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# Remark on the Paper Entitled Lattice Points of a Cubic Diophantine Equation $11(x+y)^{2}=4\left(x y+11 z^{3}\right)$ 

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Abstract: In this paper, new sets of solutions to the cubic equation with three unknowns given by $11(x+y)^{2}=4 x y+44 z^{3}$ are presented.

## Keywords: Ternary cubic, Integer solutions

## I. INTRODUCTION

When a search is made for cubic diophantine equations, the authors noticed a paper by Manju Somanath, J. Kannan, K. Raja [1] in which they have presented lattice points of the cubic diophantine equation $11(x+y)^{2}=4 x y+44 z^{3}$. However, there are other interesting sets of solutions to the above equations that are exhibited in this paper.

## II. METHOD OF ANALYSIS

Consider the cubic equation with three unknowns given by

$$
\begin{equation*}
11(x+y)^{2}=4 x y+44 z^{3} \tag{1}
\end{equation*}
$$

To start with, the substitution

$$
\begin{equation*}
y=(2 k-1) x \tag{2}
\end{equation*}
$$

in (1) gives

$$
\left(11 k^{2}-2 k+1\right) x^{2}=11 z^{3}
$$

which is satisfied by

$$
\begin{align*}
& x=121\left(11 k^{2}-2 k+1\right) \alpha^{3}  \tag{3}\\
& z=11\left(11 k^{2}-2 k+1\right) \alpha^{2} \tag{4}
\end{align*}
$$

Note that (2) - (4) satisfies (1)
Again, the substitution

$$
\begin{equation*}
y=2 k x \tag{5}
\end{equation*}
$$

in (1) leads to

$$
\left(44 k^{2}+36 k+11\right) x^{2}=44 z^{3}
$$

whose solutions are

$$
\begin{align*}
& x=242\left(44 k^{2}+36 k+11\right) \alpha^{3}  \tag{6}\\
& z=11\left(44 k^{2}+36 k+11\right) \alpha^{2} \tag{7}
\end{align*}
$$

Thus, (5)-(7) satisfy (1)
Further,
Introduction of the linear transformations

$$
\begin{equation*}
x=u+v, \quad y=u-v, \quad z=u \tag{8}
\end{equation*}
$$

in (1) leads to

$$
\begin{equation*}
v^{2}=u^{2}(11 u-10) \tag{9}
\end{equation*}
$$

After performing some algebra, it is noted that (9) is satisfied by the following two choices of $u$ and $v$ :

1) $u=11 k^{2}-2 k+1, v=(11 k-1)\left(11 k^{2}-2 k+1\right)$
2) $u=11 k^{2}+2 k+1, v=(11 k+1)\left(11 k^{2}+2 k+1\right)$

In view of (8), the corresponding two sets of values to $x, y, z$ satisfying (1) are represented below:
a) Set 1: Consider choice (i). The values of $x, y, z$ are:
$x=11 k\left(11 k^{2}-2 k+1\right)$
$y=(2-11 k)\left(11 k^{2}-2 k+1\right)$
$z=11 k^{2}-2 k+1$
b) Set 2: Consider choice (ii). The values of $x, y, z$ are:
$x=(11 k+2)\left(11 k^{2}+2 k+1\right)$
$y=-11 k\left(11 k^{2}+2 k+1\right)$
$z=11 k^{2}+2 k+1$

## REFERENCE

[1] Manju Somanath, J.Kannan, K.Raja, "Lattice Points Of A Cubic Diophantine Equation $11(x+y)^{2}=4\left(x y+11 z^{3}\right)$ ", IJRASET, Volume 5, Issue 5,1797-1800, 2017.

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