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## **Remark on the Paper Entitled Lattice Points of a Cubic Diophantine Equation** $11(x + y)^2 = 4(xy + 11z^3)$

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Abstract: In this paper, new sets of solutions to the cubic equation with three unknowns given by  $11(x + y)^2 = 4xy + 44z^3$  are presented.

Keywords: Ternary cubic, Integer solutions

### I. INTRODUCTION

When a search is made for cubic diophantine equations, the authors noticed a paper by Manju Somanath, J. Kannan, K. Raja [1] in which they have presented lattice points of the cubic diophantine equation  $11(x + y)^2 = 4xy + 44z^3$ . However, there are other interesting sets of solutions to the above equations that are exhibited in this paper.

#### II. METHOD OF ANALYSIS

| Consider the cubic equation with three unknowns given by                                                       |     |
|----------------------------------------------------------------------------------------------------------------|-----|
| $11(x+y)^2 = 4xy + 44z^3$                                                                                      | (1) |
| To start with, the substitution                                                                                |     |
| y = (2k - 1)x                                                                                                  | (2) |
| in (1) gives                                                                                                   |     |
| $(11k^2 - 2k + 1)x^2 = 11z^3$                                                                                  |     |
| which is satisfied by                                                                                          |     |
| $x = 121(11k^2 - 2k + 1)\alpha^3$                                                                              | (3) |
| $z = 11 \left( 11k^2 - 2k + 1 \right) \alpha^2$                                                                | (4) |
| Note that $(2) - (4)$ satisfies $(1)$                                                                          |     |
| Again, the substitution                                                                                        |     |
| y = 2kx                                                                                                        | (5) |
| in (1) leads to                                                                                                |     |
| $(44k^2 + 36k + 11)x^2 = 44z^3$                                                                                |     |
| whose solutions are                                                                                            |     |
| $x = 242(44k^2 + 36k + 11)\alpha^3$                                                                            | (6) |
| $z = 11(44k^2 + 36k + 11)\alpha^2$                                                                             | (7) |
| Thus, (5)-(7) satisfy (1)                                                                                      |     |
| Further,                                                                                                       |     |
| Introduction of the linear transformations                                                                     |     |
| x = u + v,  y = u - v,  z = u                                                                                  | (8) |
| in (1) leads to                                                                                                |     |
| $v^2 = u^2 (11u - 10)$                                                                                         | (9) |
| After performing some algebra, it is noted that (9) is satisfied by the following two choices of $u$ and $v$ : |     |
| 1) $u = 11k^2 - 2k + 1, v = (11k - 1)(11k^2 - 2k + 1)$                                                         |     |
| 2) $u = 11k^{2} + 2k + 1, v = (11k + 1)(11k^{2} + 2k + 1)$                                                     |     |



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In view of (8), the corresponding two sets of values to x, y, z satisfying (1) are represented below:

*a)* Set 1: Consider choice (i). The values of x, y, z are:

$$x = 11k(11k^{2} - 2k + 1)$$
  

$$y = (2 - 11k)(11k^{2} - 2k + 1)$$
  

$$z = 11k^{2} - 2k + 1$$
  

$$k = 2$$
  
Consider choice (1)

b) Set 2: Consider choice (ii). The values of x, y, z are:

 $x = (11k + 2)(11k^{2} + 2k + 1)$   $y = -11k(11k^{2} + 2k + 1)$  $z = 11k^{2} + 2k + 1$ 

#### REFERENCE

[1] Manju Somanath, J.Kannan, K.Raja, "Lattice Points Of A Cubic Diophantine Equation  $11(x + y)^2 = 4(xy + 11z^3)$ ", IJRASET, Volume 5, Issue 5,1797-1800, 2017.











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