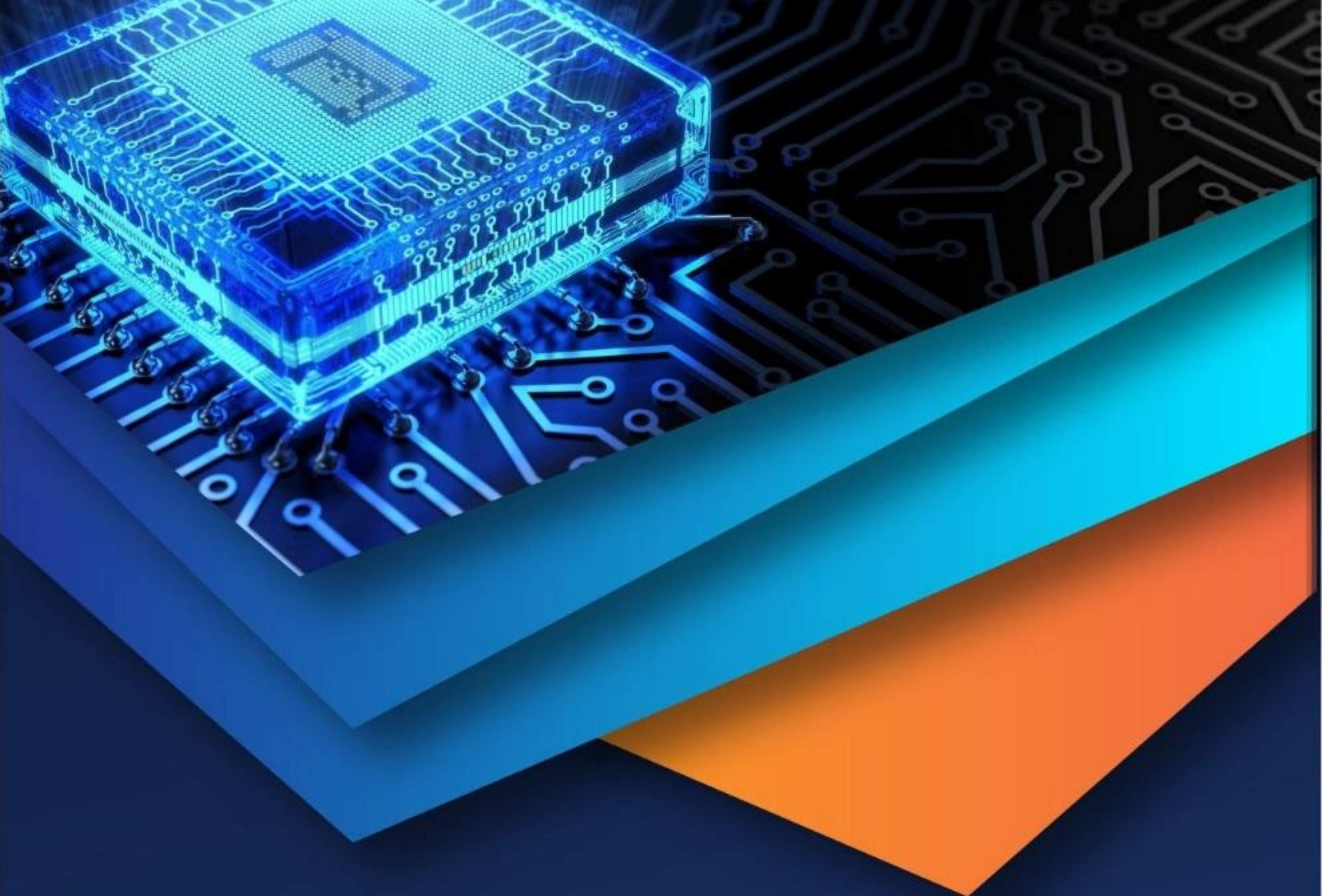

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