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Simulation Analysis of MIMO/OFDM Evaluation with Enhanced Non-Zero FBMC/OQAM Modulation Scheme

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Abstract: FBMC is a multicarrier transmission scheme that introduces a filter-bank to enable efficient pulse shaping for the signal conveyed on each individual subcarrier. This additional element represents an array of band-pass filters that separate the input signal into multiple components or subcarriers, each one carrying a single frequency sub-band of the original signal. As a promising variant of filtered modulation schemes, FBMC, originally proposed in [4] and also called OFDM/OQAM [5] or staggered modulated multitone (SMT) [6], can potentially achieve a higher spectral efficiency than OFDM since it does not require the insertion of a CyclicPrefix (CP). Additional advantages include the robustness against highly variant fading channel conditions and imperfect synchronization by selecting the appropriate PF type and coefficients [3]. Such a transceiver structure usually requires a higher implementation complexity related not only to the filtering steps but also to the applied modifications to the modulator/demodulator architecture. However, the usage of digital poly phase filter bank structures [5], [7], together with the rapid growth of digital processing capabilities in recent years have made FBMC a practically feasible approach. Existing: Filter-bank multi-carrier with offset quadrature amplitude modulation (FBMC/OQAM) is considered by recent research projects as one of the key enablers for the future 5G air interface. It exhibits better spectral shape and improves mobility support compared to orthogonal frequency-division multiplexing (OFDM) thanks to the use of a time and frequency localized prototype filter. The choice of this filter is crucial for FBMC/OQAM, due to its substantial impact on achieved performance and complexity levels. In the context of 5G, short frame sizes are foreseen in several communication scenarios to reduce system latency, and therefore short filters are preferred. In this context, a novel short filter allowing for near perfect reconstruction and having the same size as one OFDM symbol is proposed. Using frequency-spread (FS) implementation for the FBMC/OQAM receiver, analytical analysis and simulation results show that the proposed filter exhibits better robustness to several types of channel impairments when compared to state-of the-art prototype filters and OFDM modulation. In addition, FS-based hardware architecture of the filtering stage is proposed, showing lower complexity than the classical polyphase network-based implementation. Proposed-In this proposed the impact of nonlinear distortion induced by High Power Amplifiers (HPA) for medium and high power signals is analyzed for Filter Bank Multi-Carrier (FBMC) systems using Offset Quadrature Amplitude Modulation (OQAM). A closed form expression for Bit Error Rate (BER) expression is derived and analyzed for MMSE equalizer models for FBMC/OQAM system with nonlinear HPA in frequency selective Rayleigh channel by varying constellation size in OQAM modulation and Input Back-Off. The performance is compared for the models with 64 sub channels and input back-off for 6 and 8 decibels with 4*4 MIMO system. In lieu of validating the obtained simulation results, theoretical results have also been compared.

Keywords: Low Power, Sample and Hold Circuit, Current Conveyor Analogue Switches , FBMC, multicarrier transmission scheme

I. INTRODUCTION

Filter Bank Multi-Carrier (FBMC) is a multicarrier modulation where the subcarriers are passed through filters that suppress signals side lobes, provides better spectral localization. FBMC modulation is considered in Fifth Generation (5G) cellular network standards for its robustness due to network asynchronicity and frequency misalignment between users. FBMC is usually either coupled with QAM or with offset-QAM (OQAM) modulation techniques. High Power Amplifiers (HPA) used in FBMC system for radio frequency up-conversion, may enter the nonlinear region for medium and high power signals. It provides high power efficiency and increased battery life for power amplifiers and it produces peak clipping and intermodulation distortion causing Adjacent Channel Interference (ACI) and power losses leading to increased error rates. Intuitively, Chang and Saltzberg introduced FBMC modulation for parallel data transmission [1][2]. A nonlinearity for Gaussian data is analyzed [3]. Cyclic Prefix (CP)-based Orthogonal Frequency Division Multiplexing (OFDM) modulation offers better robustness to multipath channel effects is used widely in current technologies. However, the use of CP and the high side lobes of the rectangular pulse shape induce a loss of the spectral efficiency. OFDM signals may exhibit large peak-to-average-power ratio values and frequency synchronization among subcarriers [4][5]. A theoretical analysis have been carried out for the analysis of an OFDM system using a memory-less HPA [6]. Effect of nonlinearity on multiple input multiple output system is observed [7]. The FBMC system with TWTA Saleh model of HPA is analyzed [9] in the presence of nonlinear phase distortion is performed. The objective of this paper is to analyse the effect of nonlinear distortion induced by memory-less HPA models such as TWTA, SSPA and SEL on bit error rate performance of FBMC systems with OQAM modulation. This paper is organized as follows. Section II presents the system model of FBMC-OQAM system and HPA models. Section III evaluates analytical expression for FBMC system with nonlinear HPA. Simulation results are discussed in section IV and section V concludes the paper.

II. SYSTEM MODEL

Consider an FBMC system with OQAM modulation using TWTA [8], SSPA and SEL models with 6dB Input Back-Off (IBO) and knee voltage $\beta = 1$ in frequency selective Rayleigh fading channel as shown in figure 1. The polar coordinates of the input signal is $s = re^{j\theta}$ .Symbol at the output of the non-linear HPA is $\hat{s} = f_A(r)\, e^{j(f_P(r)+\theta)}$, where $r, \theta \rightarrow$ amplitude and phase of input signal. $f_A(r) \rightarrow$ AM/AM Conversion of the HPA is a measure of how closely the input and output transfer characteristic of an amplifier matches with a straight line. $f_P(r) \rightarrow$ AM/PM Conversion of the HPA is a measure of the time delay or shift in phase angle

1) *Twta Saleh Model:* The TWTA amplifies signal uses an electron beam and a slow wave structure. TWTAs has the advantage of large currents and provides high output powers.AM/AM and AM/PM conversion of TWTA characterized by Saleh model [9] is

$$f_A(r) = A_{is}^2 \frac{r}{r^2 + A_{is}^2} \tag{1}$$

$$f_P(r) = \varphi_0 \frac{r^2}{r^2 + A_{is}^2} \tag{2}$$

where A_{is} represents input saturation voltage, r is the amplitude of signal at the input of HPA.

2) *SSPA Model:* SSPA is a microwave frequency amplifier uses field-effect transistors for amplification. It is a high-power amplifier using solid-state electronics that possess reduced size. The AM/AM and AM/PM conversions of the SSPA model can be expressed as

$$f_A(r) = \frac{r}{\left[1 + \left(\frac{r}{A_{os}}\right)^{2\beta}\right]^{\frac{1}{2\beta}}} \tag{3}$$

$$f_P(r) = 0 \tag{4}$$

where A_{os} represents output saturation voltage and β is the knee factor sharpness of transition from linear region to saturation region.

3) *Sel Model:* SEL model is used for modeling a HPA with a perfect predistortion system. The global transfer function of the predistortion followed by the HPA is a limiter. When knee value→∞, the SSPA model converges towards the SEL. The SEL used to model the HPA with ideal predistortion can be described by the following AM/AM and AM/PM functions:

$$f_A(r) = \begin{cases} r & , r \le A_{is} \\ A_{is} & , r > A_{is} \end{cases} \tag{5}$$

$$f_P(r) = 0 \tag{6}$$

D) FBMC - Transmitted signal at t^{th} instant is defined by

$$i(t) = \sum_{m=0}^{M-1} \sum_{n=-\infty}^{\infty} a_{m,n} \gamma_{m,n}(t) \quad (7)$$

where, M is the number of subcarriers, $a_{m,n}$ is the real valued 4-OQAM symbols on m^{th} subcarrier and n^{th} symbol frame, $\gamma_{m,n}$ is the time, frequency shifted prototype filter impulse response. The prototype filter for FBMC system is designed as [10],

$$\gamma_{m,n}(t) = g\left(t - \frac{nT}{2}\right) \exp\left(j\frac{2\pi}{T} m\left(t - \frac{D}{2}\right)\right) \exp(\phi_{m,n}) \quad (8)$$

where, g(t) is the prototype filter impulse response which is null for $i \notin \{0,1,\ldots,KM-1\}$ T is the symbol period $\mathrm{D} = \mathrm{KM} - 1$ is the delay term which depends on length of prototype filter, K is the overlapping factor $\phi_{\mathrm{m,n}} = \frac{\pi}{2}(\mathrm{m}+\mathrm{n}) - \pi\mathrm{mn}$ is the initial phase term. In a distortion-less, noise-free channel, the demodulated signal at subcarrier m_0 and time instant n_0 is given by,

$$y_{m_0,n_0} = \langle i(t), \gamma_{m_0,n_0}(t) \rangle = \int_{-\infty}^{\infty} i(t) \gamma^*_{m_0,n_0}(t) dt \quad (9)$$

Substituting Equation (7), the demodulated signal becomes

$$y_{m_0,n_0} = a_{m_0,n_0} + \sum_{m \neq m_0} \sum_{n \neq n_0} a_{m,n} \int_{-\infty}^{\infty} \gamma_{m,n}(t) \gamma^*_{m_0,n_0}(t) dt \quad (10)$$

$$y_{m_0,n_0} = a_{m_0,n_0} + ju_{m_0,n_0} \quad (11)$$

,where u_{m_0,n_0} is the pure imaginary intrinsic interference. Perfect Reconstruction is possible only when $\Re\left\{ \int_{-\infty}^{\infty} \gamma_{m,n}(t) \gamma^*_{m_0,n_0}(t) dt \right\} = \delta_{m,m_0} \delta_{n,n_0}$ According to Bussgang's theorem [11], the nonlinearly amplified signal conditioned to the transmission of a fixed symbol can be modeled as

$$s(t) = K_0 i(t) + d(t) \quad (12)$$

K_0 is a constant complex gain given by

$$K_0 = \frac{1}{2} E\left[\frac{\partial S(\rho)}{\partial \rho} + \frac{S(\rho)}{\rho} \right] \quad (13)$$

d(t) is an additive zero mean noise uncorrelated with i(t), having variance

$$\sigma_d^2 = E\left((d(t))^2\right) = E\left(|S(\rho)|^2\right) - |K_0|^2 E\left(\rho^2\right) \quad (14)$$

Received signal after nonlinear amplification is

$$y_{m_0,n_0} = K_0 H_{m_0,n_0}\left(a_{m_0,n_0} + ju_{m_0,n_0}\right) + H_{m_0,n_0} d_{m_0,n_0} + w_{m_0,n_0} \quad (15)$$

Where, H_{m_0,n_0} is the channel coefficient corresponding to the subcarrier m_0 and instant n_0. The channel is considered to be constant over neighboring symbols contributing the ISI. Zero-forcing equalized symbol when H_{m_0,n_0} and K_0 are perfectly known

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$$\hat{a} = \Re\left\{\frac{y}{K_0 H}\right\} = a + \Re\left\{\frac{d}{K_0}\right\} + \Re\left\{\frac{w}{K_0 H}\right\} \tag{16}$$

III. FBMC SYSTEM WITH HPAS

An instantaneous SNR for the FBMC/OQAM system using Equation (15),

$$\gamma = \frac{|K_0|^2 |H|^2 \sigma_x^2}{|H|^2 \sigma_d^2 + \sigma_w^2} \tag{17}$$

PDF of the instantaneous SNR is given by

$$p_\gamma(\gamma) = \begin{cases} \dfrac{\sigma_w^2 \gamma_c}{\Omega\left(\gamma_c - \sigma_d^2 \gamma\right)^2} e^{-\frac{\sigma_w^2 \gamma}{\Omega\left(\gamma_c - \sigma_d^2 \gamma\right)}}, & if\, 0 \le \gamma \le \dfrac{\gamma_c}{\sigma_d^2} \\ 0, & otherwise \end{cases} \tag{18}$$

BER of M-QAM in AWGN channel is given by

$$BER = a \int_0^{\frac{\gamma_c}{\sigma_d^2}} erfc\left(\sqrt{\gamma b}\right) \frac{\sigma_w^2 \gamma_c}{\Omega\left(\gamma_c - \sigma_d^2 \gamma\right)^2} e^{-\frac{\sigma_w^2 \gamma}{\Omega\left(\gamma_c - \sigma_d^2 \gamma\right)}} d\gamma \tag{19}$$

where a and b are modulation specific constants given by

$$a = \frac{2\left(\sqrt{M} - 1\right)}{\sqrt{M} \log_2(M)} \quad b = \frac{3 \log_2(M)}{(M-1)} \tag{20}$$

Channel noise variance is defined by

$$\sigma_w^2 = \frac{N_0}{E_b} \frac{1}{\log_2 M} P_{out} \tag{21}$$

The signal mean power at the output of the HPA is defined by

$$P_{out} = E_b \frac{2 \log_2 M}{T} \tag{22}$$

A closed form expression for theoretical bit error rate is defined as

$$BER = a\left(1 - \int_0^{\frac{\gamma_c}{\sigma_d^2}} erf\left(\sqrt{\gamma b}\right) \frac{\sigma_w^2 \gamma_c}{\Omega\left(\gamma_c - \sigma_d^2 \gamma\right)^2} e^{-\frac{\sigma_w^2 \gamma}{\Omega\left(\gamma_c - \sigma_d^2 \gamma\right)}} d\gamma\right) \tag{23}$$

$$BER = a(1 - I) \tag{24}$$

where $I = \int_0^{\frac{\gamma_c}{\sigma_d^2}} erf\left(\sqrt{\gamma b}\right) \frac{\sigma_w^2 \gamma_c}{\Omega\left(\gamma_c - \sigma_d^2 \gamma\right)^2} e^{-\frac{\sigma_w^2 \gamma}{\Omega\left(\gamma_c - \sigma_d^2 \gamma\right)}} d\gamma$ (25)

The error function is defined as

$$erf(u) = \frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n u^{2n+1}}{n!(2n+1)} \tag{26}$$

Substituting for error function and using Taylor series expansion,

$$e^{-\frac{\sigma_w^2 \gamma}{\Omega(\gamma_c - \sigma_d^2 \gamma)}} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left(\frac{\sigma_w^2}{\Omega}\right)^k \frac{\gamma^k}{(\gamma_c - \sigma_d^2 \gamma)^k} \tag{27}$$

By using table of integrals,

$$\int_0^u \frac{x^{\mu-1}}{(1+\beta x)^v} dx = \frac{u^\mu}{\mu} \, {}_2F_1(v, \mu; 1+\mu; -\beta\mu) \tag{28}$$

, which is valid for $[|\arg(1+\beta\mu)| < \pi, \Re(\mu) > 0]$ where ${}_2F_1(.,.;.;.)$ denotes the Hypergeometric function. Hypergeometric function is defined for |z|<1 by the power series,

$${}_2F_1(q, b; c; z) = \sum_{n=0}^{\infty} \frac{(q)_n (b)_n}{(c)_n} \frac{z^n}{n!} \tag{29}$$

where Pochhammer Symbol $(q)_n = \begin{cases} 1 & , n = 0 \\ q(q+1)\ldots(q+n-1) & , n > 0 \end{cases}$

$$I = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{(-1)^{k+n}}{k!n!(2n+1)} \left(\frac{\sigma_w^2}{\Omega}\right)^{k+1} \left(\frac{1}{\sigma_d^2}\right)^{n+k+\frac{3}{2}} \frac{(b\gamma_c)^{n+\frac{1}{2}}}{n+k+\frac{3}{2}} \, {}_2F_1\left(k+2, n+k+\frac{3}{2}; n+k+\frac{5}{2}; 1\right) \tag{30}$$

Closed form BER expression obtained by substituting Equation (31) into Equation (24),

$$BER = a\left(1 - \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \left[\frac{(-1)^{k+n}}{k!n!(2n+1)} \left(\frac{\sigma_w^2}{\Omega}\right)^{k+1} \left(\frac{1}{\sigma_d^2}\right)^{n+k+\frac{3}{2}} \frac{(b\gamma_c)^{n+\frac{1}{2}}}{n+k+\frac{3}{2}} \, {}_2F_1\left(k+2, n+k+\frac{3}{2}; n+k+\frac{5}{2}; 1\right)\right]\right) \tag{31}$$

Substituting Equation (21), into BER expression, the closed form expression is

$$BER = a\left(1 - \frac{2}{\sqrt{\pi}}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\left[\frac{\frac{(-1)^{k+n}}{k!n!(2n+1)}\left(\frac{\frac{N_0}{E_b}\frac{1}{\log_2 M}P_{out}}{\Omega}\right)^{k+1}\left(\frac{1}{\sigma_d^2}\right)^{n+k+\frac{3}{2}}}{\frac{(b\gamma_c)^{n+\frac{1}{2}}}{n+k+\frac{3}{2}}\,{}_2F_1\left(k+2, n+k+\frac{3}{2}; n+k+\frac{5}{2}; 1\right)}\right]\right)$$

1) Twta Model: aaa Nonlinear characteristics of TWTA amplifier is as given in Equation (1) and Equation (2),

Soft envelope $S(\rho) = F_a(\rho)e^{jF_p(\rho)}$

$$S(\rho) = A_{is}^2\left(\frac{\rho}{\rho^2 + A_{is}^2}\right)e^{j\varphi_0\left(\frac{\rho^2}{\rho^2 + A_{is}^2}\right)} \tag{33}$$

Complex constant gain as given by eq. (13), $K_0 = \frac{1}{2}E\left[\frac{\partial S(\rho)}{\partial \rho} + \frac{S(\rho)}{\rho}\right]$

$$\frac{\partial S(\rho)}{\partial \rho} = \frac{A_{is}^2\left(A_{is}^4 - \rho^4\right) + j\varphi_0 2\rho^2 A_{is}^4}{\left(A_{is}^2 + \rho^2\right)^3}\exp\left[j\varphi_0\left(\frac{\rho^2}{\rho^2 + A_{is}^2}\right)\right] \tag{34}$$

$$K_0 = \frac{1}{2}E\left[\frac{A_{is}^2\left(A_{is}^4 - \rho^4\right) + j\varphi_0 2\rho^2 A_{is}^4}{\left(A_{is}^2 + \rho^2\right)^3}\exp\left[j\varphi_0\left(\frac{\rho^2}{\rho^2 + A_{is}^2}\right)\right] + \left(\frac{A_{is}^2}{\rho^2 + A_{is}^2}\right)e^{j\varphi_0\left(\frac{\rho^2}{\rho^2 + A_{is}^2}\right)}\right] \tag{35}$$

Variance of distortion noise is given by eq. (14), $\sigma_d^2 = E\left(|S(\rho)|^2\right) - |K_0|^2 E\left(\rho^2\right)$

$$|S(\rho)|^2 = \frac{A_{is}^4\rho^2}{\left(\rho^2 + A_{is}^2\right)^2} \tag{36}$$

Substituting Equation (37) into Equation (14),

$$\sigma_d^2 = E\left(\frac{A_{is}^4\rho^2}{\left(\rho^2 + A_{is}^2\right)^2}\right) + |K_0|^2\left(\rho^2 + 2\sigma_x^2\right)e^{-\frac{\rho^2}{2\sigma_x^2}}$$

2) Sel Model: Nonlinear characteristics of SEL amplifier is as given in Equation (5) and Equation (6), Soft envelope $S(\rho) = F_a(\rho)e^{jF_p(\rho)}$ Complex constant gain given by eq. (13), $K_0 = \frac{1}{2}E\left[\frac{\partial S(\rho)}{\partial \rho} + \frac{S(\rho)}{\rho}\right]$

$$\frac{\partial S(\rho)}{\partial \rho} = \begin{cases} 1 & for\ \rho \le A_{is} \\ 0 & for\ \rho > A_{is} \end{cases} \tag{38}$$

$$\frac{S(\rho)}{\rho}=\begin{cases}1 & for\ \rho\le A_{is}\\ \dfrac{A_{is}}{\rho} & for\ \rho>A_{is}\end{cases} \tag{39}$$

Substituting Equation (38) and Equation (39) into Equation (13),

$$K_0=\begin{cases}\mathrm{E}(1) & for\ \rho\le A_{is}\\ \dfrac{A_{is}}{2}\mathrm{E}\left(\dfrac{1}{\rho}\right) & for\ \rho>A_{is}\end{cases} \tag{40}$$

Rayleigh distributed PDF of ρ is $f_\rho(\rho)=\frac{\rho}{\sigma_x^2}e^{-\frac{\rho^2}{2\sigma_x^2}}\quad,\rho\ge 0$

$$\mathrm{E}\left(\frac{1}{\rho}\right)=\int\frac{1}{\rho}f_\rho(\rho)d\rho=\frac{1}{\sigma_x^2}\int e^{-\frac{\rho^2}{2\sigma_x^2}}d\rho \tag{41}$$

From table of integrals,

$$\mathrm{E}\left(\frac{1}{\rho}\right)=j\sqrt{\frac{\pi}{2\sigma_x^2}}erf\left(\frac{-\rho}{\sqrt{2\sigma_x^2}}\right) \tag{42}$$

$$K_0=\left(1-e^{-\frac{A_{is}^2}{\sigma_x^2}}\right)+\frac{1}{2}\sqrt{\pi\frac{A_{is}^2}{\sigma_x^2}}erfc\left(\sqrt{\frac{A_{is}^2}{\sigma_x^2}}\right) \tag{43}$$

Variance of HPA noise as in eq. (14), $\sigma_d^2=E\left((d(t))^2\right)=E\left(|S(\rho)|^2\right)-|K_0|^2\ E\left(\rho^2\right)$

$$|S(\rho)|^2=\begin{cases}\rho^2 & ,\rho\le A_{is}\\ A_{is}^2 & ,\rho>A_{is}\end{cases} \tag{44}$$

$$\mathrm{E}(\rho^2)=\int\frac{\rho^3}{\sigma_x^2}e^{-\frac{\rho^2}{2\sigma_x^2}}d\rho \tag{45}$$

From table of integrals,

$$\int x^m e^{ax^n}dx=\frac{e^{ax^n}}{n}\left(\frac{x^n}{a}-\frac{1}{a^2}\right)\quad for\ \frac{m+1}{n}=2 \tag{46}$$

Using Equation (48),

$$\mathrm{E}(\rho^2)=-\left(\rho^2+2\sigma_x^2\right)e^{-\frac{\rho^2}{2\sigma_x^2}} \tag{47}$$

The variance of the nonlinear distortion noise is

$$\sigma_d^2=\sigma_x^2\left(1-e^{-\left(\frac{A_{is}^2}{\sigma_x^2}\right)}-K_0^2\right) \tag{48}$$

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3) *SSPA Model:* Nonlinear characteristics of SEL amplifier is as given in Equation (5) and Equation (6), Soft envelope $S(\rho) = F_a(\rho)e^{jF_p(\rho)}$ Complex constant gain given by eq. (13), $K_0 = \frac{1}{2}E\left[\frac{\partial S(\rho)}{\partial \rho} + \frac{S(\rho)}{\rho}\right]$ Let, $A_{os} = 1, \beta = 1$ $S(\rho) = \frac{\rho}{\sqrt{1+\rho^2}}$

$$\frac{S(\rho)}{\rho} = \frac{1}{\sqrt{1+\rho^2}} \tag{49}$$

$$\frac{\partial S(\rho)}{\partial \rho} = \frac{1}{(1+\rho^2)^{\frac{3}{2}}} \tag{50}$$

Substituting, $$K_0 = \frac{1}{2}E\left[\frac{1}{(1+\rho^2)^{\frac{3}{2}}} + \frac{1}{\sqrt{1+\rho^2}}\right] = \frac{1}{2}E\left[\frac{2+\rho^2}{(1+\rho^2)^{\frac{3}{2}}}\right] \tag{51}$$

Let $x = \rho^2, n = 2$,

Chi-square distributed pdf, $$p_x(x) = \frac{x^{\left(\frac{n}{2}-1\right)} e^{-\frac{x}{2\sigma^2}}}{\sigma^n 2^{\frac{n}{2}} \Gamma\left(\frac{n}{2}\right)} \tag{52}$$

$$K_0 = \frac{1}{2}\int \frac{x+2}{(1+x)^{\frac{3}{2}}} \left(\frac{e^{-\frac{x}{2\sigma_x^2}}}{2\sigma_x^2}\right) dx \tag{53}$$

From table of integrals,

$$\int P_m(x) e^{ax} dx = \frac{e^{ax}}{a} \sum_{k=0}^{m} (-1)^k \frac{p^k(x)}{a^k} \tag{54}$$

$P_m(x)$ is a polynomial of x in degree m

$p^k(x)$ is the k$^{\text{th}}$ derivative of $P_m(x)$ with respect to. x

$$K_0 = \frac{e^{-\frac{x}{2\sigma_x^2}}}{(4\sigma_x^2)\left(\frac{-1}{2\sigma_x^2}\right)} \sum_{k=0}^{m} (-1)^k \frac{p^k(x)}{\left(\frac{-1}{2\sigma_x^2}\right)^k} \tag{55}$$

Using binomial expansion,

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots \tag{56}$$

This equation is valid for any n positive or negative, integer or fraction provided that -1<x<1

$$(1+x)^{-\frac{3}{2}} = 1 + \left(-\frac{3}{2}\right)x + \frac{\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{2!}x^2 + \frac{\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right)}{3!}x^3 + \dots \quad for\ -1 < x < 1 \tag{57}$$

Neglecting higher powers of x,

$$K_0 = -\frac{e^{-\frac{x}{2\sigma_x^2}}}{2}\left\{\left[(x+2)\left(1-\frac{3}{2}x\right)\right] + \left[\frac{(-1)\left[\left(1-\frac{3}{2}x\right)-\frac{3}{2}(x+2)\right]}{\left(\frac{-1}{2\sigma_x^2}\right)}\right] + \left[\frac{\left(-\frac{3}{2}-\frac{3}{2}\right)}{\left(\frac{-1}{2\sigma_x^2}\right)^2}\right]\right\} \tag{58}$$

Simplifying,

$$K_0 = -\frac{e^{-\frac{x}{2\sigma_x^2}}}{2}\left\{\left(\frac{-3}{2}x^2 - 2x + 2\right) - \left(2\sigma_x^2(2+3x)\right) - \left(12\sigma_x^4\right)\right\} \tag{59}$$

Variance of the NLD noise as in eq. (14), $\sigma_d^2 = E\left(|S(\rho)|^2\right) - |K_0|^2\, E\left(\rho^2\right)$

$$\sigma_d^2 = \mathrm{E}\left(\frac{\rho^2}{1+\rho^2}\right) - |K_0|^2\, \mathrm{E}\left(\rho^2\right) \tag{60}$$

Using Equation (49), $\sigma_d^2 = \mathrm{E}\left(\frac{\rho^2}{1+\rho^2}\right) + |K_0|^2\, e^{-\frac{\rho^2}{2\sigma_x^2}}\left(\rho^2 + 2\sigma_x^2\right)$ (61)

$$\mathrm{E}\left(\frac{\rho^2}{1+\rho^2}\right) = \int\left(\frac{\rho^2}{1+\rho^2}\right)\frac{e^{-\frac{\rho^2}{2\sigma_x^2}}}{2\sigma_x^2}\,d\rho^2 \tag{62}$$

$$\mathrm{E}\left(\frac{x}{1+x}\right) = \int x(1+x)^{-1}\frac{e^{-\frac{x}{2\sigma_x^2}}}{2\sigma_x^2}\,dx \tag{63}$$

Approximating binomial expansion,

$$\mathrm{E}\left(\frac{x}{1+x}\right) = \frac{1}{2\sigma_x^2}\int x(1-x)e^{-\frac{x}{2\sigma_x^2}}\,dx \tag{64}$$

Using table of integrals for polynomial coefficient as given in Equation (56),

$$\mathrm{E}\left(\frac{x}{1+x}\right) = \frac{e^{-\frac{x}{2\sigma_x^2}}}{\left(2\sigma_x^2\right)\left(-\frac{1}{2\sigma_x^2}\right)}\sum_{k=0}^{2}(-1)^k\frac{p^k(x)}{a^k} \tag{65}$$

$$\mathrm{E}\left(\frac{x}{1+x}\right) = -e^{-\frac{x}{2\sigma_x^2}}\left[x(1-x) - \left[\frac{(1-x)-x}{\left(\frac{-1}{2\sigma_x^2}\right)}\right] + \frac{(-1-1)}{\left(\frac{-1}{2\sigma_x^2}\right)^2}\right] \tag{66}$$

$$\mathrm{E}\left(\frac{x}{1+x}\right) = -e^{-\frac{x}{2\sigma_x^2}}\left[x(1-x) + 2\sigma_x^2(1-2x) - 8\sigma_x^4\right] \tag{67}$$

Substituting, $$|K_0|^2 = \left(-\frac{e^{-\frac{x}{2\sigma_x^2}}}{2}\left\{\left(\frac{-3}{2}x^2 - 2x + 2\right) - \left(2\sigma_x^2(2+3x)\right) - \left(12\sigma_x^4\right)\right\}\right)^2 \tag{68}$$

Variance of the nonlinear distortion noise is given by,

$$\sigma_d^2 = -e^{-\frac{x}{2\sigma_x^2}}\left[x(1-x) + 2\sigma_x^2(1-2x) - 8\sigma_x^4\right] + |K_0|^2\, e^{-\frac{\rho^2}{2\sigma_x^2}}\left(\rho^2 + 2\sigma_x^2\right) \tag{69}$$

IV. RESULTS AND DISCUSSION

In this section, The Bit Error Rate (BER) performance of FBMC/OQAM system with nonlinear HPA in frequency selective Rayleigh channel is investigated by varying constellation size $\mathrm{M} = 4,16,64$ in OQAM modulation, number of sub-channels $\mathrm{N} = 64,128,256$ and Input Back-Off $\mathrm{IBO} = 6,8\,\mathrm{dB}$ for frequency selective Rayleigh channel through Monte Carlo simulations. This analysis is also compared with the analytical results. The parameters considered are Knee factor for SSPA is kept as $\beta = 1$, output saturation voltage $A_{os} = 1$.

64 subcarriers is considered for OFDM and FBMC simulations are shown in Figure 2. This figure shows that the side lobe amplitude for FBMC system is considerably lower than that of OFDM system. Figures 3, 4 and 5 show that BER is plotted as a function of various values of SNR for TWTA, SSPA and SEL models. It is shown that for all the models, by increasing the number of symbols in OQAM modulation, the bit error rate is increased. It is also shown that the bit error rate is minimum for SEL model when compared with the other models is shown in figure 6. It is due to the fact that the SEL Model provides better linear amplitude characteristics. Figure 7 shows that if the number of subcarriers increases, bit error rate is also increases due to reduced power per bandwidth and thereby causing estimation errors.

V. CONCLUSIONS

Analytical BER expression is derived and analysed for SSPA and SEL models in FBMC systems using OQAM. Theoretical and simulated BER performance of FBMC/OQAM system with nonlinear HPA in frequency selective Rayleigh channel is simulated by varying constellation size in OQAM modulation, number of sub-channels and Input Back-Off. BER performance is observed to be superior for SEL model, as it has more linear amplitude characteristics than other models. Increasing knee factor value in SSPA reduces the nonlinear distortion and thereby decreases BER. Side lobe amplitude for FBMC system is considerably lower than that of OFDM system. With increasing number of symbols in OQAM modulation, BER increases due to reduced distance between constellation points. As the number of subcarriers increases, BER increases due to reduced power per bandwidth, causing estimation errors.

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